Seiberg Witten Map and the Axial Anomaly in Noncommutative Field Theory

Rabin Banerjee [1]

S. N. Bose National Centre for Basic Sciences,
Block JD, Sector III, Bidhannagar, Kolkata 700098, India.

and

Subir Ghosh [2]

Physics and Applied Mathematics Unit,
Indian Statistical Institute, 203 B. T. Road, Kolkata 700035, India.

Abstract :
Using the point-splitting regularisation, we calculate the axial anomaly in an arbitrary even dimensional Non-Commutative (NC) field theory. Our result is (star) gauge invariant in its *unintegrated* form, to the leading order in the NC parameter. Exploiting the Seiberg Witten map, this result gets transformed to the familiar Adler-Bell-Jackiw anomaly in ordinary space-time. Furthermore, using this map, we derive an expression for the unintegrated axial anomaly for constant fields in NC space-time, that is valid to all finite orders of the NC parameter.

Keywords: Axial anomaly, noncommutative field theory, Seiberg-Witten map

The subject of anomalies occurring in gauge theories on ordinary space-time has a long history [1]. Perhaps the most familiar of these is the axial $U(1)$ anomaly [2], which usually goes by the name of Adler-Bell-Jackiw (ABJ) [3] anomaly. Ever since the importance of Non-Commutative (NC) manifolds was realised [4] , it has been natural to investigate the structure of anomalies in such a setting. Various results have been reported [5] in this context. The structure of the axial anomaly in the NC spacetime is expected to play a pivotal role just as the ABJ anomaly does in ordinary space-time. We feel however that the study of this NC axial anomaly is *incomplete* and a reappraisal of the problem is necessary. This is principally because expressions for the NC anomaly are given only in their *integrated* versions ($\int d^4x \ < \partial_\mu \hat{j}^{\mu(5)} >$) and not in their basic unintegrated forms. Strictly speaking, therefore, the analogue of the gauge invariant ABJ anomaly in usual spacetime with the corresponding $*$-gauge invariant anomaly in the NC spacetime is missing.

In this paper we first show that, by properly accounting for the various divergences, a $*$-gauge invariant result, up to the first non-trivial order in θ (the noncommutative parameter), can be obtained for the $U(1)$ anomaly in its *unintegrated* form. The computation is done for any even $d = 2n$ dimensions. It should be stressed that the present analysis has been done only till the first nontrivial order in θ. For other computations in the NC spacetime that have been done for the first nontrivial order in θ, we refer to [6].

Now it is known that the Seiberg Witten (SW) map [7, 8] connects gauge equivalent sectors of NC theory to its conventional counterpart. Since our result for the unintegrated anomaly is $*$-gauge invariant, we expect that an appropriate application of the SW map should transform this expression to the usual gauge invariant ABJ anomaly. This is indeed so, as proved explicitly for two and four dimensions.

Once we are convinced that it is possible to get a $*$-gauge invariant result for the anomaly, which is connected to the usual ABJ anomaly by the SW map, we can go further and exploit the inverse of this map to get the unintegrated NC anomaly from the conventional result, valid to all finite orders in θ, albeit for a constant field tensor. This is because the general SW map obeys such a condition [7].

We have also discussed another feature related to anomalies. It is shown that a redefinition of the axial current is possible so that it has a vanishing divergence (zero anomaly). This modified current is, however, no longer $*$-gauge invariant. This is the exact analogue of what happens in usual space-time.

To fix our notations and definitions, consider the ABJ anomaly [3] in usual space-time,

$$< \partial_\mu j^{\mu(5)} > \equiv \mathcal{A} = -\frac{g^2}{16\pi^2}\epsilon_{\mu\nu\lambda\rho}F^{\mu\nu}F^{\lambda\rho}, \tag{1}$$

where $j^{\mu(5)} = \bar{\psi}\gamma^\mu\gamma^5\psi$ is the fermionic current and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field tensor. Obviously the anomaly is gauge invariant under the usual Gauge Transformations (GT) $\delta A_\mu = -\partial_\mu \Lambda$.

