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ON THE NOTION OF RESIDUAL FINITENESS FOR *G*-SPACES

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Abstract

We define equivariant completion of a G-complex and define residually finite G-spaces. We show that the group of G-homotopy classes of G-homotopy self equivalences of a finite, residually finite G-complex, is residually finite. This generalizes some results of Roitberg.

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1. Introduction

The notion of profinite completion in group theory is well understood and it is well known that profinite completion of a group is residually finite. The notion of profinite completion of Sullivan [8] in homotopy theory motivated Roitberg to introduce the notion of residual finiteness in the homotopy category [7]. He showed that the profinite completion of a path connected CW-complex is residually finite [7, Theorem 1 (a)]. He further showed that for a finite CW-complex X which is residually finite, $\mathscr{E}(X)$, the pointed homotopy classes of self homotopy equivalences is residually finite [7, Theorem 3]. This is the homotopy theoretic analogue of the well-known result of Baumslag that the automorphism group of a finitely generated residually finite group is residually finite. The aim of this paper is to prove equivariant versions of the above results of Roitberg.

Let G be a finite group and $G\mathscr{H}$ denote the category of G-path connected G-CW-complexes (which we abbreviate to G-complexes) with base point. All maps and homotopies are based. Following Sullivan, we define the profinite completion

$\hat{X}_G$ of a G-complex X (for equivariant completion, generalizing the non equivariant completion of Bousfield-Kan, see [3]). We also introduce the notion of residual finiteness for G-spaces and show that for any $X \in G\mathscr{H}$, the profinite completion $\hat{X}_G$ is residually finite. Let $\mathscr{E}_G(X)$ denote the group of G-homotopy classes of equivariant homotopy self equivalences of X. One of the main results of the paper is

THEOREM 1.1. *Let $X \in G\mathscr{H}$ be finite. Assume that X is residually finite. Then $\mathscr{E}_G(X)$ is a residually finite group.*

Recall that a theorem of Sullivan [9] and Wilkerson [11] says that if X is a nilpotent finite complex, then $\mathscr{E}(X)$ is commensurable with an arithmetic group and hence, is finitely presented. Thus if X is a finite, nilpotent complex which is also residually finite, then $\mathscr{E}(X)$ being residually finite and finitely presented, is Hopfian. The equivariant analogue of the Sullivan-Wilkerson theorem is proved in [10]. We use this to prove

THEOREM 1.2. *If $X \in G\mathscr{H}$ is finite and nilpotent, then $\mathscr{E}_G(X)$ is Hopfian.*

Convention Throughout, G will denote a finite group and all spaces, maps and homotopies are based and '$X \in G\mathscr{H}$ is finite' is meant that X is a finite G-CW-complex.

2. Equivariant completion and residual finiteness

Recall that a space F is *totally finite* if the homotopy groups $\pi_n(F)$, $n \geq 1$ are finite and if in addition there exists a positive integer n_0 such that $\pi_n(F) = 0$ for $n > n_0$. A space is of *finite type* if all its homotopy groups are finitely generated.

A G-space X is *totally finite* if for every subgroup H of G, the H fixed point set X^H is totally finite.

DEFINITION 2.1. A G-space X is *residually finite* if for any finite G-complex W and $\alpha, \beta \in [W, X]_G$, $\alpha \neq \beta$ there exists a G-map $f : X \to Z$ with Z totally finite such that $f_*(\alpha) \neq f_*(\beta)$ where $f_* : [W, X]_G \to [W, Z]_G$ is the map induced by f.

A G-map $f : X \to Y$ between G-spaces is a $\mathbb{F}$-*monomorphism* if for every finite $W \in G\mathscr{H}$ the induced map $f_* : [W, X]_G \to [W, Y]_G$ is a monomorphism.

Here is an example of a residually finite space.

