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VARIATION OF PHYSICAL CONSTANTS, REDSHIFT AND THE ARROW OF TIME

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Theories of fundamental physics as well as cosmology must ultimately not only account for the structure and evolution of the universe and the physics of fundamental interactions, but also lead to an understanding of why this particular universe follows the physics that it does. Such theories must ultimately lead to an understanding of the values of the fundamental constants themselves. However, all such efforts have failed, leaving fundamental constants outside of any physical theories. In this paper we take a different approach than the usual evolutionary picture where the physics itself is assumed invariant. We study numerical relations among fundamental constants starting from relationships first proposed by Weinberg. We have shown that they turn out to be equivalent to the relations found by Dirac. Then a new scaling hypothesis relating the speed of light c and the scale of the universe R is explored. The "coincidences" of Dirac and Eddington concerning large numbers and ratios of fundamental constants do not need to be explained in our view, rather they are accepted as premises and in the process, they yield a fundamentally different view of the cosmos. We develop an axiomatic approach and the fundamental constants can be assumed to vary and this variation leads to an apparent expansion of the universe. Also the variation of constants leads to change in the parameters like permittivity and refractive index of the quantum vacuum. This gives rise to a possibility of explaining some of anomalies found in the observations of high redshift quasars. The variations of the fundamental constants lead to a changing universe, *i.e.*, the number of nucleons varies, *etc.* The increase of the number of nucleons and the redshift of the spectral lines appear to be related to the emergence of an arrow of time as perceived by an observer in the present universe. Possible implications of this new approach in astrophysical domains are discussed.

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1. Introduction

At least several decades ago, starting with Milne [1, 2] and Dirac [3], Edidington [4], Weinberg [5] questions have been arising from time to time whether the Newtonian gravitational constant, G, is varying in cosmological time. Thus cosmological consequences of allowing some of these constants of nature to change, have been studied [6] to evaluate the effects of time-evolution of 'constants' in generalizing the framework of the general theory of relativity with the purpose of allowing them to become space-time variables. Through the Scalar–Tensor theory of gravity proposed by Brans–Dicke [7], the variation of the gravitational 'constant' G has been studied extensively. Bounds of a possible variation of the fundamental physical constants at the epoch of primordial nucleosynthesis are determined by Ivanchik *et al.* [8].

Currently, following Bekenstein [9], Sandvik, Barrow and Magueijo [10–12] have also developed a theory which describes the space-time variation of the fine structure constant. This provides a framework for the rigorous study of simultaneous variations of their three dimensional counterparts [13–17]. New observational limits have been stimulated by high-quality astronomical data [17–19]. Marciano, Barrow and Damour [14–16] have shown in their three dimensional subspace theory that "constants" will vary at the same rate as any change which is occurring at the scale lengths of the extra compact dimensions.

Damour *et al.* [15] showed that cosmological variation of α may proceed at different rates at different points in space-time. Various functional forms for time variations of α/G have been derived using the Kaluza–Klein theory and the assumption of constant masses. Marciano discussed the self-consistency relations required if there are simultaneous variations of different constants in unified gauge theories and examined any possible non-monotonic variation in α with t, using a running coupling dependence of strong, weak and electromagnetic interactions to produce self-consistent predictions for the simultaneous variation of more than one coupling or mass ratio. This has been discussed in detail by Drinkwater *et al.* [17], where the variations of G and α could be linked by relations of the form $\dot{\alpha}/\alpha^2 \sim \dot{G}/G$. Considering high energy physics, featuring additional dimensions of space and new dilation fields, they have provided motivations for studying variations in the gravitational, strong and electroweak coupling constants, [20–22].

Theories unifying gravity and other kinds of interactions, such as string theory and M theory where, the existence of the additional compact dimensions of space have been considered [23, 24], suggest the possibility of spatial and temporal variation of physical "constants" in the Universe. The currently popular scenarios for M theory [25–27] suggest that the gravitational force needs to be assumed to act in all (> 3) spatial dimensions (the 'bulk') whilst all other interactions act only in three-dimensional space (the 'brane'). Thus observations of the constancy of three-dimensional non-gravitational constants in 3-dimensions(like α) could therefore be of *crucial importance* in testing these theoretical scenarios.

A few interesting theories have recently been proposed, namely, a kind of *fine tuning* has been attempted to be established between the variation in the fine structure constant α and the possible change of the light propagation speed [28–31]. The so called minimal Varying Speed of Light(VSL) theories offer possible explanation for the different cosmological problems: the horizon, flatness, cosmological constant, entropy and, homogeneity problems. They developed successfully, based on the bimetric gravity theory, the vector field mediated models of dynamical light [31], where they applied minimally a coupled Einstein Klein–Gordon equation in order to solve some of the cosmological problems, for example, horizon and flatness problem, generation of the seeds of galaxy formation [29] and also the recently established dimming problem of supernovae [30,32]. In their approach, the speed of light is considered as dynamical in nature, in the limit of large $c(t)$, the radiation density and entropy in the very early universe becomes significantly diluted, resulting in an Einstein Klein–Gordon wave equation and a phase transition period. This ultimately controls the behavior of the perturbative wave modes and finally the possible explanation for the horizon and flatness problems are given.

Another group [33–35], using a more or less same basic approach, used exact theories incorporating time variations in α and G and showed how the presence of negative spatial curvature and a positive cosmological constant might play an essential role in bringing to an end variations in the scalar fields that drive the time changes, also emphasizing the existence of a set of duality transformations between these two representations. To complete with, we mention the earlier work by Levin and Freese [36] which discussed the inflationary-type cosmologies resulting from a dynamic Planck's constant. However, in our approach, we shall consider the variation of speed of light in FLRW metric itself and, therefore, no observable consequences are found, in contrast to VSL cosmologies.