Now the corresponding expressions for the field tensor and GT in NC space-time are given by [4] [3]

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + ig[\hat{A}_\mu, \hat{A}_\nu]_*$$

[3] We denote all variables in NC space-time by a *Caret*.

$$\delta \hat{A}_\mu = -\partial_\mu \hat{\Lambda} + ig[\hat{\Lambda}, \hat{A}_\mu]_*, \tag{2}$$

where g denotes the gauge coupling and the Moyal $(*)$ bracket is defined as

$$[A, B]_* = A * B - B * A,$$

$$A(x) * B(x) = e^{\frac{i\theta_{\mu\nu}}{2}\frac{\partial}{\partial \eta_\mu}\frac{\partial}{\partial \zeta_\mu}} A(x+\eta)B(x+\zeta) \mid_{\zeta=\eta=0} = AB + \frac{i}{2}\theta^{\mu\nu}\partial_\mu A \partial_\nu B + O(\theta^2).$$

It is easy to see that $\hat{F}_{\mu\nu}$ transforms covariantly under the $*$-GT (2),

$$\delta \hat{F}_{\mu\nu} = ig[\hat{\Lambda}, \hat{F}_{\mu\nu}]_*. \tag{3}$$

The general consensus is that, adopting a $*$-gauge invariant regularization, the $U(1)$ anomaly in NC space-time can be computed paralleling the usual ABJ analysis. However, in order to preserve the $*$- gauge invariance, it is essential to perform an integration over space-time. This leads to an integrated expression of the axial anomaly [5],

$$\int d^4x \; \hat{\mathcal{A}} = \int d^4x \; < \partial_\mu \hat{j}^{\mu(5)} > = -\frac{g^2}{16\pi^2}\int d^4x \; \epsilon_{\mu\nu\lambda\rho}\hat{F}^{\mu\nu} * \hat{F}^{\lambda\rho}. \tag{4}$$

where the fermionic current is defined by a $*$-multiplication, $\hat{j}^{\mu(5)} = i\bar{\psi}\gamma^\mu\gamma^5 * \psi$.

Obviously, in the unintegrated version, the anomaly is no longer $*$- gauge invariant, because of the covariant transformation law (3) for $\hat{F}^{\mu\nu}$. To proceed with our computations of the $*$-gauge invariant anomaly, consider the following lagrangean [5]

$$\mathcal{L} = i\hat{\bar{\psi}}\gamma^\mu * (\partial_\mu + ig\hat{A}_\mu) * \hat{\psi}. \tag{5}$$

The axial current in any $d = 2n$ space-time dimensions is point-split regularized as

$$\hat{j}_\mu^{(2n+1)}(x, \epsilon) = \hat{\bar{\psi}}(x_+)\gamma_\mu\gamma^{2n+1} * \hat{U}(x_+, x_-) * \hat{\psi}(x_-), \tag{6}$$

where $x_\pm = x \pm \frac{\epsilon}{2}$ and the Schwinger line integral,

$$\hat{U}(x_+, x_-) = (e^{-ig\int_{x_-}^{x_+} dz^\mu \hat{A}_\mu(z)})_* \approx 1 - ig\epsilon^\mu \hat{A}_\mu + -\frac{g^2}{2}(\epsilon.\hat{A}) * (\epsilon.\hat{A}) + O(\epsilon^3),$$

is inserted to preserve the invariance of (6) under the following $*$-GT,

$$\hat{\psi} \to (e^{ig\hat{\Lambda}})_* * \hat{\psi},$$

$$\hat{U}(x_+, x_-) \to (e^{ig\hat{\Lambda}(x_+)})_* * \hat{U}(x_+, x_-) * (e^{ig\hat{\Lambda}(x_-)})_*.$$