EXAMPLE 2.2. Let $X = S^1 \vee S^1$. Then X can be given the structure of a $\mathbb{Z}_2$-complex as follows. X has one 0-cell of the type $\mathbb{Z}_2/\mathbb{Z}_2$ and one 1-cell of the type $\mathbb{Z}_2/e$. X can then be readily recognized as an equivariant Eilenberg-MacLane space $K(\lambda, 1)$

where λ is the $O_{\mathbb{Z}_2}$-group $\lambda(\mathbb{Z}_2/e) = F_2$, the free group of rank two, and $\lambda(\mathbb{Z}_2/\mathbb{Z}_2)$ is the trivial group. We claim that X is residually finite as $\mathbb{Z}_2$-space. First note that if W is a finite G-complex then

$$[W, K(\lambda, 1)]_G \cong \mathrm{Hom}_{O_G}(\underline{\pi}_1(W), \lambda).$$

(This is true more generally [6]). Now let $\alpha, \beta \in [W, K(\lambda, 1)]_G$ be such that $\alpha \neq \beta$. Then clearly $\alpha(\mathbb{Z}_2/e) \neq \beta(\mathbb{Z}_2/e) : \pi_1(W^e) \to \lambda(\mathbb{Z}_2/e)$. Since F_2 is residually finite there exists a finite group F and a homomorphism $\mu : \lambda(\mathbb{Z}_2/e) \to F$ such that $\mu \circ \alpha(\mathbb{Z}_2/e) \neq \mu \circ \beta(\mathbb{Z}_2/e)$. Define an O_G-group λ' by $\lambda'(\mathbb{Z}_2/e) = F$ and $\lambda'(\mathbb{Z}_2/\mathbb{Z}_2)$ to be the trivial group. Then, the map $\mu : \lambda \to \lambda'$ defined by $\mu(\mathbb{Z}_2/e) = \mu$ and $\mu(\mathbb{Z}_2/\mathbb{Z}_2)$ being the trivial homomorphism, defines a natural transformation. This gives rise to a G-map $h : K(\lambda, 1) \to K(\lambda', 1)$ of equivariant Eilenberg-MacLane spaces. Clearly $h_*(\alpha) \neq h_*(\beta)$. Observe that $K(\lambda', 1)$ is totally finite. Note that X is not nilpotent as a $\mathbb{Z}_2$-space (compare Proposition 2.9).

PROPOSITION 2.3. *If X is residually finite as a G-space, then X^G is residually finite.*

PROOF. Let $\alpha, \beta \in [W, X^G]$, $\alpha \neq \beta$ with W a finite CW-complex. Then endowing W with the trivial G-action, α, β can be considered as elements of $[W, X]_G$ and it is easy to see that $\alpha \neq \beta$, as elements of $[W, X]_G$. Hence there is a totally finite G-space Z and a G-map $f : X \to Z$, such that, $f_*(\alpha) \neq f_*(\beta)$. Then, it follows that $f_*^G(\alpha) \neq f^G(\beta)$. □

We can now construct a G-space X which is residually finite, if one forgets the group action but is not residually finite when considered as a G-space.

EXAMPLE 2.4. Let $G = \mathbb{Z}_2$. Let $f : \mathbb{Q} \to \mathbb{Z}$ denote the only homomorphism between the additive group of rationals and the integers. This map is then realized as a map $f : K(\mathbb{Q}, 1) \to S^1$ of Eilenberg-MacLane spaces. Consider the O_G-space T, defined by, $T(G/G) = K(\mathbb{Q}, 1)$ and $T(G/e) = S^1$, with all structure maps as the identity, except the map $T(\hat{e}) : T(G/G) \to T(G/e)$, which equals f. Then, by the Elmendorf construction [2], there exists a G-space CT, such that, CT has the homotopy type of S^1, whereas CT^G has the homotopy type of $K(\mathbb{Q}, 1)$. Corollary 1 of [7] shows that CT^G is not residually finite, but the underlying space of the G-space CT, is clearly residually finite. It follows from the above proposition that, CT is not residually finite, as a G-space.

We now turn to the definition of equivariant completion. Recall [4, Theorem 3.1, page 134] that, a contravariant functor from $G\mathscr{H}$ to the category of sets, is *representable*, if and only if, it satisfies the Brown's axioms (the wedge and the Mayer-Vietoris axioms). A functor satisfying the wedge and the Mayer-Vietoris axioms will be called a *Brownian functor*. A *compact Brownian functor* is a Brownian functor taking values in compact Hausdorff spaces.

