Sagdeev's approach to study the effect of the kinematic viscosity on the dust ion-acoustic solitary waves in dusty plasma

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(Received 11 November 2004; accepted 9 February 2005; published online 7 April 2005)

Sagdeev's technique is used to study the dust ion-acoustic solitary waves (DIASWs) in a dusty plasma comprising ions, electrons, and charged dust grains taking into account the ion kinematic viscosity. Exact analytical results for the solitary wave solutions were obtained for small amplitude DIASW. The effects of the ion kinematic viscosity and the ion temperature on the feature of DIASW have been investigated.

During the last two decades a lot of interest has been shown[1–15] in the study of dusty plasmas. The motivation for studying dusty plasma is the following. Dusty plasmas occur in nature in various forms such as planetary rings,[16–18] cometary tails, intrastellar clouds, etc. On the other hand, experiments in the laboratory have been made on dusty plasmas.[19–22] Studies on solitary waves and double layers in dusty plasma have also been made by a number of workers.[1–11] Nonlinear structures such as solitary waves and shock waves in dusty plasma have been observed in the laboratory especially in condensed plasmas[23] and colloidal suspension.[24] Now the two normal modes of unmagnetized and weakly coupled dusty plasmas are the following: One is the dust acoustic mode (DAW) and the other is the dust ion-acoustic wave (DIAW). These waves were first theoretically predicted by Rao, Shukla, and Yu,[1] and Shukla and Silin,[2] respectively. Usually the nonlinear waves in dusty plasma are studied using the reductive perturbation technique (RPT) which generally gives rise to the famous Korteweg–de Vries (KdV) equation.[13] Several authors have used this technique to obtain weakly nonlinear KdV solitons. Recently El-Labany *et al.*[25] studied the dust ion-acoustic solitary waves (DIASWs) in a medium of fully ionized, collisonless, unmagnetized dusty plasma with positive ions, warm negatively charged dust grains, and nonisothermal electrons. They studied the effect of trapped electron temperature, dust charge variation, and grain radius on the nonlinear DIAWs. Li *et al.*[26] studied dust-ion acoustic soliton in collisonal inhomogeneous plasma. Using RPT they have shown that the characteristic properties of the soliton change as it propagates in the inhomogeneous dusty plasma. However if one takes into account the the dust kinematic viscosity,[27] one obtains the Korteweg–de Vries–Burgers equation which, under certain condition, produces the shock wave solution. Nakamura and Sarma[20] have investigated the dissipation of ion-acoustic solitary waves in a dusty plasma considering the ion dust collision and kinematic viscosity of ions. Also, Watanabe has studied the dissipation effect on ion-acoustic solitons experimentally.[28] To explain dissipation effect due to the presence of dust particles on the ion-acoustic solitary waves, many improvements of KdV equation have been proposed.[20] However the RPT is applicable only to small amplitude solitary waves. For arbitrary amplitude solitary waves a nonperturbative[29] approach is necessary. Unfortunately, except in simple cases, an exact analytical expansion for the Sagdeev's pseudopotential cannot be obtained. For example, if one takes into account the full nonlinearity, the exact analytical form of the Sagdeev's potential cannot be derived. However, it is shown that a single ordinary second-order differential equation for $\int n d\phi$ can be derived which when solved would give the pseudopotential. Though for nonzero kinematic viscosity, this equation cannot be solved analytically, one can still obtain the pseudopotential upto any order in ϕ. This is the advantage over the RPT in this case. In the present paper we will adopt this approach to study solitary waves in a plasma comprising immobile charged dust grains, warm electrons, and warm ions. We shall also see that for small η, the ion kinematic viscosity, KdV-Burger type equation can be derived using our formalism, thus reproducing the result of RPT in the small amplitude case.

Here we consider a dusty plasma whose constituents are electrons, ions, and dust grains. On the ion-acoustic time scale the electrons are taken Boltzmanian so that the density is given by

$$n_e = \exp \phi, \qquad (1)$$

where n_e is normalized by the unperturbed electron density n_{e0}. Equations governing the ion dynamics are

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0, \qquad (2)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial \phi}{\partial x} - 3\sigma n\frac{\partial n}{\partial x} - \eta\frac{\partial^2 u}{\partial x^2}, \qquad (3)$$

$$\frac{\partial^2 \phi}{\partial x^2} = \exp \phi - n\delta_1 + z_d\delta_2. \qquad (4)$$

Here

$$\eta = \frac{\mu_{id}}{\omega_{pi}\lambda_{de}^2}, \quad \sigma = \frac{T_i}{T_e}, \quad \delta_1 = \frac{n_{i0}}{n_{e0}}, \quad \delta_2 = \frac{n_{d0}}{n_{e0}}; \qquad (5)$$