The recent observations of astrophysical events at high redshifts [37,38] can be used to place severe limits on the variation of the speed of light itself ($\Delta c/c$), as well as on the photon mass (m_γ). Schaefer [37] presented new limits on $\Delta c/c < 6.3 \times 10^{-21}$ and the lowest limit on $m_\gamma \sim 4.2 \times 10^{-44}$ from explosive events at high redshifts. Lehnert and Roy [39] discussed, from the point of the possible effect of fluctuation of permittivity and permeability in vacuum, that, photons may be gaining mass, in the case that photons have non-zero masses. Again, the variation of physical constants lead to fluctuations of the refractive index with time, the possible consequences of which *i.e.*, the particle production by time varying refractive index, have been studied in detail by many groups [79–81] and is now a well known effect in cosmology. Recently, Ranada [40] proposed that due to variation of physical constants, there will be change of permittivity and permeability of the quantum vacuum, the effect of which will lead to the change of refractive index of the vacuum. In that case, there should be an additive effect on the rest mass of photon as well as it can give rise to the shift of the frequency of the photon propagating through this kind of vacuum.

In this paper, in Section 2, we mention some important recent experimental observations as well as theoretical developments regarding the variation of the physical constants, occurring in a single way or simultaneously. In Section 3, we discuss the deviation of numerical relations and the concept of scaling. Finally, the possible implications for astrophysical observations and cosmology are dealt in Section 4 and discussions are in Section 5.

2. Fine tuning as implied by experimental and cosmological observations

There are a number of observations which must be applied in any cosmological theory that attempts to explain the observed structure of the universe. Since we presently have no understanding of why the constants of Nature assume the values they do in our universe, whether they are logically independent, or, even whether they are truly constant, it is difficult to realize whether only one fundamental constant, one at a time, is varying, or all of them do simultaneously vary *i.e.*, if there is a real sense of fine tuning in the amount of variation these constants follow.

2.1. Experimental observations

Generally, the direct laboratory measurements provide interesting constraints on time-varying α [41], where $\alpha = e^2/\hbar c$. By comparing the rates of two clocks associated with different atoms (H-maser and Hg^+) over a 140-d period, measurements constrained $3.7\times10^{-14}\mathrm{yr}^{-1} \leq |\dot{\alpha}/\alpha| \leq 1.4\times10^{-14}$). These limits are significantly weaker than those derived from geophysics and astrophysics because of the billions of years of look back time over which the latter two fields can gather data.

An analysis of the observed anomalous abundance of Sm^{149} at OKLO-phenomenon — a natural nuclear fission reactor that operated at Gabon, West Africa, ~ 1.8 billion years ago, also points to this limit of variation of α with time. Shlyakhter [42], following the nuclear resonance level in the Sm^{150} isotope, put on an upper bound on $|\Delta\alpha/\alpha|$. Though his estimate of

$|\Delta\alpha_{\rm s}/\alpha_{\rm s}| \leq 5 \times 10^{-10}{\rm y}^{-1}$, or $|\dot{\alpha}/\alpha| \leq 2.5 \times 10^{-19}{\rm y}^{-1}$ gave substantial improvement of the upper bound, this has received a limited acceptance, may be, due to the reason that the derivation steps are much less direct than in estimates based on quasar spectra or clock standards.

Damour and Dyson [43] analyzed the variability issue using a different approach and found more stringent bounds. They concluded that the relative change of α from then to now is in an interval, given by $-0.9 \times 10^{-7} < (\alpha^{\rm Oklo} - \alpha^{\rm now})/\alpha < 1.2 \times 10^{-7}$ and $-6.7 \times 10^{-17}{\rm yr}^{-1} < \dot{\alpha}/\alpha < 5.0 \times 10^{-17}{\rm yr}^{-1}$, obtained from the constancy of the K^{40} decay rate [44], comparable to the limit derived from Big Bang Nucleosynthesis (BBN): $|\beta^{\rm BBN} - \beta^{\rm now}/\beta < 0.06|$. More recently, Fujii *et al.* [45] obtained somewhat tighter constrains taking new samples from the Oklo reactor: $\Delta\alpha/\alpha = (-0.04 \pm 0.15) \times 10^{-7}$. However, the Oklo limit corresponds to variations at very low "redshift", $z \sim 0.1$, *i.e.*, in local or in a non-cosmological environment. There are several other studies which set bounds on the variation of $|\Delta\alpha|$, using a number of different data [46].

Experimentally, quasar (QSO) absorption lines, and particularly the detection of high-redshift absorption systems which are intersecting the lines of sight towards distant quasars provide an ideal and powerful tool in a cosmological setting where one can search for possible temporal or even spatial variations in the assumed fundamental constants of Nature.

Savedoff [47] first analyzed doublet separations seen in galaxy emission spectra to obtain constraints on the variation of the most observationally sensitive constant α. Various propositions and ideas, since 1930, together with the first constraints from spectroscopy of QSO absorption systems, starting from the 1960s, are discussed in detail by Varshalovich and Potekhin [48]. Tight constraints on $\Delta\alpha/\alpha$ come from optical absorption-line studies. Drinkwater *et al.* [17] and Carilli *et al.* [49] considered the bounds that can be placed on the variation of the fine structure constant and proton g factor from radio observations of atomic and molecular transitions in high-redshift quasars which have further been constrained to smaller values for α at higher redshifts.