We shall need the following two properties of compact Brownian functors.

(1) Suppose k' is a contravariant functor defined on the subcategory of $G\mathscr{H}$ consisting of finite G-complexes taking values in compact Hausdorff spaces. Suppose that k' satisfies the Brown's axioms, whenever they make sense. Then, there is a unique extension of k' to a compact Brownian functor k, defined by, $k(X) = \operatorname{inv}\lim_\alpha k'(X_\alpha)$, where the inverse limit is over the finite G-subcomplexes X_α of X.

(2) The arbitrary inverse limit of compact Brownian functors, over a small filtering category, is a compact Brownian functor.

The proofs of both these facts are analogous to the nonequivariant case [8, page 36] and are therefore omitted. We shall use the above properties of compact Brownian functors to introduce equivariant completion as follows.

Step 1 For $X \in G\mathscr{H}$, let $\mathscr{F}_X$ denote the category whose objects are G-maps $X \to F$ with F a totally finite G-space and morphisms are homotopy commutative diagrams.

LEMMA 2.5. *$\mathscr{F}_X$ is a small filtering category.*

PROOF. Recall ([8]) that, to show that the category $\mathscr{F}_X$ is small filtering we need to check the smallness, the directedness of $\mathscr{F}_X$ and the essential uniqueness of maps in $\mathscr{F}_X$. The first condition is clear since we can replace $\mathscr{F}_X$ by an equivalent small category, by picking a representative from each G-homotopy type of F's. The second property is also clear as given objects $f_1 : X \to F_1$ and $f_2 : X \to F_2$ in $\mathscr{F}_X$ we can imbed them in $f_1 \times f_2 : X \to F_1 \times F_2$. The essential uniqueness of maps in $\mathscr{F}_X$ follows from the co-equalizer construction in equivariant homotopy theory, which is given by a suitable pushout diagram [4, page 39]. Explicitly, for two morphisms from $\pi' : X \to F'$ to $\pi : X \to F$ in $\mathscr{F}_X$ given by G-maps $f_1, f_2 : F' \to F$, consider the G-space

$$\{(p, x) \in F^I \times F' : p(0) = f_1(x),\ p(1) = f_2(x)\}$$

with diagonal action, where the G-action on F^I is induced by the action on F. Let F'' be the component of the above G-space containing the base point, the base point being the constant path at the base point of F in the first factor and the base point of F' in the second factor. Then, as in the non-equivariant case [4, page 40], we have an exact sequence

$$\cdots \to \pi_i(F'')^H \to \pi_i(F')^H \to \pi_i(F^H) \to \cdots,$$

for every subgroup H of G. From this exact sequence it follows that F'' is a totally finite G-space. Now, one gets the required co-equalizer by using a G-homotopy from $f_1 \circ \pi'$ to $f_2 \circ \pi'$. $\square$

Step 2 Let $Z \in G\mathcal{H}$ be finite and F a totally finite G-space. Then by equivariant obstruction theory [1], it is easy to see that, the homotopy set $[Z, F]_G$ is finite. This yields a contravariant functor defined on the sub category of $G\mathcal{H}$ consisting of finite G-complexes and taking values in compact Hausdorff spaces. A direct verification shows that this functor satisfies the Brown's axioms whenever they make sense. Then by property (1), we get a compact Brownian functor defined by $S_F(Y) = \operatorname{inv\,lim}_\alpha [Y_\alpha, F]_G = [Y, F]_G$, where the inverse limit is taken over the finite G-subcomplexes of Y.

From Step 1 and Step 2 we get a functor on $\mathcal{F}_X$ which assigns to each object $\pi : X \to F$, the compact Brownian functor S_F obtained as in Step 2. By property (2) of compact Brownian functors $\operatorname{inv\,lim}_{\mathcal{F}_X} S_F$ is again a compact Brownian functor, which assigns, to each $Y \in G\mathcal{H}$, the compact Hausdorff space $\operatorname{inv\,lim}_{\mathcal{F}_X}[Y, F]_G$. Therefore, by Brown's representation theorem [4, Theorem 3.1, page 134], there exists a space $\hat{X}_G$ in $G\mathcal{H}$ such that for every G-complex Y there is a bijection

$$[Y, \hat{X}_G]_G \longleftrightarrow \operatorname{inv\,lim}_{\mathcal{F}_X}[Y, F]_G.$$

DEFINITION 2.6. The space $\hat{X}_G$ is called the *equivariant profinite completion* of X.

