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Gas dynamical approach to study dust acoustic solitary waves

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Dust acoustic nonlinear waves are studied using gas dynamical approach. The structure equation for dust fluid has been obtained using the conservation laws for mass flux and momentum. The role of dust sonic point for the formation of soliton has been discussed. Conditions for the existence of soliton have been derived in terms of collective Mach number, taking into account the dust charge variation.

Solitary waves in plasma have been studied extensively during the last three decades. Ikezi, Taylor, and Baker[1] were the first to discover ion acoustic solitons in plasma. Since then nonlinear waves in plasma have been studied using different methods,[2–7] both perturbative and nonperturbative. Among the perturbative methods the most popular one is the reductive perturbative technique,[8] which uses stretched coordinates. The famous Korteweg–de Vries (KdV) equation was derived in plasma using this method. Among the nonperturbative methods, the most widely used is Sagdeev's pseudopotential method. This is usually applied in a fluid mechanics framework on the assumption that the motion of soliton is analogous to that of a particle moving in a pseudopotential well. A number of people[9–11] have used this method to find out soliton solutions like ion acoustic soliton,[12] dust acoustic solitons,[13] etc. It has been found that Sagdeev's method yields the KdV soliton solution for small amplitude waves. This method has the advantage that it can be applied to arbitrary amplitude waves and is also useful to study large amplitude double layers. Another nonperturbative approach is the gas dynamic description which has been recently developed by Mckenzie[14] and others. It differs from Sagdeev's approach in its viewpoint and uses a reference frame that moves with the nonlinear structure and relies on the construction of various Bernoulli-type constraints of motion. This, in turn, puts constraints on the existence of solitary wave like structures and provides more physical insights that are rather nonexplicit in Sagdeev's approach.

To study the flow, the generalized momentum function and the energy functions for each species are deduced. Here the equilibrium point corresponds to the zero of the momentum function and the charge neutral point gives value of u_d, the dust velocity, where the electric stress vanishes. Sonic points for different species limit the strength of the flow through the phenomenon of choked flow.[14] For the formation of a soliton the charge neutral point should lie between the equilibrium point and the initial point. In the pseudopotential method the criterion for the existence of soliton is derived in terms of the plasma parameters which determine the shape of the Sagdeev potential function. Here also, criterion for the existence of solitons is derived in terms of combined Mach number and density ratios. Mckenzie *et al.*[15] studied compresive and rarefactive ion-acoustic solitons in a bi-ion plasma using this method. They also studied electron-acoustic solitons[16] in complex plasma. Here we study dust acoustic nonlinear waves using the gas dynamic description. Similar study has been made by Mckenzie[17] using the fluid dynamic approach. The main difference between the problem undertaken here and the one studied by Mckenzie is that in the latter case dust charge variation has been ignored. However as has been found recently, dust charge variation plays an important role in determining the parametric conditions for the existence of solitary waves. For the dust charging we have taken electrostatic probe model.[18] Ma and Liu[19] calculated the characteristic time for dust motion equal to 2 ms for micrometer sized dust grains, while the dust charging time is 95 ns. So, on the hydrodynamic time scale, the dust charge can quickly reach local equilibrium and the electron and ion currents balance each other. Simplifying the current balance equation $I_e+I_i\approx 0$ we obtain z_d as a function of plasma potential ϕ. z_{d0} is obtained from the current balance equation. Since we do not use the charging equation we take the ion or electron continuity equation without any source or sink term.

The one-dimensional normalized equations governing the dust dynamics in a dusty plasma consisting of warm dust particles and electrons and ions are

$$\partial n_s/\partial t + (\partial/\partial x)(n_s u_s) = 0, \tag{1}$$

$$m_s n_s[\partial/\partial t + u_s(\partial/\partial x)]u_s = q_s n_s E_x - \partial p_s/\partial x, \tag{2}$$

$$\partial E_x/\partial x = 4\pi(q_e n_e + q_i n_i + q_d n_d), \tag{3}$$

$$I_e + I_i = 0, \tag{4}$$

where

$$I_e = -\pi a_d^2 e(8T_e/\pi m_e)^{1/2} n_e \exp(eq_d/a_d T_e), \tag{5}$$

$$I_i = \pi a_d^2 e(8T_i/\pi m_i)^{1/2} n_i(1 - eq_d/a_d T_i), \tag{6}$$

$$p_s = n_s^{\gamma_s}. \tag{7}$$

Here γ_s are the adiabatic indices for the electron ($s=e$), ion ($s=i$), and dust ($s=d$), respectively. m_s, q_s are the mass, charge for the sth species, n_s is the number density, p_s is the pressure, E_x is the electric field in the x direction. T_i, T_e are, respectively, the ion and electron temperatures. Before obtaining the conservation equations we normalize different quantities as follows. Densities of electron and ion are normalized to their initial values n_{e0}, n_{i0}, respectively. Similarly the dust density n_d and pressure p_d are normalized to n_{d0}, p_{d0}, respectively, and the grain charge number z_d is normalized to its initial value z_{d0}. Distance x, time t, dust particle velocity u_d, characteristic potential ϕ are normalized to λ_{Dd}, ω_{pd}^{-1}, u_0, and T_e/e, respectively, where