Observations of Webb *et al.* [50,51] confirmed these results with improved techniques and extended previous results to a higher-redshift sample of damped Lyman-α systems. They studied relativistic transitions to different ground states using absorption line QSO spectra by exploiting the extra sensitivity of many-multiplet technique. The trend of all these results appears to be that the value of α was *lower in the past*, with $\Delta\alpha/\alpha = -0.72 \pm 0.18 \times 10^{-5}$ over $z \approx 0.5 - 3.5$ (spanning $\sim 23\%$ to 87% of the age of the universe).

The most precise constraint, obtained by Murphy *et al.* [52], is $\Delta\alpha/\alpha = (-0.5 \pm 1.3) \times 10^{-5}$, by analyzing 21 SiIV doublets ($2 < z < 3$) in 13 QSO spectra which thus provide strong evidence that the fine structure constant might be changing with cosmological time [50,51,53,54]. They also considered the implications of simultaneous variations of several "constants" and showed how these observational limits can be used to constrain a class of inflationary universe theories in which small fluctuations in the fine-structure constant are also predicted to occur.

Other investigations [55–57] have claimed preferred nonzero values of $\Delta\alpha/\alpha < 0$ to best fit the Cosmic Microwave Background (CMB) and Big Bang Nucleosynthesis (BBN) data at $z \approx 10^3$ and $z \approx 10^{10}$, respectively, but result in much larger variations. Another group [58] studied this problem of possible variation of the fundamental physical constants at the epoch of quasar spectra formation (*i.e.*, $\sim$ 10 billion years ago). They calculated the upper limits of this variation on the basis of the analysis of absorption spectra of Quasars with high redshifts and also applied a number of systemic effects, which can simulate variation of the constants.

2.2. Cosmological observations

We now turn our attention to several cosmological observations and discuss some of their implications on the nature of the universe.

(a) The universe appears to be quite flat, in other words the density of the universe is very close to the so-called closure or critical density,

$$\rho_{\rm crit} = 2 \times 10^{-29} \left(\frac{H_0}{100\ {\rm kms^{-1}Mpc^{-1}}} \right)^2 {\rm gr\ cm^{-3}}\,, \tag{1}$$

where H_0 is the Hubble constant defined as the apparent rate of expansion with distance, $\dot{R}/R$ and R being the scale of the universe. The observed density is not really equal to the closure density when one observes regular, luminous matter. In big bang cosmology, the so-called "Hubble constant" is actually a function of cosmic time, *i.e.*, it is a variable. Its present-day value seems to be $\sim$ 75 $\rm kms^{-1}Mpc^{-1}$. The universe appears to be close (but still off by factor of $\sim$ 10–100 from the closure limit, at present) to a flat, Euclidean, Einstein–de Sitter state as indicated from (1), and yet it is still not clear what the geometry of the universe is, *i.e.*, whether exactly flat (which would be required by the inflationary scenario); open (yielding a forever-expanding, negatively curved space-time); or closed (yielding a maximum expansion and a positively curved space-time).

(b) If one is to assume that the universe followed an inflationary period in the distant past, then the universe must have been exactly flat to one part in 10^{50} near the time of Big Bang. This is so-called *flatness* problem: This is such a remarkable requirement that the usual interpretation proposed in the early 80's was that — early on, the universe was in an inflationary state, washing out any departures from flatness on time scales of 10^{-35} sec. The inflationary model proposed by Guth [59] and others has been developed in various forms to account for the flatness of the universe and also is proposed to solve the horizon problem, or apparent homogeneity of the 2.73 K black body radiation seen by COBE [60]. The latter problem involves the observation that although the 2.73 K radiation was emitted $\sim 10^5$ years after the beginning, opposite sides of the sky at that time were out of causal contact, separated by $\sim 10^7$ light years. Other structures involving large-scale correlations in the universe exist such as very large structures in the distribution of matter [61]. These structures may be progressively hierarchical all the way to the scale of the universe itself.

(c) If the universe is indeed flat, observations indicate that baryons (or luminous matter) can only contribute at most ~ 0.05 of the closure density at present. We should ultimately be able to detect the other 90% or more of the matter required to give closure density, presumed to be in the form of cold dark matter [62]. Nevertheless, attempts to detect such exotic matter in the laboratory have, so far, failed. Moreover, the recent realization that the cosmological constant Λ may have to be re-introduced [63], to account for the possibility of an accelerating universe, has also led to the probability of Λ itself varying and other similar notions [64]. Barrow & Magueijo [65] developed a particular theory for varying c (or α) in which the stress contributed by the cosmological constant varies through the combination Λc^2. They also showed how the observed non-zero cosmological acceleration [32, 66] might be linked to a varying α. According to them, the case of varying c theories is based on the fact that the effect is driven by a scalar field, coupled to the gravitational effect of pressure. The very slow variation of the scalar field makes possible for slow variation of c which at the radiation era converted the Λ energy density into radiation, thus preventing Λ dominance; but at the pressureless matter era the situation reversed.

This kind of theory allows variations of c or α to be $\sim 10^{-5} H_0$ at $z \sim 1$ and yet the associated Λ term can be dominant today and produce the much needed acceleration. Inflationary universe models provide a possible theoretical explanation for proximity to flatness but no explanation for the smallness of the cosmological constant itself. Nevertheless, without some direct laboratory verification or overwhelming requirements imposed by particle theory (neither of which presently exists), the nature of dark matter remains elusive. This is clearly a very unsatisfying situation.