Clearly, $\hat{X}_G$ comes equipped with a G-map $i : X \to \hat{X}_G$, which is determined by the objects of $\mathcal{F}_X$ and is called the completion map.

We now prove an important property of equivariant completion. First recall that a G-space X is nilpotent if every fixed point set is nilpotent. An equivariant Postnikov decomposition for a G-space B consists of G-maps $\alpha_n : B \to B_n$ and $r_{n+1} : B_{n+1} \to B_n$, $n \geq 0$ such that B_0 is a point and α_n induces an isomorphism $\underline{\pi}_q(B) \to \underline{\pi}_q(B_n)$ for $q \leq n$, $r_{n+1}\alpha_{n+1} = \alpha_n$, and r_{n+1} is the G-fibration over a $K(\underline{\pi}_{n+1}(B), n+2)$ by a map $k^{n+2} : B_n \to K(\underline{\pi}_{n+1}(B), n+2)$. On passage to H-fixed points, a Postnikov system for B gives a Postnikov system for B^H. Moreover, every nilpotent G-space admits a Postnikov decomposition [4, 2].

PROPOSITION 2.7 (Hasse principle). *Let $Y \in G\mathcal{H}$ be finite and $B \in G\mathcal{H}$ be a nilpotent space of finite type. If $f, g : Y \to B$ are G-maps such that $i \circ f$ is G-homotopic to $i \circ g$, then f is G-homotopic to g.*

PROOF. The proof is by induction over the stages in the equivariant Postnikov system of B and is parallel to the nonequivariant case. Let $K \to B_{n+1} \to B_n$ be a part of the equivariant Postnikov decomposition of B (see [4, 2]), where $K = K(\underline{\pi}, n+1)$ and $\underline{\pi} = \underline{\pi}_{n+1}(B_{n+1})$. Suppose $f_n : Y \to B_n$ and $f_{n+1} : Y \to B_{n+1}$ are the G-maps constructed from f. Now consider the G-fibration

$$\operatorname{Map}(Y, K) \to \operatorname{Map}(Y, B_{n+1}) \xrightarrow{r} \operatorname{Map}(Y, B_n)$$

with the obvious action on the function spaces so that

$$\mathrm{Map}(Y, B_{n+1})^G = \mathrm{Map}_G(Y, B_{n+1}).$$

We then have an ordinary fibration

$$\mathrm{Map}_G(Y, K) \to \mathrm{Map}_G(Y, B_{n+1}) \xrightarrow{r} \mathrm{Map}_G(Y, B_n).$$

Consider the homotopy exact sequence of the above fibration

$$\cdots \to \pi_1(\mathrm{Map}_G(Y, B_n), f_n) \xrightarrow{I} \pi_0(\mathrm{Map}_G(Y, K), f_{n+1})$$
$$\xrightarrow{\tilde{f}_{n+1}} \pi_0(\mathrm{Map}_G(Y, B_{n+1}), f_{n+1}) \xrightarrow{r} \pi_0(\mathrm{Map}_G(Y, B_n), f_n).$$

Note that $\pi_0(\mathrm{Map}_G(Y, K), f_{n+1}) = H_G^{n+1}(Y; \underline{\pi})$ where $H_G^{n+1}(Y; \underline{\pi})$ denotes the Bredon cohomology group with coefficients in the O_G-group $\underline{\pi}$ [1]. Here $\tilde{f}_{n+1}$ denotes the map given by the action of $H_G^{n+1}(Y, \underline{\pi})$ on $(f_{n+1}) \in \pi_0(\mathrm{Map}_G(Y, K), f_{n+1})$ obtained by equivariant obstruction theory [5]. Clearly, the image $I = I(f_{n+1})$ is the isotropy subgroup of the point (f_{n+1}) and the map r collapses the orbits of the action of $H_G^{n+1}(Y, \underline{\pi})$. Thus we get an exact sequence

$$0 \to I(f_{n+1}) \to H_G^{n+1}(Y, \underline{\pi}) \to \mathrm{orbit}(f_{n+1}) \to 0.$$

