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An economic inspection interval for control of defective items in a hot rolling mill

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ABSTRACT *The article addresses a real-life problem on determining the optimum sampling interval for control of defective items in a hot rolling mill. Having observed that the pattern of appearance of mill defects indicates a geometric process failure mechanism, an economic model is developed in line with the method suggested by Taguchi and critically examined by Nayebpour & Woodall. An expression for the expected loss per product as a function of the sampling interval is derived and the optimum interval is obtained by minimizing this loss function. The practical issues involved in this exercise, such as estimation of various cost components, are also discussed and the effect of erroneous estimation of cost components is studied through a sensitivity analysis.*

1 Introduction and background

In most manufacturing processes that produce a continuous stream of products at a high speed, there is a category of defects that occur due to specific process faults and continue to be found in 100% of items until the source is detected and rectified. Such processes are usually monitored by inspecting the product at predetermined intervals and launching an investigation the moment a defect is found. Determination of the optimum sampling interval has always been a matter of concern for industries having such processes. On one hand, frequent inspection requires more cost, time and manpower whereas, on the other hand, reduced frequency of inspection may lead to the risk of rejection of a large number of items. This problem of developing economically based online control methods for attributes has been addressed in detail by Taguchi (1981, 1984, 1985), Taguchi *et al.* (1989) and Nayebpour & Woodall (1993). Whereas Taguchi *et al.* (1989) did not explicitly assume a specific process failure mechanism (PFM), Nayebpour &

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Woodall (1993) considered a geometric PFM. This article presents a case study where a similar situation was encountered, and a lowest-cost sampling interval was determined by developing a statistical model for the appearance of defects.

The study was taken up in a hot rolling mill where steel billets were hot rolled to produce wire rods in the form of coils. The process produced a continuous stream of coils that were subjected to online inspection at-regular-intervals. Every mth coil (the existing-value of m at the time of study was 10) was sampled and inspected to detect the presence of defects.

Defects appearing in coils could be categorized as follows.

(i) Defects of a random nature, which appeared in single items and disappeared without any corrective action. Such defects were usually attributed to defects in individual billets.
(ii) Defects of a systematic nature, which appeared due to some special causes related to the rolling process and continued until the proper corrective action was taken.

Defects of type (ii) call for prompt detection, as the longer it takes to detect the defect, the greater is the loss associated with the production of defective items. It was evident that the shorter the inspection interval, more prompt would be the detection. However, the cost of sampling and inspection, which would increase with reduction in the inspection interval, was also an important consideration. Thus, it was important to determine an optimum inspection interval that would minimize the costs associated with the occurrences of defects of type (ii). The case presented here is thus similar to case I as mentioned by Nayebpour & Woodall (1993), where the process shifts from producing no defective items to producing all defective items. Henceforth, by 'defect' we shall mean only a type (ii) defect.

The approach taken to solve this problem, described in Section 2, is similar to the one developed by Nayebpour & Woodall (1993). However, certain aspects of this particular case, as explained in Section 3, required some modifications in the Nayebpour & Woodall (1993) cost function. The optimum sampling interval was obtained by minimizing this cost function. Taguchi's loss function was also optimized and it was seen, as pointed out by Nayebpour & Woodall, that the results were almost the same despite some changes in the basic framework of the problem.

A sensitivity analysis was performed by studying the impact of change of cost ratios on the optimum sampling interval. The results were also used to motivate managers by showing how inspection efforts can be reduced and costs saved if the process is improved.

2 The approach

As in most applications of economic models for designing control schemes, an expression for expected loss per product has been derived as $E(L) = E(C)/E(T)$, where $E(C)$ denotes the cost per production cycle (starting with the beginning of production or after an adjustment and ending with removal of the assignable cause) and $E(T)$ denotes the expected number of units produced per cycle. This is possible since the sequence of production, monitoring and adjustment, with accumulation of losses over the cycle, can be represented by a renewal reward process as described in Ross (1997). The expressions for both $E(C)$ and $E(T)$, as functions of the inspection interval m, have been derived using a geometric PFM after finding that the data on appearance of defects fit well to such a statistical model. The optimum sampling interval has been derived by using a direct search method.