(d) As we saw, present-day approximate flatness yields to an exact flatness in the distant past (this was one of the main reasons why the inflationary scenario was introduced to begin with). The alternative is to accept *fine tuning* in the universe. In fact, the flatness of the universe is not the only fine tuning. In considering other fundamental observed facts, the universe appears to be extremely fined tuned. It was Eddington [4, 67] and Dirac [3] who noticed that certain cosmic "coincidences" occur in nature linking microscopic with macroscopic quantities [68]. A most unusual relationship is the ratio of the electric force to gravitational force (this ratio is presumably a constant in an expanding universe where the physics remains constant), or

$$\frac{e^2}{Gm_e m_p} \sim 10^{40}\,, \tag{2}$$

while the ratio of the observable size of the universe to the size of an elementary particle is

$$\frac{R}{\left(\frac{e^2}{m_e c^2}\right)} \sim 10^{40}\,. \tag{3}$$

Here, in this relationship, the numerator is changing as the universe expands because the scale of the universe R is constantly changing in an expanding universe.

Dirac formulated the so-called *Large Number Hypothesis* which simply states that the two ratios in (2) and (3) are in fact equal for all practical purposes and postulates that this is not a mere coincidence. Various attempts were made to account for the apparent equality: a possibility that constants such as the gravitational constant G may be varying was proposed by Dirac [3] himself and others [44]. Other ratios such as the ratio of the size associated to an elementary particle, like the electron, to the Planck length,

$$\frac{\left(\frac{e^2}{m_e c^2}\right)}{\left(\frac{\hbar G}{c^3}\right)^{1/2}} \sim 10^{20} \tag{4}$$

can also be constructed [69] yielding to the conclusion that fine tuning is prevalent in our universe. These relationships may be indicating the existence of some deep, underlying harmonies involving the fundamental constants and linking the microcosm to the macrocosm. Physical theory has not, however, accounted for these in a self-consistent way, waiting perhaps for the anticipated unification of all physical forces at the quantum gravity or superstring levels.

(e) Other, less traditional ways, such as the Anthropic Principle emerged from attempts by Whitrow [70] and Barrow *et al.* [71] to understand why it is not surprising that we find space to have three dimensions, and by Dicke [72, 73] to understand the inevitability of Dirac's "Large number" coincidences in cosmology for the above fine tuning properties of the universe which provides some novel anthropic perspectives on the evolution of our universe. There have been many investigations of the apparent, might be termed "finely tuned", coincidences that allow complexity to exist in the universe [74–76].

Recently, a phenomenological and Newtonian model has been proposed by Ranada [40] to explain the recently observed cosmological variations of the fine structure constant as an effect of the quantum vacuum. He assumes a flat universe with cosmological constant Λ in the cases $(\Omega_M, \Omega_\Lambda)$ equal to $(0.3, 0.7)$ and $(1, 0)$ [32], respectively. This model predicts that $\Delta\alpha/\alpha$ is proportional to

$$\left(\Omega_M\left[R(t)^{-1}\right] - 2\Omega_\Lambda\left[R(t)^2 - 1\right]\right) ,$$

$R(t)$ being the scale factor, and shows some kind of agreement with the observations [51]; however, limitations at the present state of development of his theory remain.

3. Numerical relations and concept of scaling

The critical density of the universe in (1) is defined as

$$\rho_{\text{crit}} = \frac{3H_0^2}{8\pi G} \,. \tag{5}$$

Let N_p be the number of nucleons in the universe, then writing the mass of a particle in terms of cosmological quantities, we have

$$m_p \;=\; \frac{M}{N_p} = \frac{R\dot{R}^2}{2GN_p} \,, \tag{6}$$

where m_p and M are the mass of the nucleon and mass of the universe, respectively.

Weinberg [1], on the other hand, noticed that one can find a relationship linking the masses of elementary particles, such as pions, to the Hubble constant and other fundamental constants; for example

$$m_\pi \sim \left(\frac{8\hbar^2 H_0}{Gc}\right)^{1/3} \qquad \text{and} \qquad m_e \sim \left(\frac{\hbar e^2 H_0}{(8\pi)^3 G c^2}\right)^{1/3},$$

where m_π and m_e are the pion and electron masses, respectively. These relations can be rewritten as

$$m_p \sim \chi_{p\pi}\left(\frac{8\hbar^2\left(\frac{\dot{R}}{R}\right)}{Gc}\right)^{1/3} \qquad \text{with} \quad \chi_{p\pi} = \frac{m_p}{m_\pi}, \tag{7}$$

$$m_p \sim \chi_{pe}\left(\frac{\hbar e^2\left(\frac{\dot{R}}{R}\right)}{Gc^2(8\pi)^3}\right)^{1/3} \qquad \text{with} \quad \chi_{pe} = \frac{m_p}{m_e}. \tag{8}$$

From equation (6) and the above relations one can easily get

$$G^2\hbar^2 c^{-1} \sim \chi_{p\pi}^{-3} N_p^{-3} \frac{R^4 \dot{R}^5}{64} \tag{9}$$

and

$$m_p = \chi_{p*}\sqrt{\frac{\hbar c}{G}} \tag{10}$$

for $\chi_{p*} = \frac{m_p}{m_*}$, and m_* being the Planck mass. Suffix $*$ indicates in general Planck quantities. Combining (10) and (6), yields

$$cG\hbar \sim \frac{1}{4} N_p^{-2} \chi_{p*}^{-2} R^2 \dot{R}^4. \tag{11}$$

Similarly from (9) and (10), we can have

$$c \sim 2^{2/3} N_p^{-1/3}\ \chi_{p*}^{-4/3} \chi_{p\pi} \dot{R}. \tag{12}$$

The multiplying factor for $\dot{R}$ in (12) is of the order of unity, or

$$2^{2/3} N_p^{-1/3} \chi_{p*}^{-4/3} \chi_{p\pi} \sim 1.$$

Conversely, if we choose to set the required condition $2^{\frac{2}{3}} N_p^{-\frac{1}{3}} \chi_{p*}^{-\frac{4}{3}} \chi_{p\pi} = 1$, one gets the simple relationship linking the speed of light to $\dot{R}$, *i.e.*, $c = \dot{R}$ with $N_p \sim 3.7 \times 10^{79}$, which is a good estimate of the number of particles in the current universe. The relationship $c = \dot{R}$ could be interpreted as the Hubble Law $\dot{R} \sim c$, although we emphasize that this is just a relationship and might not imply that an expansion is indeed taking place. We can arrive at the similar conclusion if one works with the above relations using electrons. Now, if we start by assuming a heuristic relation

$$c \equiv \dot{R}\,,$$

i.e., the speed of light is identical to the rate of change of the scale of the universe, we can construct an axiomatic approach equivalent to the Hubble Law. This axiomatic approach can be considered as an alternative approach to the mysterious coincidences of Eddington and Dirac which Weinberg called "so far unexplained ... a real, though mysterious significance."