We proceed as in the non-equivariant case and repeat the above argument for maps into completions $\widehat{B}_G$, to get a ladder whose top row being the above exact sequence, the base row being the exact sequence

$$0 \to I(\hat{f}_{n+1}) \to H_G^{n+1}(Y, \hat{\underline{\pi}}) \to \mathrm{orbit}(\hat{f}_{n+1}) \to 0,$$

and with induced maps $c_0 : I(f_{n+1}) \to I(\hat{f}_{n+1}), c : H_G^{n+1}(Y, \underline{\pi}) \to H_G^{n+1}(Y, \hat{\underline{\pi}})$ and $c_1 : \mathrm{orbit}(f_{n+1}) \to \mathrm{orbit}(\hat{f}_{n+1})$. Here, the O_G-group $\hat{\underline{\pi}}$ is defined by the group completion $\hat{\underline{\pi}}(G/H) = \widehat{\underline{\pi}(G/H)}$. Also note that by property (1) of compact Brownian functor the map $c : H_G^{n+1}(Y, \underline{\pi}) \to H_G^{n+1}(Y, \hat{\underline{\pi}})$, is a finite completion. With this at our disposal the rest of the proof is exactly similar to the non-equivariant case. □

Equivariant completion yields, as in the nonequivariant case ([7, Theorem 1]), examples of residually finite spaces.

PROPOSITION 2.8. *If $X \in G\mathscr{H}$, then $\hat{X}_G$ is residually finite.*

Suppose that $f : X \to Y$ is a G-map with Y residually finite. If f is a $\mathbb{F}$-monomorphism, then X is residually finite. The Hasse principle implies that if $X \in G\mathscr{H}$ is nilpotent and of finite type, then the completion map $i : X \to \hat{X}_G$ is a $\mathbb{F}$-monomorphism. Both these facts put together imply

PROPOSITION 2.9. *If $X \in G\mathscr{H}$ is nilpotent and of finite type, then X is residually finite.*

3. Proof of the main theorem

In this section we prove our main theorem which gives a sufficient condition for the group $\mathcal{E}_G(X)$ to be Hopfian. The main step in proving this (as in the non-equivariant case) is showing that, under suitable conditions, the group $\mathcal{E}_G(X)$ is residually finite.

DEFINITION 3.1. Let $[f] : X \to Y$ be a morphism in $G\mathcal{H}$. f is said to *represent an epimorphism in* $G\mathcal{H}$ if for any two maps $\alpha, \beta : Y \to Z$ in $G\mathcal{H}$, $\alpha \circ f$ is G-homotopic to $\beta \circ f$ implies α is G-homotopic to β.

Suppose that X and Y_0 are in $G\mathcal{H}$ and $[X, Y_0]_G = \{[f_1], \ldots, [f_r]\}$. Define $Y = Y_0 \times \cdots \times Y_0$ with r factors. Then Y is a G-complex with the diagonal G action. Consider the G-map $f : X \to Y$ by $f = (f_1, \ldots, f_r)$. Let $M(Y)$ denote the monoid of equivariant self homotopy equivalences of Y preserving the base point. Each element of the symmetric group S_r induces a self map of the G-space Y by permuting its coordinates. This gives an embedding of S_r into $M(Y)$.