In section 3, we discuss a few aspects of the problem that call for certain modifications in the expressions derived by Nayebpour & Woodall (1993). The analysis of data related to the appearance of defects is presented in Section 4. In Section 5 we derive the expressions for $E(C)$, $E(T)$ and consequently $E(L)$. Section 6 addresses the non-mathematical but perhaps the most difficult part—estimation of the cost components. Optimization of the cost function and a sensitivity analysis with respect to the cost components are described in Sections 7 and 8 respectively.

3 Some aspects of the problem

The problem, although by and large similar to the one addressed in case I by Nayebpour & Woodall (1993), had some minor differences. First, it had been implicitly assumed by Taguchi *et al.* (1981) and Nayebpour & Woodall (1993) that after detection of the defect, the process is stopped, and a search is initiated to detect the assignable cause. This is reflected in the expression for an expected length of a cycle (which is the sum of the expected number of units produced until the defect is detected and the time lag l). However, in this case, detection of a defect would not lead to a stoppage of the process, instead, the mill speed would be reduced, corrective actions would be taken on a trial and error basis, and 100% inspection would be employed until removal of the assignable cause. The number of defective items produced after the detection of the defect was thus a random variable. The cycle length here is thus defined as $U+V$, where U is the length (in terms of number of units) of a cycle from the beginning of production (or after an adjustment) to the point of detection of a defect (including the time lag) and V is the number of defective units produced as an outcome of the trial-and-error based search for special causes.

Secondly, the online inspection consisted mainly of visual checks and hot upset checks (creating a bulge on the sampled portion of a coil by pressing it from both ends at a high temperature and examining the surface closely for defects). However, to confirm whether coils segregated as defective by online inspection would have to be rejected or downgraded, micro testing was done in the laboratory later. Although the issue related to extra inspection cost has been addressed by Nayebpour & Woodall (1993), a separate cost component is considered for this, since the expected number of units to be inspected online and those to be inspected later in the laboratory are not the same.

Instead of three cost components considered by Taguchi *et al*. (1989) and Nayebpour & Woodall (1993), the following four components of cost have been considered for optimization of the inspection interval.

(1) C_{oi}, the cost of sampling and inspecting (online) one unit of products.
(2) C_{mi}, the cost of subjecting one unit of product to micro test in the laboratory.
(3) C_d, the loss caused by producing one unit of defective product.
(4) Adjustment cost expressed as $C_a = C_1 + C_2 + C_r$, where C_1 is the cost of slowing down the process for finding the assignable cause, C_2 is the direct recovery cost including labour, material and equipment and C_r is the retrospective inspection cost. Retrospective inspection was carried out using the procedure recommended by Taguchi *et al*. (1989) of successively inspecting the item halfway in the sequence of items that could contain the first defective item.

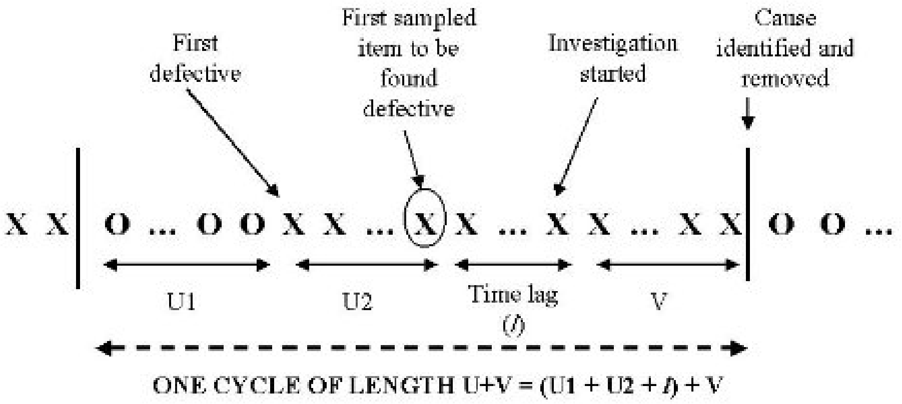

FIG. 1. A production cycle.

4 The distributions of appearance and removal of defects

As stated in the previous section, we define a cycle as a sequence of coils starting with the first defect-free coil after rectification and ending with the last defective coil before rectification (see Fig. 1).