It can be further shown that all lengths, such as the Planck length, l_*, the classical electron radius, r_e, *etc.*, are all proportional to the scale of the universe, *i.e.*,

$$l_*, r_e, \sim \ (\cdots)R\,. \tag{13}$$

For example,

$$l_* \ \sim (2^{-\frac{7}{3}} N^{-\frac{1}{3}} \chi_{p*}^{\frac{5}{3}} \kappa_{p\pi}^{-2})R\,.$$

Similar relations can be formed for r_e and r_p where r_e and r_p are the electron and proton radii, respectively. From (11) and (13) we obtain

$$G\hbar = \frac{R^2\dot{R}^3}{4} N_p^{-2}\chi_{p*}^{-2} \ \sim \ 3.4 \times 10^{-122} R^2 \dot{R}^3 \tag{14}$$

a relationship linking the gravitational and Planck's constant to R and $\dot{R}$ and where the last relationship (14) holds for the current values of $N_p^{-2}\chi_{p*}^{-2}$. Let us now set the following initial conditions, *i.e.*, $R \rightarrow l_*$ and $\dot{R} \rightarrow \frac{l_*}{t_*}$. Here l_* and t_* are the Planck length and Planck time, respectively. Then

$$\begin{aligned} &N_p^{-2}\chi_{p*}^{-2}/4 \ \rightarrow 1 && \text{at those initial conditions,} \\ &N_p^{-2}\chi_{p*}^{-2}/4 \ \sim 3.4 \times 10^{-122} && \text{for the present universe.} \end{aligned}$$

The limit $N_p \rightarrow 1$ indicates that in our model *in the beginning* there was only one bubble-like object or a *cosmic egg* [76]. Moreover, $R \rightarrow l_*$ and $N_p \rightarrow 1$ imply that $\chi_{p*} \rightarrow 1$ as well (similarly for all ratios of masses χ's), which in turn indicates that the masses of all particles were equal to each other at these initial conditions.

In the beginning

$$\frac{R}{\frac{e^2}{m_e c^2}} \sim \frac{\frac{e^2}{m_e c^2}}{\frac{G}{c^3}} \sim 1$$

rather than the large values of 10^{40} and 10^{20} which these ratios are equal to, respectively, today and also, all lengths were equal, all masses were equal and there was only one particle or *cosmic egg*. Today, these ratios are not unity, as there is a very large number of particles in the universe and R is equal to $\sim 10^{28}$ cm. However, scale-invariant relationships such as $c \equiv \dot{R}$, all lengths are proportional to each other, *etc.* still hold.

Israelit and Rosen [77] proposed a cosmological model where the universe emerges from a small bubble (*cosmic egg*) at the bounce point of a de Sitter model filled with a cosmic substrate (*prematter*). In other words, $c \equiv \dot{R}$, at the *initial time* when $N_p \to 1$ and all $\chi \to 1$, and this relationship remains invariant even at the present universe (*cf.* equations (12) and (13)). The self-consistency is obtained by calculations for the value of N_p from (12) and (14). This relation is a type of a scaling law and connects the microcosm to the macrocosm. Now, if irrespective of the presence or absence of expansion of the universe, R itself is changing from the Planck scale to the size of the observable universe, then the fundamental constants like $G, \hbar$ and c are changing simultaneously.

Note, however, that we cannot deduce the actual variation or the initial value of c and other constants from observations: The relationship $c \equiv \dot{R}$ is not enough to tell us the actual variation or even over *how long* it takes place. It is a scale invariant relationship. If we re-write it as a scale-invariant relationship,

$$\frac{c(t_*)}{c(t_0)} = \frac{\dot{R}(t_*)}{\dot{R}(t_0)},$$

where t_* and t_0 could be conveniently taken as the Planck time and the present *age* of the universe, then this relationship is not enough to give us the evolution of $\dot{R}$ or even the values of t_* and t_0. It should be mentioned that though the condition $c = \dot{R}$ does not necessarily imply $c = c(t)$, they are not contradictory to each other.

Hence it cannot tell us how c itself is varying or even if it is varying. If we wanted to insist that c is *constant*, then all the other "constants" like G and $\hbar$ *are really constant as well.* But if c is not constant, then all the other "constants" are varying as well. In both cases, however, the number of particles is changing, the ratios of masses are changing and the ratios of scales or lengths are also changing. An arrow of time *could*, therefore, be introduced. In this picture, invariant relationships hold and from unity, there is an evolution into diversity. One cannot, though, conclude how these variations are taking place, over what timescales they are taking place or even how old the universe is. The universe could be 10^{10} years old or 5×10^{-44} sec (the Planck time) old, or any time in between. *Time is strictly* *a parameter* that can be introduced in the scale-invariant relationships. It has no meaning by itself. The universe appears to be evolving as the number of particles and ratios are varying.

4. Implications

Here, we shall consider the possible implications of considering variations of the physical constants like speed of the light, fine structure constant *etc.* in the domain of cosmology and astrophysics.