$$\lambda_{Dd}=(T_e/4\pi n_{d0}z_{d0}e^2)^{1/2}, \tag{8}$$

$$\omega_{pd}^{-1}=(m_d/4\pi m_{d0}z_{d0}^2e^2)^{1/2}, \tag{9}$$

and

$$z_d=-q_d/e. \tag{10}$$

We obtain the equations of motion in the wave frame of reference in which the flow appears steady. We assume u_0 to be the streaming velocity of different species. From continuity equations number density flux is conserved:

$$n_s u_s=1, \quad s=i,e,d. \tag{11}$$

From the equations of motion for electrons, ions, dust and Eq. (3) the following equations for conservation of momentum flux are obtained:

$$u_0^2\sum m_s n_{s0}P_s(u_s)=\tfrac{1}{2}T_e z_{d0}n_{d0}(d\phi/dx)^2, \tag{12}$$

where

$$P_s(u_s)=(u_s-1)+(1/\gamma_s M_s^2)(1/u_s^{\gamma_s}-1), \tag{13}$$

$s=i,e,d$ Mach number M_s is defined as

$$M_s^2=u_0^2 m_s/\gamma_s T_s, \tag{14}$$

where, as mentioned by McKenzie, the first term on the right-hand side of Eq. (13) gives the change in dynamic pressure and the second term gives the change in thermal pressure. The electric stress is proportional to $(d\phi/dx)^2$. Thus the sum of the total pressure is balanced by the electrical stress. The energy equations for ion, electron, and dust in normalized form are given by

$$\epsilon(u_i)=-T_e\phi/m_i u_0^2, \tag{15}$$

$$\epsilon(u_e)=T_e\phi/m_e u_0^2, \tag{16}$$

$$\epsilon(u_d)=z_{d0}F(\phi)(T_e/m_d u_0^2), \tag{17}$$

where

$$\epsilon(u_s)=\tfrac{1}{2}(u_s^2-1)+[1/(\gamma_s-1)M_s^2](1/u_s^{\gamma_s-1}-1) \quad (s=i,e,d) \tag{18}$$

and

$$F(\phi)=\int_0^{\phi}q_d d\phi. \tag{19}$$

From the equation of motion, Eq. (2), E_x can be written as

$$E_x=-(1/T_e q_d z_{d0})u_d(\partial u_d/\partial x)(m_d u_0^2-\gamma_d T_d/u_d^{\gamma_d+1}), \tag{20}$$

and the dust sonic points for the sth species are given by

$$u_s=1/M_s^{2/(\gamma_s+1)}. \tag{21}$$

The momentum functions $P_s(u_s)$ and the energy functions $\epsilon(u_s)$ have a minimum at the sonic points given by Eq. (21). From Eq. (4), we can deduce

$$\left(\frac{T_e}{m_e}\right)^{1/2}n_e e^{x_0 z_d}-\left(\frac{T_i}{m_i}\right)^{1/2}\frac{n_{i0}}{n_{e0}}n_i(1-y_0 z_d)=0. \tag{22}$$

Using Eqs. (22) and (19) we can deduce the analytical form of $F(\phi)$. From Eq. (22) and assuming $\gamma_e=\gamma_i=3$ we obtain, assuming a linear approximation,

$$z_d=1+p_1\phi, \tag{23}$$

p_1 is given in the Appendix. Using Eqs. (15), (16), (18), and (22) we can get p_1. The linear approximation is justified as the coefficient of the higher order term is small if

$$\left[1+\beta\left(1-\frac{e^2 z_{d0}}{a_d T_i}\right)\right]^2 \gg \frac{1}{3\beta(M_i^2-1)}+\frac{1}{3(M_e^2-1)}, \tag{24}$$

where

$$\beta=T_i/T_e. \tag{25}$$

Using (19) and (23)

$$F(\phi)=\phi+p_1\phi^2/2 \tag{26}$$

and From Eq. (12), using Eq. (20),

$$\left(\frac{du_d}{dx}\right)^2=u_0^2 q_d^2 T_e z_{d0}\frac{\sum m_s n_{s0}P_s(u_s)}{\frac{1}{2}n_{d0}u_d^2\left(m_d u_0^2-\gamma_d T_d/u_d^{\gamma_d+1}\right)^2}. \tag{27}$$