The equations of motion following from (5) are

$$\gamma^\mu(\partial_\mu\hat{\psi} + ig\hat{A}_\mu) * \hat{\psi} = 0 \;, \quad (\partial_\mu\hat{\bar{\psi}} - ig\hat{\bar{\psi}} * \hat{A}_\mu)\gamma^\mu = 0. \tag{7}$$

Taking the divergence of the current ([6]) and using the above equations of motion, we find that the vacuum expectation value, valid for the first nontrivial order in θ, is given by,

$$< \partial_\mu \hat{j}^{\mu(2n+1)} >= \hat{\mathcal{A}} \equiv \hat{\mathcal{A}}_1 + \hat{\mathcal{A}}_2, \tag{8}$$

where

$$\hat{\mathcal{A}}_1 =< \bar{\hat{\psi}}(x_+)\gamma_{(2n+1)}\gamma_\mu \hat{\psi}(x_-) > [\{ig\hat{U}(\hat{A}^\mu(x_-) - \hat{A}^\mu(x_+)) - \partial^\mu \hat{U}\} + g\theta^{\alpha\beta}\partial_\beta \hat{U}\partial_\alpha \hat{A}^\mu],$$

$$\hat{\mathcal{A}}_2 = \theta^{\alpha\beta}\partial_\beta[< \partial_\alpha \bar{\hat{\psi}}(x_+)\gamma_{(2n+1)}\gamma_\mu \hat{\psi}(x_-) > \{g(\hat{A}^\mu(x_+) - \hat{A}^\mu(x_-)) - i\partial^\mu \hat{U}\}]. \tag{9}$$

In the above expressions, anti-symmetry of $\theta^{\mu\nu}$ has been used. It can be verified that only linear and up to quadratic divergences survive in the VEVs in $\hat{\mathcal{A}}_1$ and $\hat{\mathcal{A}}_2$ respectively. The presence of $\gamma_{(2n+1)}$ and the nature of the terms besides the VEVs conspire to produce the above result. Thus, regarding the factors multiplying the VEVs, in $\hat{\mathcal{A}}_1$ only terms up to $O(\epsilon)$ are retained whereas in $\hat{\mathcal{A}}_2$, terms up to $O(\epsilon^2)$ also contribute. This should be contrasted with the point splitting calculation of axial anomaly in the usual case where only $O(\epsilon)$ terms are necessary [3]. Note that in the NC anomaly computations in the literature [5], the $\hat{\mathcal{A}}_2$-type of term being a surface term, is driven away by the overall space-time integration. Herein lies the crucial difference between our results and the usual analysis where the integrated anomaly is computed.

We provide some details of the computation of $\hat{\mathcal{A}}_1$. The factor in the parenthesis yields $F^{\nu\mu}\epsilon_\nu$, while the VEV leads to a trace so that,

$$\hat{\mathcal{A}}_1 = -g\hat{F}^{\nu\mu}\epsilon_\nu \; tr[\gamma_{(2n+1)}\gamma_\mu \int \frac{d^{2n}p}{(2\pi)^{2n}} \frac{e^{i\epsilon.p}}{\gamma^\sigma(p_\sigma - g\hat{A}_\sigma)}]. \tag{10}$$

The propagator in NC space-time is defined as

$$(\gamma^\mu(p_\mu - g\hat{A}_\mu))^{-1} = (\gamma^\mu(p_\mu - g\hat{A}_\mu)) * (\gamma^\nu(p_\nu - g\hat{A}_\nu))^{-1} * (\gamma^\lambda(p_\lambda - g\hat{A}_\lambda))^{-1}. \tag{11}$$