4.1. Cosmological

As mentioned earlier, Variable Speed of Light (VSL) [28–31, 33, 34, 38] cosmologies have generated lot of interest among the community for the last one decade or so which appears amenable to observational tests. The geometric consistency of VSL with that of Einstein theory of general relativity has been maintained in the following way. Within Einstein's framework the constancy of speed of light is built into it in a very fundamental level. One way of doing this is to start with FLRW metric

$$ds^2 = -c^2 dt^2 + a(t)^2 h_{ij} dx^i dx^j \,.$$

Now, in natural or orthonormal basis, the Einstein tensor can be written as

$$\begin{aligned} G_{\hat{t}\hat{t}} &= \frac{3}{a(t)^2}\left[\frac{\dot{a}(t)^2}{c^2} + K\right], \\ G_{\hat{i}\hat{j}} &= -\frac{\delta_{\hat{i}\hat{j}}}{a(t)^2}\left[2\frac{a(t)\ddot{a}(t)}{c^2} + \frac{\dot{a}(t)^2}{c^2} + K\right] \end{aligned}$$

with the spatial curvature $K = 0, \pm 1$. Then, replacing $c \to c(t)$ in that metric, it can be shown that the physics in it does not change as the concept of variable speed of light remains unchanged even after the application of coordinate transformation , *i.e.*, $c d\tau \simeq c(t) dt$, τ being a new time coordinate. This kind of coordinate change will affect the components of the metric and coordinate components of Einstein tensors but not any physical observable (*i.e.*, coordinate invariant) or the orthonormal components [78]. However, the observable consequences have also been considered in detail by several authors [79–81]. They replaced $c \to c(t)$ directly in the Einstein tensor. In such cases the modified Einstein tensor is not covariantly conserved and cannot be obtained from the curvature tensor of any space-time metric. In fact, this violation is shown to be the source of solutions of the flatness problem in VSL cosmologies. But, in our framework, as we are interested primarily in scaling, we consider the first approach of implementing variable speed of light in Einstein's general theory of relativity. At this point, instead of considering the process of matter production in an expanding universe as such, we are emphasizing the existence of scaling relationships at all levels which means that the scaling prevails at all levels starting from microcosm to macrocosm due to the variability of these constants only, while different constant changes might exist at different level, following different epoch (for example, N, c *etc.*). In fact we treat N as one of the constants. It itself is changing and this is all related to the appearance of time evolution. At present, we replace $c \to c(t)$ in the space-time metric and no observable changes are observed. Again as all the fundamental constants are changing simultaneously, our model gives identical results with evolving universe model. However, the discussion about the origin of Cosmic Background Radiation (CMBR) and the production of light elements are left for future developments.

4.2. Astrophysical implications

The variation of physical constants may change the permittivity and permeability of the underlying quantum vacuum which play significant role in the astronomical domain. According to the phenomenological Newtonian model, presented by Ranada [40], the cosmological variation of the fine structure constant is considered to be due to the combined effect of the fourth Heisenberg relation and the gravitational interaction of the virtual pairs in the zero-point radiation with all the universe. More precisely, it is argued that, because of the fourth Heisenberg relation, the density of the sea of virtual particles in the quantum vacuum must change in a gravitational field, with a corresponding variation of permittivity and permeability that depends on the average gravitational potential of the universe (ϕ). Here, the quantum vacuum is treated as a transparent optical medium characterized by its permittivity. As a result, the contribution due to change of α in the frequency shifts of the spectral lines is

$$\frac{\triangle\omega}{\omega} = 4\beta' \frac{\triangle\phi}{c^2}\,,$$

β' being certain coefficient. Though they might appear as similar, this is different from the redshift expected due to gravitational redshift which is

$$\frac{\triangle\omega}{\omega} = \frac{\triangle\phi}{c^2}\,,$$

ϕ being the Newtonian gravitational potential. Hence the total redshift, expected to be observed, is due to these two combined effects and is given by,

$$\frac{\triangle\omega}{\omega} = (1+4\beta')\frac{\triangle\phi}{c^2}\,. \tag{15}$$

However, Ranada pointed out a necessary condition for the compatibility of his results and that of gravitational redshift experiments as $\xi \neq 4\times 10^{-3}$, ξ being a a parameter related to renormalization effects of the quantum vacuum. It is interesting to note that according to this model, light is also effected by the gravitational potential ϕ so that it was slower in the past and the optical density of the quantum vacuum increases towards the past and decreases as the universe ages. However, he had to put a boundary on the value of β and consequently on ξ in order to make his results compatible with that of gravitational redshift experiments *i.e.*, $\xi \sim 1.3\times 10^{-5}$ for his model and 1.9×10^{-5} for taking the two cases togather. It is interesting to note that the best confirmation of the gravitational redshift, those of Pound, Rebeka and Snider agree with the prediction of General Relativity up to about 1%, but they also *refer* to nuclear levels in which the electromagnetism plays a part.

We think that this incompatibility is due to the fact that he did not consider the added effect due to non-zero photon mass. One of the main difficulties with all these approaches is that no change in width of the spectral lines have been observed. This is contrary to the present astronomical findings, especially, in quasar astronomy where one can get many broaddened lines as the redshift becomes higher.

Lehnert and Roy [39] showed that if the light is propagated through Maxwell vacuum with different permittivity, permability and refractive index than the ususal vacuum, then the photon will loose its energy and there will be a shifting in the spectral lines as well as the photon gaining mass. From the estimated values of permittivity and permeability one can estimate the lower bound of non-zero rest mass of the photon [82,83] whose presence could be manifested in laboratory experiments. It is also evident now, from the above discussions, that the redshift due to these three effects is an indication of arrow of time to an observer within the universe.