Taking $z_d=1+p_1\phi$, Eqs. (19) and (17) imply

$$\epsilon(u_d)=z_{d0}[1+(p_1/2)\phi](T_e\phi/m_d u_0^2). \tag{28}$$

The normalized electron velocity u_e, and ion velocity u_i can be expressed in terms of energy function for dust as

$$u_e=\left[1+(\gamma_e-1)\frac{M_{ed}^2\epsilon(u_d)}{z_{d0}[1+(p_1/2)\phi]}\right]^{-1/(\gamma_e-1)}, \quad \gamma_e\neq 1, \tag{29}$$

$$=\exp\left[-\frac{M_{ed}^2\epsilon(u_d)}{z_{d0}[1+(p_1/2)\phi]}\right], \quad \gamma_e=1, \tag{30}$$

$$u_i=\left[1-(\gamma_i-1)\frac{M_{id}^2\epsilon(u_d)}{z_{d0}[1+(p_1/2)\phi]}\right]^{-1/(\gamma_i-1)}, \quad \gamma_i\neq 1, \tag{31}$$

$$=\exp\left[\frac{M_{id}^2\epsilon(u_d)}{z_{d0}[1+(p_1/2)\phi]}\right], \quad \gamma_i=1, \tag{32}$$

where

$$M_{sd}^2=M_s^2 m_d/m_s=u_0^2 m_d/\gamma_s T_s. \tag{33}$$

From (28),

$$\phi=\frac{-1+\sqrt{1+2p_1(m_d u_0^2/z_{d0}T_e)\epsilon(u_d)}}{p_1}, \tag{34}$$

where $p_1\neq 0$. We define the normalized plasma momentum function $R(u_d)$ as

$$R(u_d) = P_d(u_d) + \frac{m_i n_{i0}}{m_d n_{d0}} P_i(u_i) + \frac{m_e n_{e0}}{m_d n_{d0}} P_e(u_e). \quad (35)$$

Then

$$\frac{n_{d0} z_{d0} T_e C_z^2}{2q_d^2} u_d^2 \left(\frac{du_d}{dx}\right)^2 \left(1 - \frac{1}{M_d^2 u_d^{\gamma_d+1}}\right)^2 = R(u_d), \quad (36)$$

$$\frac{dR}{du_d} = - m_d n_{d0} u_0^2 \left(1 - \frac{1}{M_d^2 u_d^{\gamma_d+1}}\right)\left(\frac{\delta_1}{u_e} - \frac{\delta_2}{u_i} + \frac{z_d}{u_d}\right)\frac{u_d}{z_d}. \quad (37)$$

When the term in the first parentheses in Eq. (37) is equated to zero it gives the dust sonic point and when the term in the second parentheses is equated to zero it gives the charge neutral point. At the charge neutral point the electric stress is maximum. At the dust sonic point

$$u_d = 1/M_d^{2/(\gamma_d+1)}, \quad (38)$$

the flow becomes choked as $du_d/dx \to \infty$. The root of $R(u_d) = 0$ gives the equilibrium point. An equilibrium point must be attained before the dust sonic point is reached. Assuming the dust particles to be massive so that m_e/m_d, $m_i/m_d \to 0$, the plasma momentum function is simplified as

$$R(u_d) = P_d(u_d) + (n_{i0}/\gamma_i n_{d0} M_{id}^2)(1/u_i^{\gamma_i} - 1) + (n_{e0}/\gamma_e n_{d0} M_{ed}^2)(1/u_e^{\gamma_e} - 1). \quad (39)$$

For $\delta = u_d - 1 \ll 1$, keeping terms up to δ^4, $R(u_d)$ can be expanded as

$$R(u_d) = \delta^2 E_1 + \delta^3 E_2 + \delta^4 E_3. \quad (40)$$

For $\delta = u_d - 1 \ll 1$, the structure equation (27) reduces to

$$d\delta/dx = \pm\sqrt{K^2}\,\delta, \quad (41)$$

where

$$K^2 = -\frac{1}{1 - 1/M_d^2} - 3p_1 C_z + \frac{n_{e0} M_{ed}^2 + n_{i0} M_{id}^2}{n_{d0} z_{d0}^2}. \quad (42)$$