This is nothing but the conventional definition of the propagator in ordinary space-time, with the products replaced by $*$-products and $A_\mu \to \hat{A}_\mu$. Using the identity

$$(\gamma^\mu(p_\mu - g\hat{A}_\mu)) * (\gamma^\nu(p_\nu - g\hat{A}_\nu)) = (g^{\mu\nu} + \frac{1}{2}\sigma^{\mu\nu})(p_\mu - g\hat{A}_\mu) * (p_\nu - g\hat{A}_\nu)$$

$$= (p_\mu - g\hat{A}_\mu)(p^\mu - g\hat{A}^\mu) + \frac{i}{4}g(\sigma.\hat{F}) + O(\theta^2), \tag{12}$$

where $\sigma.\hat{F} = [\gamma_\mu, \gamma_\nu]\hat{F}^{\mu\nu}$, and expanding the propagator in powers of $(p^2)^{-1}$, one finds that a single term from the expansion survives, leading to,

$$\hat{\mathcal{A}}_1 = -g\hat{F}^{\nu\mu}\epsilon_\nu \; tr[\gamma_{(2n+1)}\gamma_\mu\gamma_\lambda \int \frac{d^{2n}p \; p^\lambda}{(2\pi)^{2n}p^2}(\frac{-ig\sigma.\hat{F}}{4p^2})^{n-1}]e^{i\epsilon.p}. \tag{13}$$

The remaining terms in the perturbative expansion of the propagator do not contribute in $\hat{\mathcal{A}}_1$ either due to the trace properties of $\gamma_{(2n+1)}$ or on finally taking the $\epsilon \to 0$ limit. Using the momentum integral,

$$\int \frac{d^{2n}p}{(p^2)^n} e^{i\epsilon.p} = i \frac{\pi^n}{(n-1)!} \; ln \mid \epsilon \mid^2,$$

we arrive at

$$\hat{\mathcal{A}}_1 = \frac{(-i)^{n+1} g^n}{4^{2n-1}\pi^n \; (n-1)!} \; tr[\gamma_{2n+1}\gamma_\mu\gamma_\nu(\sigma.\hat{F})^{n-1}]\hat{F}^{\lambda\mu} \; 2\frac{\epsilon_\lambda \epsilon^\nu}{\mid \epsilon \mid^2}. \tag{14}$$

Finally, taking the trace of the γ-matrices and incorporating the symmetric limit $\frac{\epsilon_\mu \epsilon_\nu}{\epsilon^2} \mid_{\epsilon \to 0} = \frac{1}{2n}\delta_{\mu\nu}$, we obtain

$$\hat{\mathcal{A}}_1 = \frac{(-1)^{n+1}}{2^{2n-1}n!}(\frac{g}{\pi})^n \epsilon_{\mu_1\nu_1...\mu_n\nu_n}\hat{F}_{\mu_1\nu_1}...\hat{F}_{\mu_n\nu_n}. \tag{15}$$

Now we need to compute $\hat{\mathcal{A}}_2$ in (9) where the VEV is qualitatively different from that of $\hat{\mathcal{A}}_1$. This is because here one of the extra derivatives that have appeared from the θ term acts on the $\hat{\psi}$ which in the momentum integral generates an extra momentum thus making this VEV quadratically divergent as compared to the VEV in $\hat{\mathcal{A}}_1$ in (9). The rest of the computation follows identical steps as above and using the momentum integral,

$$\int \frac{d^{2n}p}{(p^2)^{n+1}} e^{i\epsilon.p} = -i\frac{\pi^n}{4(n)!} \mid \epsilon \mid^2 \; (ln \mid \epsilon \mid^2 -1),$$

we get,

$$\hat{\mathcal{A}}_2 = -g\frac{(-1)^{n+1}}{2^{2n-1}n!}(\frac{g}{\pi})^n \epsilon_{\mu_1\nu_1...\mu_n\nu_n}\theta^{\alpha\beta}\partial_\beta(\hat{A}_\alpha\hat{F}_{\mu_1\nu_1}...\hat{F}_{\mu_n\nu_n}). \tag{16}$$