T_e, T_i are, respectively, the electron and ion temperatures and ϕ is the plasma potential. μ_{id} and λ_{de} are, respectively, the kinematic viscosity for ions and electron Debye length. The ion density n, velocity u, and plasma potential ϕ are normalized to n_{i0}, the ion acoustic speed C_s, T_e/e, respectively. The displacement x and time t are normalized to λ_{de} and ω_{pi}^{-1}. n_{i0}, n_{e0}, and n_{d0} are, respectively, the ion, electron, and dust densities in the equilibrium state. λ_{de}, ω_{pi}, C_s are given as

$$\lambda_{de} = \sqrt{\frac{T_e}{4\pi n_{e0}e^2}}, \quad \omega_{pi} = \sqrt{\frac{4\pi n_{e0}e^2}{m_i}}, \quad C_s = \sqrt{\frac{T_e}{m_i}}. \tag{6}$$

Here z_d is the dust charge number. It is to be noted that the dust grains were assumed to be massive, immobile, and negatively charged.[27] To obtain solitary wave solution $\xi = x - Vt$ is introduced, where V is the solitary wave velocity. From Eq. (2)

$$n = \frac{V}{V-u} = \frac{V}{z}, \tag{7}$$

where

$$z = V - u. \tag{8}$$

From Eq. (3)

$$\frac{z^2}{2} = \frac{V^2}{2} - \phi - \frac{3\sigma}{2}n^2 + \eta\frac{dz}{d\xi} + \frac{3\sigma}{2}. \tag{9}$$

Assuming

$$F(\phi) = \int_0^{\phi} n\,d\phi \tag{10}$$

and using Eq. (4), we have

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 = e^{\phi} - \delta_1 F(\phi) + z_d\delta_2\phi - 1. \tag{11}$$

From Eqs. (7) and (10), we have

$$F'(\phi) = \frac{V}{z}, \tag{12}$$

$$\frac{dz}{d\xi} = -\frac{V}{F'(\phi)^2}F''(\phi)\frac{d\phi}{d\xi}. \tag{13}$$

Taking

$$w = F'(\phi) = n, \tag{14}$$

and from Eqs. (9) and (13), we have

$$w' = \frac{1}{\eta V}\frac{\frac{w^2V^2}{2} - \phi w^2 - \frac{3\sigma}{2}w^4 - \frac{V^2}{2} + w^2\frac{3\sigma}{2}}{\sqrt{2e^{\phi} - 2\delta_1 F(\phi) + 2z_d\delta_2\phi - 2}}. \tag{15}$$

Equations (14) and (15) together will yield $F(\phi)$ which in turn would give the Sagdeev's pseudopotential. This is our main analytical result. Before we take up the conventional expansion of the pseudopotential, let us show how the KdV-Burger (KdVB) type equation can be obtained from Eqs. (14) and (15) assuming η to be small. For the sake of simplicity we will assume, for the present purpose, σ to be zero. Extension of the result to nonzero σ is trivial. It is seen from Eq. (15) that the equation holds only when $\eta \neq 0$. For $\eta = 0$, w^2 is given by

$$w^2(V^2 - 2\phi) = V^2. \tag{16}$$

So, $n = w = V/\sqrt{V^2 - 2\phi}$ (we take the + sign here). For $\eta \neq 0$, but small, we write

$$w^2 = \frac{V^2}{(V^2 - 2\phi)} + \eta f(\phi), \tag{17}$$

where the function $f(\phi)$ is to be determined from Eq. (15). Neglecting terms of $O(\eta^2)$, $f(\phi)$ turns out to be

$$f(\phi) = \frac{2V^2\frac{d\phi}{d\xi}}{(V^2 - 2\phi)^{5/2}}. \tag{18}$$

From Eq. (17), using Eq. (4) we have

$$\frac{d^2\phi}{d\xi^2} = e^{\phi} - \frac{\delta_1 V}{\sqrt{V^2 - 2\phi}} - \frac{V\delta_1\eta\frac{d\phi}{d\xi}}{(V^2 - 2\phi)^2} + z_d\delta_2. \tag{19}$$

Differentiating Eq. (19) with respect to ξ and assuming ϕ/V^2 to be small, we have

$$\frac{d^3\phi}{d\xi^3} = \frac{d\phi}{d\xi}\left(1 - \frac{\delta_1}{V^2}\right) + \phi\frac{d\phi}{d\xi} - \frac{\delta_1\eta}{V^3}\frac{d^2\phi}{d\xi^2}, \tag{20}$$

which is the KdVB equation written in terms of the single variable $\xi = x - Vt$. As far as our knowledge goes KdVB equation has not been deduced before from pseudopotential approach. Following the procedure of Sahu and Roychoudhury[30] a particular solution of the KdVB equation (20) is obtained as follows. Integrating Eq. (20) and using the initial condition $\phi = 0$, when $d\phi/d\xi = 0$, we get

$$\frac{d^2\phi}{d\xi^2} = \phi\left(1 - \frac{\delta_1}{V^2}\right) + \frac{\phi^2}{2} - \frac{\delta_1\eta}{V^3}\frac{d\phi}{d\xi}. \tag{21}$$