LEMMA 3.2. *With the above notation, if $e : X \to X$ represents an epimorphism in $G\mathcal{H}$, then e determines a unique $\sigma \in S_r \subseteq M(T)$ such that $f \circ e$ is G-homotopic to $\sigma \circ f$. The assignment $e \mapsto \sigma$ induces a monoid homomorphism $\psi : E(X) \to S_r \subseteq M(T)$, where $E(X)$ is the monoid of equivariant self epimorphisms of the G-space X.*

PROOF OF THEOREM 1.1. Let $\theta \in \mathcal{E}_G(X)$, $\theta \neq id$. We shall exhibit a homomorphism $\eta : \mathcal{E}_G(X) \to F$ with F a finite group such that $\eta(\theta) \neq id$. Since X is residually finite, we have a map $f : X \to Y_0$ of with Y_0 totally finite such that $f_*(\theta) \neq f_*(id)$. Since X is finite and Y_0^H is totally finite one observes using equivariant obstruction theory [1] that the equivariant homotopy set $[X, Y_0]_G$ is finite. Thus by Lemma 3.2 there is a $r > 1$ and a $\sigma \in S_r \subseteq M(Y_0)$ such that $f \circ \theta$ is G-homotopic to $\sigma \circ f$ and $f_*(\theta) \neq f_*(id)$. Hence $\sigma \neq 1$. Now the monoid homomorphism $\psi : E(X) \to S_r$ of Lemma 3.2 restricted to $M(X)$ induces a group homomorphism $\eta : \mathcal{E}_G(X) \to S_r$ such that $\eta(\theta) \neq id$. This completes the proof. $\square$

PROOF OF THEOREM 1.2. Recall that by Proposition 2.9, X is residually finite. Thus $\mathcal{E}_G(X)$ is a residually finite group. Moreover it follows from the work of Triantafillou [10, Theorem 1.2] that $\mathcal{E}_G(X)$ is commensurable with an arithmetic subgroup of $\mathcal{E}_G(X_0)$, where X_0 is the equivariant rationalisation of X. Thus $\mathcal{E}_G(X)$ is finitely generated. The theorem now follows as finitely generated residually finite groups are Hopfian. This completes the proof. $\square$

There are situations where it is not difficult to recognize the group $\mathcal{E}_G(X)$ as being residually finite.

EXAMPLE 3.3. Let λ be an O_G-group. Let $n \geq 1$. If $n > 1$, then λ is abelian. Then if λ has the property that $\lambda(G/H)$ is finitely generated residually finite group for all subgroups H, then it is not difficult to see that $\mathcal{E}_G(X)$ is residually finite where X is the equivariant Eilenberg-MacLane space $K(\lambda, n)$.

EXAMPLE 3.4. As another example, suppose that $X \in G\mathcal{H}$ is a finite nilpotent space such that for any G-homotopy equivalence $f : X \to X$ which is not G-homotopic to identity, there exists a subgroup H of G such that $f^H : X^H \to X^H$ is not homotopic to the identity. Then $\mathcal{E}_G(X)$ is residually finite (compare Proposition 3.5).

We end with the following

PROPOSITION 3.5. *Suppose $X \in G\mathcal{H}$ is a finite and nilpotent. Further assume that for each subgroup H, K of G*

(1) $[X^K, X^H]$ *is a group and*
(2) $[X^K, \Omega^n X^H]$ *is trivial for $n \geq 1$.*

Then $\mathcal{E}_G(X)$ is residually finite.

PROOF. First note that for every subgroup H of G, X^H is nilpotent of finite type and hence X^H is residually finite [7]. Now let $[f] \in \mathcal{E}_G(X)$ such that $[f] \neq [id]$. Then there exists a subgroup H of G such that $[f^H] \neq [id]$, otherwise, by [2, Theorem 3], the natural family $\{[f^H]\}$ would correspond to $id : X \to X$ and this would mean $f \simeq_G id$. The group $\mathcal{E}(X^H)$ is residually finite by [7, Theorem 3]. Using the obvious homomorphism $\mathcal{E}_G(X) \to \mathcal{E}(X^H)$ one sees that the group $\mathcal{E}_G(X)$ is also residually finite. This completes the proof. □

COROLLARY 3.6. *Suppose $X \in G\mathcal{H}$ is a finite and nilpotent. Moreover suppose that the G-action on X is free outside the base point. Then $\mathcal{E}_G(X)$ is residually finite.*

EXAMPLE 3.7. Let $X = S^2 \vee S^2$. Then X can be given a $\mathbb{Z}_2$-complex structure by interchanging the copies of S^2. Then X satisfies the hypothesis of the corollary and hence $\mathcal{E}_G(X)$ is residually finite. It is easy to see that this group is non-zero.

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