Hence the cherished expression of the NC axial anomaly is

$$\hat{\mathcal{A}} = \hat{\mathcal{A}}_1 + \hat{\mathcal{A}}_2 = (-1)^{n+1}\frac{1}{2^{2n-1}n!}(\frac{g}{\pi})^n \epsilon_{\mu_1\nu_1...\mu_n\nu_n}[\hat{F}_{\mu_1\nu_1}...\hat{F}_{\mu_n\nu_n} - g\theta^{\alpha\beta}\partial_\beta(\hat{A}_\alpha\hat{F}_{\mu_1\nu_1}...\hat{F}_{\mu_n\nu_n})]. \tag{17}$$

The second term $\hat{\mathcal{A}}_2$, being a total derivative, is absent in the integrated anomaly expression in [5]. Indeed, it is precisely this piece that renders $\hat{\mathcal{A}}$ $*$-gauge invariant, without the space-time integral prescription,

$$\hat{\delta}\hat{\mathcal{A}} = \hat{\delta}\hat{\mathcal{A}}_1 + \hat{\delta}\hat{\mathcal{A}}_2 = 0. \tag{18}$$

Next we relate our result for the anomaly (17) in NC space-time with the usual ABJ anomaly (1) by using the SW map, which provides, to $O(\theta)$, the following identifications between variables in NC and usual space-time,

$$\hat{A}_\mu = A_\mu + \frac{g}{2}\theta^{\alpha\beta}A_\alpha(2\partial_\beta A_\mu - \partial_\mu A_\beta) + O(\theta^2),$$

$$\hat{F}_{\mu\nu} = F_{\mu\nu} - g\theta^{\alpha\beta}(F_{\mu\alpha}F_{\nu\beta} - A_\alpha\partial_\beta F_{\mu\nu}) + O(\theta^2). \tag{19}$$

The SW transformation maps $*$-gauge invariant expressions (in NC space-time) to gauge invariant expressions (in usual space-time). Hence it is expected that our result (17) for the NC anomaly will reduce, through the SW map, to the anomaly in the usual space-time. This will be explicitly shown for $1+1$ and $3+1$ dimensions.

From (17), the two dimensional NC anomaly is,

$$\hat{\mathcal{A}}_2 = \frac{g}{2\pi}\epsilon_{\mu\nu}[\hat{F}^{\mu\nu} - g\theta^{\alpha\beta}\partial_\beta(\hat{A}_\alpha\hat{F}^{\mu\nu}] + O(\theta^2). \tag{20}$$

Substituting the SW relations from (19) we get,

$$\begin{aligned}\hat{\mathcal{A}} \to \mathcal{A} &= \frac{g}{2\pi}\epsilon_{\mu\nu}F^{\mu\nu} - \frac{g^2}{2\pi}\epsilon_{\mu\nu}\theta_{\alpha\beta}[F^{\mu\alpha}F^{\nu\beta} - A^\alpha\partial^\beta F^{\mu\nu} + \partial^\beta(A^\alpha F^{\mu\nu})] \\ &= \frac{g}{2\pi}\epsilon_{\mu\nu}F^{\mu\nu} - \frac{g^2}{2\pi}\epsilon_{\mu\nu}\theta_{\alpha\beta}[F^{\mu\alpha}F^{\nu\beta} + \frac{1}{2}F^{\beta\alpha}F^{\mu\nu}]. \end{aligned}\tag{21}$$

Exploiting the fact that in two dimensions $\theta_{\mu\nu}$ and $\epsilon_{\mu\nu}$ are proportional, we can use the following identity,

$$\theta_{\alpha\beta}\epsilon_{\mu\nu} \equiv \theta\epsilon_{\alpha\beta}\epsilon_{\mu\nu} = \theta(\delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\nu}\delta_{\beta\mu}).$$