To solve Eq. (21) we use the direct method given in Ref. 30. We just quote the result here. Details are given in Ref. 30:

$$\phi = \frac{A^2g^2}{(e^{-Ag/2} - Bg)^2}, \quad A = -\frac{2\delta_1\eta}{5V^3}, \quad B = \pm\sqrt{\frac{1}{3}}, \tag{22}$$

where g is the integration constant which depends on the initial condition and V is determined from the equation $25(V^6 - \delta_1 V^4) + 6\delta_1^2\eta^2 = 0$. It can be shown[30] that when $Bg = -1$, the above solution will be identical with the solution obtained by the tanh method.[31] However for η not too small one cannot derive Eq. (20). For small amplitude but arbitrary η, we expand $F(\phi)$ up to ϕ^4 terms,

$$F(\phi) = \phi + b\phi^2 + c\phi^3 + d\phi^4. \tag{23}$$

Here b is determined from the equation

$$2b(V^2 + 3\sigma) - 1 - 12b\sigma - 2b\eta V\sqrt{1 - 2\delta_1 b} = 0, \tag{24}$$

and c and d are given by

$$c = \frac{b\eta V + 3\sqrt{1 - 2\delta_1 b}(4b - 2b^2V^2 + 30\sigma b^2)}{3(3V^2 - 9\sigma)\sqrt{1 - 2\delta_1 b} + \eta V(42b\delta_1 - 18)}, \tag{25}$$

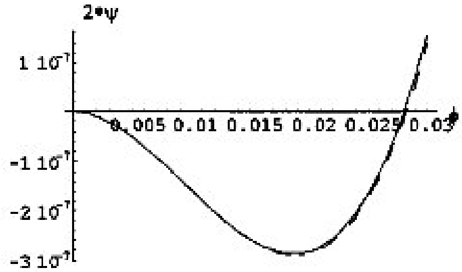

FIG. 1. Sagdeev's potential $2\psi(\phi)$ vs ϕ is plotted, where the dust-plasma parameters are η=0.06, σ=0, δ_2=0.0001, V=1.5, δ_1=2.24. Dotted and solid curves correspond to the higher and lower order approximations to $\psi(\phi)$, respectively.

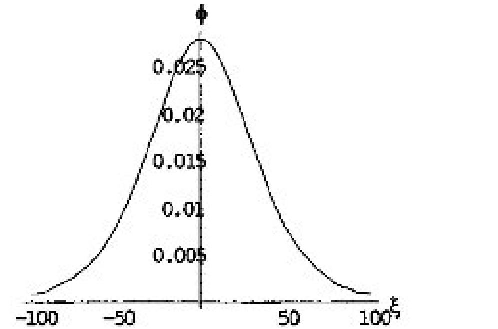

FIG. 2. ϕ vs ξ is plotted using the expression (32) for $\psi(\phi)$ with parameters same as those used in Fig. 1.

$$d\left[12\sqrt{1-2\delta_1 b}\,.\,\eta V-\frac{2b\delta_1\eta V}{\sqrt{1-2\delta_1 b}}-4V^2+12\sigma\right]$$
$$=\frac{-b\,\eta V}{12\sqrt{1-2\,\delta_1 b}}+\frac{b\,\eta V\sqrt{1-2\delta_1 b}}{36}\left(\frac{1-6\delta_1 c}{1-2\delta_1 b}\right)^2+d_1, \quad (26)$$

where

$$d_1=-c\frac{1-6\delta_1 c}{\sqrt{1-2\delta_1 b}}\eta V+6bcV^2-(4b^2+6c)$$
$$-3\sigma(16b^3+30bc). \quad (27)$$

Writing

$$\psi=-\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2, \quad (28)$$

we obtain from Eq. (11)

$$\psi=-e^{\phi}+\delta_1 F(\phi)+1-z_d\delta_2\phi. \quad (29)$$

Keeping terms up to ϕ^2,

$$\frac{d^2\phi}{d\xi^2}=A_1\phi+A_2\phi^2, \quad (30)$$

where

$$A_1=1-2b\delta_1, \quad A_2=-3c\delta_1+\tfrac{1}{2}. \quad (31)$$

Integrating Eq. (30), we obtain

$$\phi=-\frac{3(1-2\delta_1 b)}{1-6c\delta_1}\,\mathrm{Sech}^2\left(\frac{\sqrt{1-2\delta_1 b}\,\xi}{2}\right). \quad (32)$$