According to Lehnert and Roy, Maxwell's equations in vacuum are modified by assigning a small nonzero conductivity (σ). As a first step, we can extend the modified Maxwell's equations, assigning a nonzero space-charge *in vacuo* to it. Then, if a nonzero conductivity coefficient is assigned to this Maxwell vacuum instead of space-charge then the photon looses its energy when it propagates through such a vacuum. At first, let us consider the Maxwell equations with $\sigma \neq 0$, *i.e.*,

$$\begin{aligned} \mathrm{div}\boldsymbol{E} &= 0\,, \qquad \mathrm{curl}\boldsymbol{H} = \sigma\boldsymbol{E} + \epsilon_0\chi_e\frac{\partial \boldsymbol{E}}{\partial t}\,, & (16)\\ \mathrm{div}\boldsymbol{H} &= 0\,, \qquad \mathrm{curl}\boldsymbol{E} = \mu_0\chi_m\frac{\partial \boldsymbol{H}}{\partial t}\,, & (17) \end{aligned}$$

where,
$\mu_0 =$ the vacuum permeability constant,
$\chi_e =$ the relative dielectric constant,
$\chi_m =$ the relative permeability constant.
Here, the four-current is given by

$$j = (\boldsymbol{j}, j_0) \quad \text{with} \quad \boldsymbol{j} = \sigma\boldsymbol{E}; \quad j_0 = 0\,.$$

Again,

$$\nabla\times\nabla\times\boldsymbol{E} = -\nabla^2\boldsymbol{E} \tag{18}$$

which together with the above Maxwell's equations (16) and (17) gives

$$\nabla^2\boldsymbol{E} = -\frac{\epsilon_0\chi_e\chi_m}{c^2}\mu_0\frac{\partial^2\boldsymbol{E}}{\partial t^2} + \sigma\mu_0\chi_m\frac{\partial\boldsymbol{E}}{\partial t}\,. \tag{19}$$

This equation is not time invariant. The second term on the right-hand side indicates that there will be a dissipation of energy during the propagation of a photon. Considering a plane wave in the z-direction

$$\begin{aligned} E_x &= be^{i\omega(t-z/v)}\,, & (20)\\ H_y &= b\left(\frac{\epsilon_0\chi_e}{\mu_0\chi_m}\right)^{1/2}\exp i\omega\left(t-\frac{z}{v}\right), & (21) \end{aligned}$$

and putting ($q = \frac{1}{v}$), we get

$$q^2 = \frac{\epsilon_0\chi_e\chi_m}{c^2}\left(1 - \frac{i\sigma}{\omega\epsilon\chi_e}\right). \tag{22}$$

The velocity defined by v above will give rise to a complex refractive index in vacuum. The real part of q^2 gives rise to a phase velocity of propagation of the disturbance through the underlying vacuum. Taking the real and imaginary part as α and β, respectively, E_x and H_y can be shown to be proportional to

$$\exp(-\omega\beta z)\exp(t-\alpha z)$$

and the complex quantity q can be written as

$$q = \alpha - i\beta$$

with

$$\alpha^2 = \frac{\chi_e\chi_m}{2c^2}\left[\left(1+\left(\frac{\sigma}{\epsilon_0\chi_e\omega}\right)^2\right)^{1/2}+1\right],$$

$$\beta^2 = \frac{\chi_e\chi_m}{2c^2}\left[\left(1+\left(\frac{\sigma}{\epsilon_0\chi_e\omega}\right)^2\right)^{1/2}-1\right].$$

Then the following situation arises:

(a) Plane waves are progressively damped with the factor $\exp(-kz)$, where $k = \omega\beta$.

(b) The phase velocity of propagation of the wave is $1/\alpha$ and varies with the frequency.

In the limit $\sigma/\omega \longrightarrow 0$, we have

$$\alpha \simeq 1 + \frac{1}{8}\left(\frac{\sigma^2}{\epsilon_0^2\chi_e^2}\frac{1}{\omega^2}\right) + \mathcal{O}(\frac{\sigma^4}{\omega^4})\,; \qquad \beta^2 \simeq \frac{1}{2}\,\frac{\sigma^2}{(\epsilon_0\chi_e)^2}\,\frac{1}{\omega^2}\,. \tag{23}$$

Then the phase velocity v_p and the group velocity v_g of propagation of the disturbance through the underlying vacuum, after some calculations become

$$v_p = \frac{c}{(\chi_e\chi_m)^{\frac{1}{2}}}\left(1-\frac{1}{8}\,\frac{\sigma^2}{(\epsilon_0\chi_e)^2}\,\frac{1}{\omega^2}\right), \tag{24}$$

$$v_g = \frac{c}{(\chi_e\chi_m)^{\frac{1}{2}}}\left[\sqrt{1+\frac{1}{4}\frac{\sigma^2}{(\epsilon_0\chi_e)^2}\,\frac{1}{\omega^2}}\right]. \tag{25}$$

However, in the limiting case, when $\sigma \longrightarrow 0$, we have $v_p = v_g = c$.