It is easy to see that the θ-terms drop out leaving the usual anomaly,

$$\mathcal{A} = \frac{g}{2\pi}\epsilon_{\mu\nu}F^{\mu\nu}. \tag{22}$$

On the other hand, establishing the above mapping for four dimensional space-time is more complicated. The four dimensional NC anomaly from (17) is

$$\hat{\mathcal{A}} = -\frac{g^2}{16\pi^2}\epsilon_{\mu\nu\rho\lambda}[\hat{F}^{\mu\nu}\hat{F}^{\rho\lambda} - g\theta_{\alpha\beta}\partial^\beta(\hat{A}^\alpha\hat{F}^{\mu\nu}\hat{F}^{\rho\lambda})]. \tag{23}$$

Again substituting the NC fields in terms of the usual fields from (19), we find,

$$\begin{aligned}\hat{\mathcal{A}} \to \mathcal{A} = -\frac{g^2}{16\pi^2}\epsilon_{\mu\nu\rho\lambda}F^{\mu\nu}F^{\rho\lambda} \\ + \frac{g^3}{16\pi^2}\epsilon_{\mu\nu\rho\lambda}\theta_{\alpha\beta}(2F^{\mu\nu}F^{\rho\alpha}F^{\lambda\beta} + \frac{1}{2}F^{\beta\alpha}F^{\mu\nu}F^{\rho\lambda}).\end{aligned}\tag{24}$$

Utilising the following tensor identity for antisymmetric matrices $F_{\rho\lambda}$ and $\tilde{F}_{\rho\lambda} \equiv 2\epsilon_{\rho\lambda\mu\nu}F^{\mu\nu}$,

$$\tilde{F}_{\rho\sigma}F^{\sigma\beta}F^{\rho\alpha} = -\frac{1}{4}F^{\beta\alpha}\tilde{F}_{\mu\nu}F^{\mu\nu},$$

we find that the θ contribution vanishes, leaving the usual ABJ anomaly

$$\mathcal{A} = -\frac{g^2}{16\pi^2}\epsilon_{\mu\nu\rho\lambda}F^{\mu\nu}F^{\rho\lambda}. \tag{25}$$

The above analysis constitutes the explicit matching of the NC anomaly and the usual anomaly via the SW map, to the leading order in θ.

Next, we move on to the construction of the modified NC current,

$$\hat{J}_\mu = \hat{\bar{\psi}}\gamma_\mu\gamma_{2n+1} * \hat{\psi} + \hat{\Delta}_\mu, \quad (26)$$

such that it is free of divergence anomaly,

$$\partial_\mu \hat{J}^\mu = 0, \quad (27)$$

modulo terms of $O(\theta^2)$. Indeed, the modified current would no longer be $*$-gauge invariant. The explicit expressions for the modifications in $1+1$ and $3+1$ dimensions respectively are given below;

$$\hat{\Delta}_\mu = -\frac{g}{\pi}\epsilon_{\mu\nu}[\hat{A}^\nu + \frac{g}{2}\theta_{\alpha\beta}\hat{A}^\alpha(2\partial^\beta\hat{A}^\nu - \partial^\nu\hat{A}^\beta)], \quad (28)$$

$$\hat{\Delta}_\mu = \frac{g^2}{8\pi^2}\epsilon_{\mu\nu\rho\sigma}[\hat{A}^\nu\hat{F}^{\rho\sigma} - g\theta_{\alpha\beta}(\hat{A}^\alpha\hat{F}^{\beta\nu}\hat{F}^{\rho\sigma} - \hat{A}^\nu\hat{F}^{\rho\alpha}\hat{F}^{\sigma\beta} + \frac{1}{2}\hat{A}^\alpha\hat{F}^{\rho\sigma}\partial^\nu\hat{A}^\beta + \hat{A}^\nu\hat{A}^\alpha\partial^\beta\hat{F}^{\rho\sigma})]. \quad (29)$$