For $K^2 > 0$, we need $M_{dc}^2 > 1/(1 - 1/M_d^2) + 3p_1 C_z$, where M_{dc} is the "collective" heavy Mach number

$$M_{dc}^2 = (n_{e0} M_{ed}^2 + n_{i0} M_{id}^2)/n_{d0} z_{d0}^2. \quad (43)$$

$K^2 > 0$ corresponds to evanescent type solution of the form $\exp(\pm Kx)$. For $M_d = \infty$, which corresponds to the case when the dust component is cold, we have

$$P_d(u_d) = (u_d - 1), \quad (44)$$

$$P_i(u_i) = (1/\gamma_i M_i^2)(1/u_i^{\gamma_i} - 1), \quad (45)$$

$$P_e(u_e) = (1/\gamma_e M_e^2)(1/u_e^{\gamma_e} - 1), \quad (46)$$

$$R_d(u_d) = (u_d - 1) + \frac{r + z_{d0}}{\gamma_i M_{id}^2}\left(\frac{1}{u_i^{\gamma_i}} - 1\right) + \frac{r}{\gamma_e M_{ed}^2}\left(\frac{1}{u_e^{\gamma_e}} - 1\right). \quad (47)$$

The equation for determining the charge neutral point is

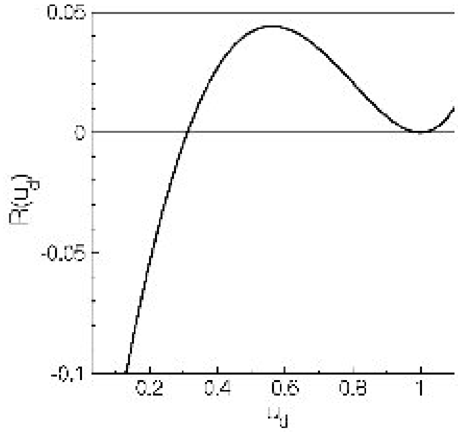

FIG. 1. $R(u_d)$ vs u_d is plotted where the dust-plasma parameters are $n_{i0} = 1.3\times10^{10}$ cm^{-3}, $n_{e0} = 10^{10}$ cm^{-3}, $n_{d0} = 10^5$ cm^{-3}, $z_{d0} = 3\times10^4$, $T_e = T_i = 10$ eV, $M_{id} = M_{ed} = 13\,304.34$, $T_d = 0.1$ eV, $M_d = 1153.44$, $a_d = 13.8186\times10^{-5}$ cm, $m_i = 1837\times m_e$.

$$\delta_1/u_e - \delta_2/u_i + u_d/z_d = 0. \quad (48)$$

For $M_d = \infty$, the structure equation can be written as

$$u_d \frac{du_d}{dx} = \frac{1}{\sqrt{C_z}}(u_d - 1) \times \sqrt{-1 + \frac{(u_d+1)^2}{4}(M_{dc}^2 - 3p_1 C_z) + 2p_1 C_z (u_d + 1)}. \quad (49)$$

In the small amplitude limit the region of existence of soliton is derived as

$$1 < p_1 C_z + M_{dc}^2 < 4. \quad (50)$$

In case of dust acoustic wave the dust mass provides the inertia whereas the restoring force is due to the thermal pressure of the electrons and ions. The plasma momentum function $R(u_d)$ is simplified to Eq. (40) when we assume the dust to be massive. Plasma momentum function is plotted in Fig. 1. The deviations of the ion and electron velocities (normalized) from the equilibrium value are plotted against dust ve-

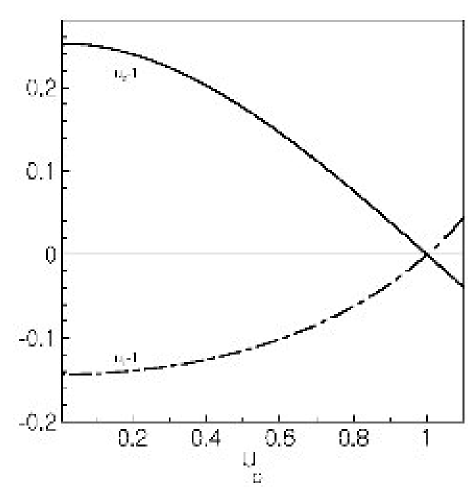

FIG. 2. $u_i - 1$, $u_e - 1$ vs u_d are plotted where the parameters are the same as those used in Fig. 1.

