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Minimum Distance Estimation for Some Stochastic Partial Differential Equations

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ABSTRACT

Asymptotic properties of minimum distance estimators for the parameter involved for a class of stochastic partial differential equations are investigated following the techniques in Kutoyants and Pilibossian (1994).

Keywords. Minimum distance estimation, stochastic partial differential equation.

AMS 2000 subject classifications. Primary 62M40; Secondary 60H15.

1. Introduction

In their recent monograph, Kallianpur and Xiong (1995) discuss the properties of solutions of stochastic partial differential equations (SPDE's). They indicate that SPDE's are being used for stochastic modelling for instance in the study of neuronal behaviour in neurophysiology and in building stochastic models of turbulence. The theory of SPDE's is investigated in Ito (1984) and more recently in Rozovskii (1990) and Da Prato and Zabczyk (1992). Huebner *et al.* (1993) started the investigation of maximum likelihood estimation of parameters of two types of SPDE's and extended their results for a class of parabolic SPDE's in Huebner and Rozovskii (1995). Bayes estimation of parameters for such classes of SPDE are discussed in Prakasa Rao (2000).

One can construct maxiumum likelihood estimators (MLE) in the models for SPDE discussed in Sections 2 to 4 and it is known that these estimators are consistent and asymptotically normal and asymptotically efficient as the amplitude of the noise ε decreases to zero or the time of observation T increases to infinity. In spite of having such good properties, the maximum likelihood estimators have some short comings at the same time. Their calculation is often

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cumbersome as the expressions for MLE involve stochastic integrals which need good approximants for computational purposes. Furthermore MLE are not robust in the sense that a slight perturbation in the noise component say from a Wiener process to another Gaussian process with finite variation will change the properties of the MLE substantially. In order to circumvent these problems, the minimum distance approach is proposed. Properties of the minimum distance estimators (MDE) were discussed in Millar (1984) in a general frame work.

Our aim in this paper is to obtain the minimum distance estimators of parameters for some class of SPDE's and investigate the asymptotic properties of such estimators following the work of Kutoyants and Pilibossian (1994)(*cf.* Kutoyants, 1994) for the estimation of a parameter of the Ornstein-Uhlenbeck process.

2. Parabolic Stochastic PDE

Let $(\Omega, \mathcal{F}, P)$ be a probability space and consider a stochastic partial differential equation (SPDE) of the form

$$du_\varepsilon^\theta(t,x) = A^\theta u_\varepsilon^\theta(t,x)dt + \varepsilon dW(t,x),\ 0 \le t \le T,\ x \in G \tag{2.1}$$

where $A^\theta = \theta A_1 + A_0$, A_1 and A_0 being partial differential operators, $\theta \in \Theta \subset R$ and $W(t,x)$ is a cylindrical Brownian motion in $L_2(G)$, G being a bounded domain in R^d with the boundary ∂G as a C^∞-manifold of dimension $(d-1)$ and locally G is totally on one side of ∂G. For the definition of cylindrical Brownian motion, see, Kallianpur and Xiong (1995, p. 93).

The order $Ord(A)$ of a partial differential operator A is defined to be the order of the highest partial derivative in A. Let m_0 and m_1 be the orders of the operators A_0 and A_1 respectively. We assume that the operators A_0 and A_1 commute and m_1 is even.

Suppose the solution $u_\varepsilon^\theta(t,x)$ of (2.1) has to satisfy the boundary conditions

$$u_\varepsilon^\theta(0,x) = u_0(x), \tag{2.2}$$

$$D^\gamma u_\varepsilon^\theta(t,x)|_{\partial G} = 0 \tag{2.3}$$

for all multiindices γ such that $|\gamma| = m-1$ where $2m = \max(m_1, m_0)$. Here

$$D^\gamma f(\mathbf{x}) = \frac{\partial^{|\gamma|}}{\partial x_1^{\gamma_1} \cdots \partial x_d^{\gamma_d}} f(\mathbf{x}) \tag{2.4}$$

with $|\gamma| = \gamma_1 + \cdots + \gamma_d$. Suppose that

$$A_i(\mathbf{x})u = -\sum_{|\alpha|,|\beta| \le m_i} (-1)^{|\alpha|} D^\alpha (a_i^{\alpha\beta}(\mathbf{x}) D^\beta u) \tag{2.5}$$