Lastly, we are in a position to derive the above results, such as the divergence anomaly, to an *arbitrary* finite order in the NC parameter θ. From the present analysis, it can be established convincingly that, at least to the leading order in θ, even an object such as the divergence anomaly, (which appears due to the short distance singularity in the theory), when computed in a NC space-time, matches identically with the result obtained from the anomaly in the usual space-time through the SW transformation. Since the SW map, valid for arbitrary orders in θ, exists for constant field tensor, we can get the NC divergence anomaly $\hat{\mathcal{A}}$ to arbitrary orders in θ directly from the anomaly expression $\mathcal{A}$ in usual space-time, through the SW transformation given below,

$$F_{\mu\nu} = \hat{F}_{\mu\nu}\frac{1}{1+g\theta.\hat{F}}, \quad (30)$$

where $\theta.\hat{F} \equiv \theta^{\alpha\beta}\hat{F}_{\alpha\beta}$. Hence, for constant fields $F_{\mu\nu}$, to arbitrary order in θ, the unintegrated NC divergence anomaly in $1+1$ and $3+1$ dimensions are given by,

$$\hat{\mathcal{A}} = \frac{g}{2\pi}\epsilon_{\mu\nu}\hat{F}^{\mu\nu}\frac{1}{1+g\theta.\hat{F}}, \quad (31)$$

$$\hat{\mathcal{A}} = -\frac{g^2}{16\pi^2}\epsilon_{\mu\nu\rho\sigma}\hat{F}^{\mu\nu}\frac{1}{1+g\theta.\hat{F}}\hat{F}^{\rho\sigma}\frac{1}{1+g\theta.\hat{F}}. \quad (32)$$

From the fact that the SW map is dimension independent, it is obvious that all operations can be carried through in arbitrary dimensions and subsequently the above conclusions and results will also hold generically.

To conclude, we have found an *unintegrated* expression for the axial anomaly in Non-Commutative (NC) space-time that is $*$-gauge invariant, up to the first non-trivial order in the NC parameter θ. A point-splitting regularization has been adopted. Contrary to the analysis in ordinary space-time, divergences up to $O(\epsilon^{-2})$ in the point-splitting parameter ϵ are significant.

The structure of the NC anomaly displayed two terms; the first is the expected modification from the usual anomaly, obtained by replacing $F_{\mu\nu} \to \hat{F}_{\mu\nu}$ [5], but the second one is a totally new contribution. It is a total derivative and hence vanishes if the integrated form of the anomaly is computed [5].

We have also shown that an application of the Seiberg-Witten (SW) map transforms the NC anomaly to the ordinary ABJ anomaly. This serves as an *a-posteriori* justification of our results since the expressions $\hat{\mathcal{A}}$ and $\mathcal{A}$ are $*$ and usual gauge invariant in NC and ordinary space-time respectively and hence should be connected through the SW map. This is shown explicitly for $1+1$ and $3+1$ dimensions.

Having convinced ourselves that the $*$-gauge invariant anomaly in NC space-time and the gauge invariant anomaly in ordinary space-time are connected by the SW map, some immediate consequences follow. The first of these is that it is possible to redefine the axial current so that the NC anomaly vanishes. Of course the modified axial current is no longer $*$-gauge invariant. Explicit expressions for such counterterms have been provided for $1+1$ and $3+1$ dimensions. Secondly, it was possible to derive an unintegrated form of the NC anomaly for a constant field, valid to all finite orders in θ, by applying the general form of the SW map in the ordinary anomaly.

Regarding future prospects, it should be possible to evaluate the unintegrated form of the non-abelian chiral anomalies, at least to the leading order in θ, and compare the results with the usual expressions. The implications of the SW map in this context could also be discussed. Finally, the unintegrated divergence anomalies can illuminate the structure of the NC commutator anomalies along with the associated Schwinger terms. In the ordinary case, using the point-splitting regularization, such an approach proved useful [9].

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