As already stated, let U denote the number of coils produced from the beginning of a cycle to the detection of a defect and V denote the number of defective coils produced as an outcome of the trial-and-error based search for special causes. Both U and V are random variables. The current sampling interval was $m_c = 10$.

As shown in Fig. 1, one may write $U = U_1 + U_2 + l$, where U_1 is the number of coils before the first defective, $U_2 (\leqslant m_c)$ is the number of defective coils produced before the first inspection and l is the time lag.

As shown by Nayebpour & Woodall (1993), under the assumption of a geometric PFM, the maximum likelihood and method of moments based estimator of p_u (probability of a defective coil) is given by

$$\hat{p}_u = 1 - \left(1 - \frac{m_c}{\bar{u} - l}\right)^{1/m_c} \tag{1}$$

where $\bar{u}$ is the estimator of $E(U)$.

However, since through retrospective inspection it was possible to find the exact realized values of U_1, which helped to confirm that the pattern of appearance of defects was indeed geometric (see Fig. 2), p_u was estimated in a simpler way as $\hat{p}_u = 1/u_1 = 0.0139$, u_1 being the estimator of $E(U_1)$. A geometric distribution with $p_u = 0.0139$ gave an excellent fit to the data (the computed value of the Pearson chi-square statistic was 1.814 against the tabulated value 11.1 of $\chi^2_{5,0.05}$).

Data collected on the observed values of V showed that V follows a geometric distribution, with estimated probability of finding a non-defective coil $\hat{p}_v = 0.0857$. Once again, the distribution with $p_v = 0.0857$ gave a very good fit to the data (the computed value of the Pearson chi-square statistic was 3.9497 against the tabulated value 14.067 of $\chi^2_{7,0.05}$).

Data also revealed that an estimate of l can be taken as 4, which meant that l was less than m_c, the current inspection interval.

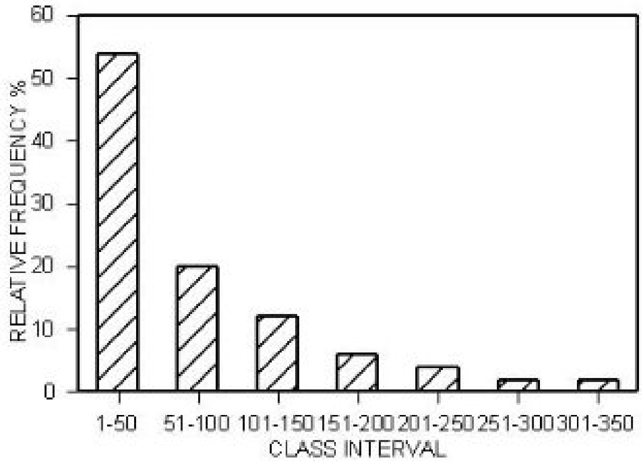

FIG. 2. Distribution of U_l.

5 The expected loss function

We have already seen that the expected loss function is given by $\mathrm{E}(L) = \mathrm{E}(C)/\mathrm{E}(T)$. The expected length of a cycle is given by

$$\mathrm{E}(T) = \mathrm{E}(U) + \mathrm{E}(V) = \frac{m}{1-q_u^m} + l + \frac{1}{p_v} \tag{2}$$

To find $\mathrm{E}(C)$, we must add the expected cost of online inspection per cycle, the expected cost of micro-tests to be done in the laboratory per cycle, the expected cost of defective products per cycle, and the expected cost of adjustment.

The expected cost of online inspection per cycle is

$$\left(\frac{1}{1-q_u^m} + \text{integer}\left(\frac{l}{m}\right) + \frac{1}{p_v}\right) C_{oi} \tag{3}$$

In this case, since $l = 4$ and $m = 10$ means integer$(l/m) = 0$, equation (3) reduces to

$$\left(\frac{1}{1-q_u^m} + \frac{1}{p_v}\right) C_{oi} \tag{4}$$

The expected cost of micro-tests to be done per cycle is given by

$$\left(m - \frac{q_u}{1-q_u} + \frac{mq_u^m}{1-q_u^m} + l + \frac{1}{p_v}\right) C_{ml} \tag{5}$$

The expected cost of defective products per cycle is

$$\left(m - \frac{q_u}{1-q_u} + \frac{mq_u^m}{1-q_u^m} + l + \frac{1}{p_v}\right) C_d \tag{6}$$

The adjustment cost per cycle is given by

$$C_a = C_1 + C_2 + C_r \tag{7}$$

where C_1, C_2, C_r are as defined earlier and $C_r = N_r C_{oi}$, N_r being the expected number of retrospective inspections done per cycle.