Keeping terms up to ϕ^3, we have

$$\frac{d^2\phi}{d\xi^2}=A_1\phi+A_2\phi^2+A_3\phi^3, \quad (33)$$

where,

$$A_3=-4d\delta_1+\tfrac{1}{6}. \quad (34)$$

Integrating Eq. (33), we obtain

$$\phi=\left[-\frac{A_2}{3A_1}+\sqrt{\frac{A_2^2}{9A_1^2}-\frac{A_3}{2A_1}}\,\mathrm{Cosh}(\sqrt{A_1}\xi)\right]^{-1}. \quad (35)$$

If $A_2^2=\frac{9}{2}A_1A_3$ then solution (35) would not be valid and a shock wave solution is obtained which is given by

$$\phi=A[1+\tanh\alpha(\xi+\xi_0)], \quad (36)$$

where ξ_0 is integration constant and

$$A=-\frac{3A_1}{2A_2}, \quad \alpha=\frac{A_2}{3\sqrt{2A_3}}, \quad (A_3>0). \quad (37)$$

This clearly shows that to get the shock wave solution one has to take higher order terms in the expansion of ψ.

Before we discuss the numerical results it may be noted that Eqs. (32) and (35) would not be valid for $\delta_1 b=\frac{1}{2}$, $\delta_1 c=\frac{1}{6}$. For these values of parameters one must take a higher order term and when $A_1<0$, but $\delta_1 c\neq\frac{1}{6}$, both these solutions will show periodic behavior. Hence for solitary wave we will assume $A_1>0$ and $A_3\neq 0$. In Fig. 1, ψ is plotted with respect to ϕ in case of small amplitude solitary wave taking terms up to ϕ^3 and ϕ^4 when η=0.06, other parameters being σ=0, δ_2=0.0001, V=1.5, δ_1=2.24. As can be seen, inclusion of ϕ^4 term does not change qualitatively the nature of solitary waves though the analytical solutions, Eqs. (32) and (35), are different. This is because the amplitude turns out to be rather small. It appears that the solutions (32) and (35) are different and under some conditions, the latter one leads to the shock wave. This means that the Sagdeev potential may lead to different coherent structure with the same amplitude and width. However to obtain a shock wave solution one has to include higher order terms. It is seen that DIASW exists for positive ϕ. In Fig. 2 the potential profile against ξ is shown using the relation (32). From Fig. 3 we find that amplitude of the soliton decreases as σ increases. Figure 4 shows that as η increases the amplitude of the soliton decreases. For large

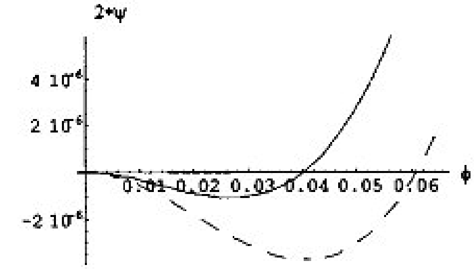

FIG. 3. Sagdeev's potential $2\psi(\phi)$ vs ϕ is plotted, where the dust-plasma parameters are η=0.04, δ_2=0.0001, V=1.5, δ_1=2.2. Two different values of σ have been used. Here σ=0.01 corresponds to the dotted curve and σ=0.012 corresponds to the solid curve.

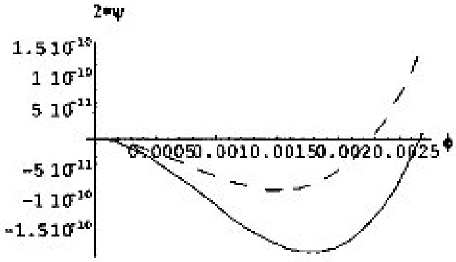

FIG. 4. Sagdeev's potential $2\psi(\phi)$ vs ϕ is plotted, where the dust-plasma parameters are $\delta_2=0.0001$, $V=1.5$, $\delta_1=2.2488$, $\sigma=0$. Two different values of η are used. Here $\eta=0.04$ corresponds to the solid curve. The dashed curve is for $\eta=0.06$.

amplitude DIASWs one has either to solve the coupled equations (14) and (15) or has to include higher order terms in the expansion. However the solution given in Eq. (35) is already a higher order solution which cannot be obtained using RPT method. To conclude, the set of equations governing the ion dynamics, when kinematic viscosity μ_{id} is taken into account, have been reduced to a set of coupled nonlinear ordinary differential equations easily amendable to numerical integration. However one can analytically expand $\psi(\phi)$ up to any order in ϕ. In the present case it has been explicitly demonstrated that though for small amplitude one can neglect term of order ϕ^4 in the pseudopotential, for a shock wave solution one has to include higher order terms. For small η and ϕ/V^2, KdV-Burger type equation is obtained; thus reproducing the RPT result.

ACKNOWLEDGMENT

The authors are grateful to the referee for his comments and suggestions which were of considerable help in improving the paper.

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