Now taking v_g as the velocity of photon and m_γ as the nonzero mass of photon, we have

$$E = h\nu = \frac{m_\gamma c^2}{\sqrt{1-\frac{v_g^2}{c^2}}} \tag{26}$$

and the mass of photon becomes

$$m_\gamma^2 = \frac{h^2\nu^2}{n^2c^4}\left[\left(n^2-1\right)-\frac{\sigma^2}{(\epsilon_0\chi_e)^2}\frac{1}{\omega^2}\right] \quad \text{for} \quad n = \sqrt{\chi_e\chi_m}\,. \tag{27}$$

But this is unphysical. If we instead introduce the phase velocity in the de Broglie relation, we get a physical solution *i.e.*, a real nonzero rest mass of the photon. Finally, we get for $n \sim 1$,

$$m_\gamma \simeq \frac{\sigma h}{\sqrt{2(\epsilon_0\chi_e)}}\,\frac{\pi}{c^2}\,.$$

Let us now consider the variation of permittivity (ϵ) and permeability (μ), following Ranada [39] in addition to the above mentioned effect. Then expressing the relative permittivity and permeability at a space time point with a weak gravitational potential ϕ, we get

$$\epsilon_r = 1 - \beta'(\phi - \phi_{\oplus})/c^2 \,, \qquad \mu_r = 1 - \gamma'(\phi - \phi_{\oplus})/c^2 \,. \tag{28}$$

β' and γ' being certain coefficients, which must be positive since the quantum vacuum is dielectric as well as paramagnetic. $\phi_{\oplus}$ here represents the present gravitational potential of the entire universe at the earth. Taking into consideration all the variations together, we can write the velocity of light, changed to

$$c' \simeq \frac{c}{\sqrt{\epsilon_r \mu_r}} = c\left[1 + (\beta' + \gamma')(\phi - \phi_{\oplus})/2c^2\right] = c^2\,\bar{m_\gamma} \tag{29}$$

and following equation (29), the effective mass of photon changed to

$$\begin{aligned} E &= m_\gamma (c')^2 = m_\gamma c^2 \left[1 + (\beta' + \gamma')\frac{\phi - \phi_{\oplus}}{2c^2}\right], \qquad (30)\\ \bar{m_\gamma} &= m_\gamma \left[1 + (\beta' + \gamma')\frac{\phi - \phi_{\oplus}}{2c^2}\right]. \qquad (31) \end{aligned}$$

Thus, the effective non-zero photon, gaining mass as a result of non-zero conductivity-coefficient in vacuum, calculated by Lehnert and Roy, will be modified further due to the variations of the physical constants, if any. It is interesting to note that in such cases, *i.e.*, if there is variation of physical constants due to the variation of permittivity and permeability, then one should get redshift due to non-zero rest mass of photon too, in addition to the above effects. It is worth to mention that the time varying refractive index is considered as the well known sources for particle creation in VSL cosmologies. These effects share many of the features calculated from inflationary scenarios.

5. Discussion and conclusions

The existence of horizons of knowledge in cosmology, indicate that as a horizon is approached, ambiguity as to a unique view of the universe sets in. It was precisely these circumstances that apply at the quantum level, requiring that complementary constructions be employed [84]. At the *initial time*, which could be conveniently taken as the Planck time, if we set the conditions like $c \equiv \dot{R}$, as proposed in this paper, we can axiomatize the numerical relations connecting the microcosm and the macrocosm. One then has scale-invariant relationships. During the *evolutionary* process of the universe, the fundamental constants are changing or they may be constant. In the former case, we do not know how or even over what timescales they are changing. In the latter case, one gets the usual evolutionary universe. This is a clear case where complementarity applies.

In other words, as N_p is changing from the initial value of 1 (unity) to the present large value of $\sim 10^{80}$ (diversity), more particles are created as R and all length scales as well as all masses are changing. This could be interpreted by an observer as an "expansion of the universe". An observer, who is inside the universe will perceive an "arrow of time" and an "evolving universe". But equivalently, as the "constants" change (they would *all* have to be changing), there appears to be an evolution. As $N_p \rightarrow 10^{80}$, the present number of the nucleons in the universe, the fundamental "constants" achieve their present values.

To recapitulate, the arrow of time can be related to a kind of complementarity between two constructs, *i.e.*, the fundamental "constants" *are truly constant*, on the one hand; and the fundamental "constants" *are changing*, on the other hand.

In summary, we found that by adopting Weinberg's relationship [1], (which can be shown to be equivalent to Dirac's relationships (2) and (3) when the latter are equated to each other), we can obtain a relationship linking the speed of light c to the rate of change of the scale of the universe. In fact, the proportionality factor is ~ 1 if one substitutes for values of fundamental quantities like the present number of particles in the universe, *etc.* The next step assumes that the relationship linking c and R is an identity, *i.e.*, $c \equiv \dot{R}$ for example, at the Planck time, one observes that this relationship still holds if the ratios of all masses $\rightarrow 1$ and the number of particles also $\rightarrow 1$.

As such, it is possible (but not necessary) to state that *all* the fundamental constants are changing and not just one of them as was assumed in past works. It is interesting that, recently, the possibility of the cosmological constant Λ itself changing [32,64] and references therein, has been suggested. As such, what we are suggesting here as a framework for the universe is — a natural extension of previous ideas. Therefore, as N_p changes from an initial value of 1 to the present value of $10^{80}(1 \rightarrow 10^{80})$, the universe would be appearing to be evolving to an observer inside it or an arrow of time would be introduced.

Again due to the variation of physical constants, the structure of quantum vacuum will also be changed as a result of which there will be a redshift as an effect of the changing permittivity and permeability of the vacuum. The evidence of this kind of redshift can be related to the arrow of time which will appear as an arrow of time to an observer within the universe similar to that due to change of number of nucleons. Finally, the outcomes of this prescription are not just that an arrow of time is introduced and the mysterious coincidences of Dirac and Eddington now can be understood as scale-invariant relationships linking the microcosm to the macrocosm; in addition, all scales are linked to each other and what one calls, *e.g.* the *fine structure constant, fundamental length*, *etc.* are purely a convention and interrelated. In the same way, time itself is not as fundamental as the scale-invariant relationships linking the microcosm to the macrocosm.

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