Thus, the expected loss per cycle is given by

$$\mathrm{E}(C) = \left(\frac{1}{1-q_u^m} + \frac{1}{p_v}\right)C_{oi} + \left(m - \frac{q_u}{1-q_u} + \frac{mq_u^m}{1-q_u^m} + l + \frac{1}{p_v}\right)(C_{mi} + C_d) + C_a \quad (8)$$

Consequently, the expected loss per product is given by

$$E(L) = E(C)/E(T)$$

$$= \left\{ \frac{\left(\frac{1}{1-q_u^m} + \frac{1}{p_v}\right)C_{oi} + \left(m - \frac{q_u}{1-q_u} + \frac{mq_u^m}{1-q_u^m} + l + \frac{1}{p_v}\right)(C_{mi} + C_d) + C_a}{\left(\frac{m}{1-q_u^m} + l + \frac{1}{p_v}\right)} \right\} \quad (9)$$

6 Estimation of the cost components

Practically speaking, estimation of the cost components is a real difficult task and can never be expected to be accurate. However, as stated by Montgomery (1996), unlike other components of an economic model, the costs need not be estimated with high precision: In fact, it can easily be seen that the ratio $C_{oi}: C_{mi}: C_d: C_a$ is adequate to optimize equation (9), which further strengthens this argument.

The costs of online inspection and micro testing were found by considering the inspectors' salaries, the proportion of time they spent on this specific inspection, the cost of the material wasted for inspection (portions from the sampled coils were cut), the cost of conducting hot upset tests, and so on.

Estimation of C_d posed some problems. It was not necessary that a defective product would be scrapped; in the majority of cases it was downgraded and sold for a different application. This would again depend on the severity of the defect. There were also situations where a product declared as defective by online inspectors would be declared OK by the subsequent micro test. Although the estimation was performed considering these factors to the extent possible, it was felt that C_d might have been slightly overestimated.

C_a had to be calculated for the entire cycle. It had three cost components, out of which C_1, the cost of slowing down the process was estimated as $t_n p_v/(t_s - t_n) \times$ price of a product, where t_n and t_s respectively denote the time taken to produce a coil at normal and slow speed and $1/p_v$ was the expected run length of products before the assignable cause could be established.

C_2, the direct recovery cost could be calculated without much difficulty. C_r, the retrospective inspection cost was calculated as $4C_{oi}$, since 3–4 retrospective inspections had to be performed in every cycle when the current sampling interval was 10, irrespective of where the first defect occurred.

This detailed analysis yielded the following estimates of the four cost components:

$$C_{oi} = \text{Rs}\,21, C_{mi} = \text{Rs}\,35, C_d = \text{Rs}\,138 \text{ and } C_a = \text{Rs}\,180$$

Note that the C_{oi}, C_{mi}, C_d are costs per item whereas C_a is a cost incurred per cycle.

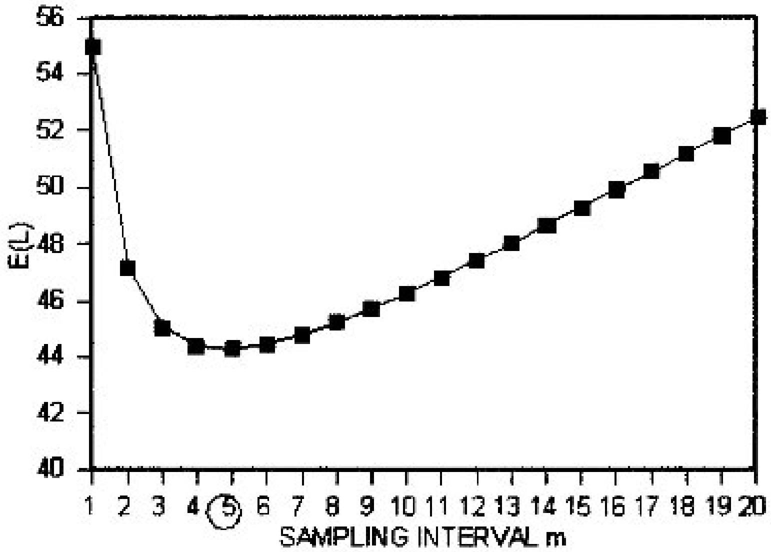

FIG. 3. The expected loss function.