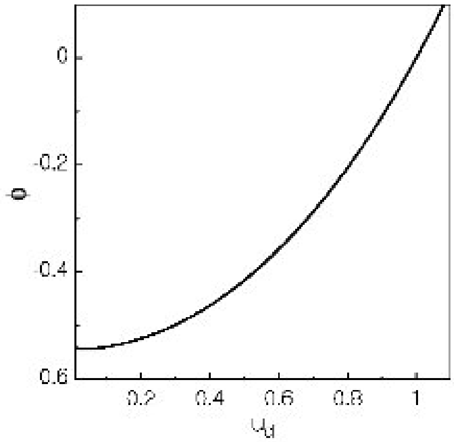

FIG. 3. ϕ vs u_d is plotted. The data are same as those used in Fig. 1.

locity in Fig. 2 using Eqs. (29) and (31), respectively. It is to be noted that we have considered here the adiabatic case $\gamma_d = \gamma_e = \gamma_i = 3$. It is seen that the charge neutral point is attained between the initial point and the equilibrium point. The sonic point occurs at $u_d = 0.030\,38$ and the second equilibrium point occurs at $u_d = 0.0111$.

To conclude, dust acoustic solitary waves have been studied using the gas dynamical approach discussed in Ref. 14 taking into account the dust charge variation and it is found that the charge variation puts further constraints [see, e.g., Eq. (50)] on the existence of dust acoustic solitary waves. The exact form of $\epsilon(u_d)$ is obtained when one considers Eq. (22). We have assumed that z_d can be taken as a linear function of ϕ. Figure 3 shows the potential profile associated with the dust acoustic soliton.

APPENDIX

$$\delta_1 = n_{e0}/n_{d0}z_{d0}, \tag{A1}$$

$$\delta_2 = n_{i0}/n_{d0}z_{d0}, \tag{A2}$$

$$r = n_{e0}/n_{d0}, \tag{A3}$$

$$x_0 = e^2 z_{d0}/a_d T_e, \tag{A4}$$

$$y_0 = e^2 z_{d0}/a_d T_i, \tag{A5}$$

$$C_z = m_d u_0^2/z_{d0}T_e, \tag{A6}$$

$$A_1 = 2/3\beta(M_i^2 - 1), \tag{A7}$$

$$A_2 = -2/3(M_e^2 - 1), \tag{A8}$$

$$p_1 = (A_1 - A_2)(1 - y_0)/2(x_0(1 - y_0) + y_0), \tag{A9}$$

$$D_1 = [n_{e0}(2 - \gamma_e)M_{ed}^4 + n_{i0}(\gamma_i - 2)M_{id}^4]/n_{d0}z_{d0}^2, \tag{A10}$$

$$D_2 = \frac{n_{e0}(\gamma_e - 2)(2\gamma_e - 3)M_{ed}^6 + n_{i0}(\gamma_i - 2)(2\gamma_i - 3)M_{id}^6}{24 n_{d0} z_{d0}^4}, \tag{A11}$$

$$E_1 = \tfrac{1}{2}(1 - 1/M_d^2)(p_1 C_z + M_{dc}^2 - 1), \tag{A12}$$

$$E_2 = -\frac{\gamma_d + 1}{3M_d^2} + \frac{1}{2}\left(1 + \frac{\gamma_d}{M_d^2}\right)\left(1 - \frac{1}{M_d^2}\right)(p_1 C_z + M_{dc}^2) - \frac{1}{2}\left(1 - \frac{1}{M_d^2}\right)^3 (p_1^2 C_z^2 + p_1 C_z M_{dc}^2 - 2D_1), \tag{A13}$$

$$E_3 = \frac{(\gamma_d + 1)(\gamma_d + 2)}{8M_d^2} + \frac{p_1 C_z + M_{dc}^2}{2}\left(\frac{1}{4}\left(1 + \frac{\gamma_d}{M_d^2}\right)^2 - \frac{\gamma_d(\gamma_d + 1)}{6M_d^2}\left(1 - \frac{1}{M_d^2}\right)\right) + g_1, \tag{A14}$$

$$g_1 = \frac{3}{4}\left(1 - \frac{1}{M_d^2}\right)^2\left(1 + \frac{\gamma_d}{M_d^2}\right)(2D_1 - p_1^2 C_z^2 - p_1 C_z M_{dc}^2) + \left(1 - \frac{1}{M_d^2}\right)^4\left(D_2 + \frac{p_1^3 C_z^3}{2} + \frac{5}{8}M_{dc}^2 p_1^2 C_z^2 - \frac{3D_1}{2}p_1 C_z\right). \tag{A15}$$

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