# Determination of the Best Mean Fill

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## ABSTRACT

This paper is concerned with a filling problem. The company has received complaints from a valuable customer that the net weight is below the lower specification limit (LSL). Data are collected to estimate the extent of variability. The observed total variation is apportioned into two parts, viz., variation due to the filling process and variation due to the measurement process. Thus, the variation due to the filling process is estimated. The problem is formulated as a profit maximization problem where the barrels meeting the LSL generate a specified profit; whereas, those below the LSL generate lesser profit, as a rework has to be done (in filling them to the desired LSL). The optimum setting is thus determined.

*Key Words:* Grease; Net weight; Process variability; Profit.

## INTRODUCTION

This study was carried out at a grease manufacturing plant. The grease is filled in barrels to be sold in the markets. Until recently, the company had a monopoly. One of its main customers is the defense establishment (DE). Not only strict quality control conditions have to be fulfilled with respect to the functional aspects of the grease, but also the net weight supplied is an important quality parameter. This paper is concerned with the net weight as the quality parameter.

## THE PROBLEM

There have been a series of complaints from the DE that the net weight of grease is below the permissible lower specification limit. The company markets the grease in barrels of net weight 182 $\pm$ 0.5 kg. However, the DE has been saying that, in a number of cases, the net weight is below 181.5 kg. In certain cases (technically, very specific grades manufactured specifically for the DE), the DE has gone to the extent of imposing a penalty (in monetary terms) for lesser net weight. This monetary loss has forced the company to resort to sampling inspection of the filled barrels in order to ensure that the net weight requirement is being satisfied. However, this cannot be a feasible long-term solution to the problem. Hence, the company is concerned on this score and would like to address this problem seriously.

## FILLING PROCESS

Before we begin to address the problem, a brief description of the filling process is in order.

Empty barrels move automatically to the filling platform. The digital display shows the tare weight of the barrel; and then it resets itself to zero. (These empty barrels are procured from a vendor whose 3quality system has been excellent and who enjoys the confidence of the DE). The valve opens; grease starts being filled in the barrel. The desired net weight is 182.00 kg. As the net weight reaches 174–175 kg, the valve gets closed to half to reduce the flow rate of the grease. As soon as the weight reaches 182.00 kg, the valve closes fully. The gross weight of the barrel is displayed. The filled barrel is pushed ahead and the next empty barrel moves on the filling platform.

It should be noted that other than setting the mean level (which in the present case is 182.00 kg), the machine cannot be interfered with during the filling operation. If the filling operator or supervisor is careful enough to note the barrel tare weight and the gross weight, then the net weight can be obtained, and the underfilled barrels can then be manually filled to the desired weight. These careful operators tend to observe the barrel tare weight and manually compensate the underfilled barrels so that the desired net weight is obtained.

## PROCESS VARIABILITY ASSESSMENT

Before attempting to make any correction in the mean level, we need to have knowledge of the variability involved in the filling process. Now, the observed total variability is the sum of two components: a) variability due to the filling operation and b) variability due to the measurement error. Mathematically,

$$\sigma^2_{\text{total}} = \sigma^2_{\text{process}} + \sigma^2_{\text{gauge}}$$

Hence, in order to have an estimate of $\sigma^2_{\text{process}}$ (variability in the filling operation), we need to have an estimate of $\sigma^2_{\text{gauge}}$ (variability due to the measurement error). A gauge capability study showed that $\sigma_{\text{gage}} = 0.0215\,\text{kg}$. This exercise also showed that the filling machine has a bias of 0.136 kg toward the higher side.

Having gotten an estimate of the gage capability, we can now proceed to get an estimate of the filling process variability.

## FILLING PROCESS VARIABILITY

The grease filling operation on an average takes two hours. It was decided to take five observations during the first half hour of the first hour of the filling time; and another five observations during the second half hour of the second hour. This gives an idea of the complete variation during the entire filling operation time of two hours. The response measured is the net weight of the filled barrels. The data collection was done for one month (24 working days). Thus, we had $5 \times 2 \times 24 = 240$ observations. Fig. 1 shows the histogram of the sample observations.

On the basis of these 240 observations, we have $\sigma^2_{\text{total}} = (0.17)^2$. But we know that $\sigma_{\text{gauge}} = 0.0215$. Therefore, $\sigma^2_{\text{process}} = \sigma^2_{\text{total}} - \sigma^2_{\text{gauge}} = 0.0284$. Thus, $\sigma_{\text{process}} = 0.1686$. Hence, the grease filling operation has a variability of $\sigma_{\text{process}} = 0.1686$.

## PROBLEM FORMULATION

For the purpose of the study, the problem was reformulated as described below.