7 Optimization of the cost function

Since, by equating $\delta E(L)/\delta m$ to zero we do not obtain a closed form expression, as done by Nayebpour & Woodall (1993), we optimize the cost function by a direct search method. The plot of $E(L)$ against m is shown in Fig. 3

By the direct search method, it was found that $m = 5$ yielded the minimum cost per cycle. The expected loss per product $E(L)$ for $m = 5$ was Rs 44.29, which was less than the expected loss for the current inspection interval $m = 10$ approximately by Rs 2. Thus, the company could reduce a cost of almost Rs 0.75 million merely by adopting an inspection interval of 5.

Substituting the values of C_{oi}, C_d and $u = 84$ in Taguchi's formulae (Taguchi *et al.*, 1989) for optimum m

$$m^{\star} - \sqrt{\left(\frac{2uC_{oi}}{C_d}\right)} \tag{10}$$

and

$$m^{\star\star} = \sqrt{\left\{\frac{2(u+1)\,C_{oi}}{C_d - \dfrac{C_d}{u}}\right\}} \tag{11}$$

we have $m^{\star} = 5$ and $m^{\star\star} = 5$.

This shows the robustness of Taguchi's results.

8 Sensitivity analysis with respect to the cost components

However diligently the cost components may have been estimated, there is always an element of doubt regarding their accuracy. A sensitivity analysis is thus performed to

TABLE 1. Effects of change of cost estimates on the optimum sampling interval

C_{oi}	C_d	C_{mi}	C_a	Opt m	E(L)	C_{oi}	C_d	C_{mi}	C_a	Opt m	E(L)
16	104	26	135	5	33.34	21	138	35	225	5	44.79
16	104	26	180	5	33.85	21	138	44	135	5	45.67
16	104	26	225	5	34.35	21	138	44	180	5	46.17
16	104	35	135	5	35.22	21	138	44	225	5	46.67
16	104	35	180	5	35.72	21	162	26	135	5	46.92
16	104	35	225	5	36.23	21	162	26	180	5	47.42
16	104	44	135	5	37.10	21	162	26	225	5	47.92
16	104	44	180.	5	37.60	21	162	35	135	4	48.79
16	104	44	225	5	38.10	21	162	35	180	4	49.29
16	138	26	135	4	40.37	21	162	35	225	4	49.80
16	138	26	180	4	40.87	21	162	44	135	4	50.62
16	138	26	225	4	41.38	21	162	44	180	4	51.13
16	138	35	135	4	42.21	21	162	44	225	4	51.63
16	138	35	180	4	42.71	26	104	26	135	6	36.14
16	138	35	225	4	43.21	26	104	26	180	6	36.64
16	138	44	135	4	44.04	26	104	26	225	6	37.14
16	138	44	180	4	44.55	26	104	35	135	6	38.06
16	138	44	225	4	45.05	26	104	35	180	6	38.56
16	162	26	135	4	45.27	26	104	35	225	6	39.06
16	162	26	180	4	46.28	26	104	44	135	6	39.98
16	162	26	225	4	47.10	26	104	44	180	6	40.48
16	162	35	135	4	47.61	26	104	44	225	6	40.98
16	162	35	180	4	48.11	26	138	26	135	6	43.39
16	162	35	225	4	48.94	26	138	26	180	6	43.89
16	162	44	135	4	49.44	26	138	26	225	6	44.39
16	162	44	180	4	49.95	26	138	35	135	5	45.27
16	162	44	225	4	34.81	26	138	35	180	5	45.77
21	104	26	135	6	35.31	26	138	35	225	5	46.27
21	104	26	180	6	35.81	26	138	44	135	5	47.14
21	104	26	225	6	36.70	26	138	44	180	5	47.64
21	104	35	135	5	37.20	26	138	44	225	5	48.14
21	104	35	180	5	37.70	26	162	26	135	5	48.39
21	104	35	225	5	38.20	26	162	26	180	5	48.90
21	104	44	135	5	38.58	26	162	26	225	5	49.40
21	104	44	180	5	39.07	26	162	35	135	5	50.27
21	104	44	225	5	39.58	26	162	35	180	5	50.77
21	138	26	135	5	41.91	26	162	35	225	5	51.28
21	138	26	180	5	42.41	26	162	44	135	5	52.15
21	138	26	225	5	42.92	26	162	44	180	5	52.65
21	138	35	135	5	43.79	26	162	44	225	5	53.15
21	138	35	180	5	44.29						

see the effect of change of these estimates on the optimum value of m. Assuming that very large errors are possible, we obtain E(L) and optimum m by substituting $\pm 25\%$ of the estimated values of the cost components in equation (8). The detailed results are presented in Table 1. As expected, C_d and C_{oi} have a stronger impact on the results than the other two cost components. It is observed that if C_d is underestimated by 25%, or C_{oi}, is overestimated by 25%, then the optimum interval reduces to 4, whereas if the reverse occurs, the optimum interval becomes 6. In the two worst possible situations (both are wrongly estimated by a margin of 25% in opposite directions) the optimum sampling intervals become 4 and 6.

Thus, the methodology is seen to be fairly robust to erroneous estimation of cost components.

9 Conclusions: advantages and disadvantages of the approach

Quite a few authors have discussed the advantages and disadvantages of the economic approach to the design of control schemes. We discuss some of these issues in the context of the current problem. Woodall, in particular, has been critical about the economic approach (Woodall, 1986, 1987) criticizing it from two angles. First, it is seen that most of the economic models of control charts have a high probability of type I error, thereby increasing the probability of false alarms. Secondly, usually economic models assign a cost to passing a defective characteristic, a cost that includes liability claims and customer dissatisfaction costs, and this is counter to Deming's philosophy that these costs cannot be measured and that customer satisfaction is necessary to staying in business.

In the current problem, however, the issue of a false alarm does not arise, as the type of defects dealt with in this exercise would necessarily arise as a consequence of some assignable cause, as discussed in the introductory section. The exercise also does not call for estimation of costs related to passing defective items, since under the scheme of retrospective inspection, no defective item is passed to the customer. There may be a few cases where downgrading of the product is done, but that is with the consent of the customer, who would, anyway, pay a lower price for that product.

It is felt by many researchers and practitioners that the amount of research done in the field of economic models is not justified by the number of practical applications. Montgomery (1996) feels that two major reasons behind the lack of practical implementation of this methodology may be the relative complexity of the mathematical models and their associated optimization and the difficulty in estimation of the cost components. Modern day computing facilities should easily remove the first hindrance. Estimation of cost components, as already discussed in Section 6 and justified by the sensitivity analysis, need not be very precise. Furthermore, most of the companies today have certified quality systems providing a reasonably good framework to perform quality cost analyses.

It is felt that the greatest advantage of the economic model is to convince top management by projecting the benefits in terms of hard cash. As stated by Montgomery (1996), it is usually not known that the arbitrary design of control schemes may often lead to huge economic penalties. A decision to reduce the current inspection interval is difficult to implement unless, and until, the cost benefits are projected and explained logically.

However, as emphasized by Taguchi *et al.* (1989) and echoed by Nayebpour & Woodall (1993), such a study should also pave the way for continuous improvement of the process. The implications of process improvement from the point of view of reduction of cost and administrative convenience may be explained nicely with the help of this model. For example, in the current exercise, p_u and u were estimated as 0.0139 and 84 respectively, which yielded an optimum inspection interval of 5. If the process is improved to achieve $p_u = 0.001$, the defect-free run length becomes 1008, and the optimum inspection interval becomes 16, resulting in a loss of Rs 5.80 per product as compared with Rs 44.29 under the existing situation. Such results can help in motivating managers to set goals for process improvement and understand the financial implications of improvements.

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