Assume that the contents ($X$) in a barrel is a normal random variable; (this is not a very stringent assumption and is supported by the data, as shown in Fig. 1), whose mean $\mu$ depends on the process setting and known variance $\sigma^2$. Let $g(X; \mu, \sigma^2)$ denote the normal density function with mean $\mu$ and variance $\sigma^2$.

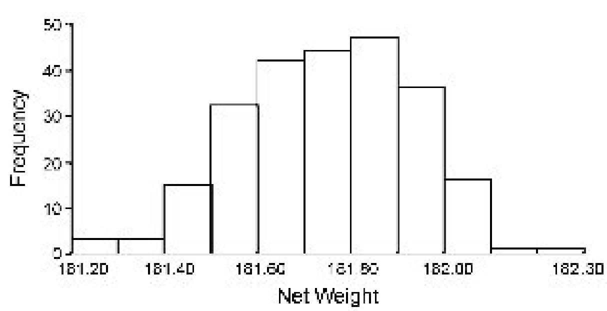


***Figure 1.*** Net weight—before study.

A filled barrel weighing less than the minimum weight specification $L$ is refilled (the contents are reused), thus incurring a reprocessing cost of Rs. $R$ per barrel. A filled barrel weighing $\geq L$ is sold in the regular market for Rs. $A$. It costs Rs $C$/unit of contents; the cost of the barrel itself is negligible.

Let $P(X;\mu)$ be the profit for a barrel filled with contents $X$, and $E[P(X;\mu)]$ its expected value. If the contents $X \geq L$, then the barrel is sold for $A$ and the net profit is $A-CX$. On the other hand, if a barrel weighs less than $L$, it is reprocessed at the cost of $R$. This refilled barrel will then realize the expected profit, $E[P(X;\mu)]$. Hence, for the reprocessed barrel, the net expected profit is $E[P(X;\mu)]-R$. The profit function per barrel is therefore

$$P(x,\mu) = \begin{cases} A - Cx & \text{if } x \geq L \\ E[P(x;\mu)] - R & \text{if } x \leq L \end{cases}$$

Then the expected profit is given by

$$E[P(X;\mu)] = \int_0^{\mathrm{L}} \{E[P(X;\mu)] - R\}g(X;\mu,\sigma^2)dX + \int_{\mathrm{L}}^{\infty} (A - CX)g(X;\mu,\sigma^2)dX \quad (1)$$

The objective is to find the optimum value for $\mu$ that will maximize the expected profit, $E(P)$, per barrel sold. Golhar[1] has shown that the best process setting $\mu^*$ that maximizes the expected profit in Eq. (1) above is given by

$$\mu^* = L + \sigma[0.712 + 0.471 \ln\{R/(C\sigma)\}] \quad (2)$$

## PROBLEM SOLUTION

In order to utilize Eq. (2), we need to know the values of $L$, $R$, $C$, and $\sigma$. In the present case, $L = 181.5$ kg; the estimates of $R$ and $C$, as supplied by the industrial engineering department of the company, are 3.50 and 3.95, respectively. The variability in the grease filling operation ($\sigma$) has been estimated (as discussed above) to be $\sigma = 0.1686$. Then using Eq. (2), we get $\mu^* = 181.752$ kg. However, since the bias in the filling machine is of the order of 0.136 kg, set the targeted best process setting $\mu_t^*$ to be 181.888 kg. As the machine is calibrated up to 0.05 kg, the best operational target value would be 181.90 kg.

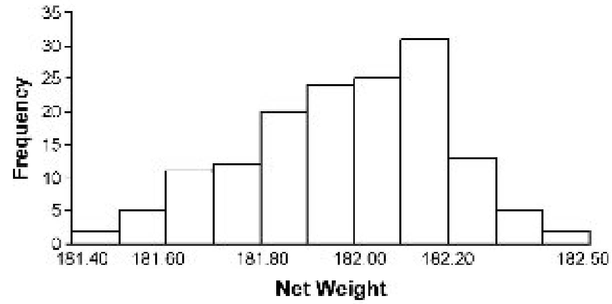


*Figure 2.* Net weight—after study.

## CONCLUSION

These findings were conveyed to the plant management. It was decided to test this optimal setting value on an experimental basis. It was tried for 15 days. Fig. 2 shows the histogram of the 150 observations. The average of these observations was found to be 181.951 kg. In view of these encouraging results, the administration decided to implement this suggested value in their standard operating procedures.

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M. Z. Anis is a faculty member of the SQC and OR (T and P) Unit of the Indian Statistical Institute. He is actively engaged in research, teaching, training, and consultancy in the fields of SQC and OR. He is a member of the Academic Council of the ISI.

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