PPS Method of Estimation Under a Transformation

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SUMMARY

An efficient probability proportional to size (PPS) method of estimation with transformed auxiliary variate is suggested for the situation when there is a negative correlation between the auxiliary variable and study variable. An analogue to the well known super population model for finite population is also suggested, using which, we compare different estimators. Finally, an empirical investigation of the performance of the proposed estimators has also been made.

Keywords: Correlation coefficient, Probability proportional to size with or without replacement scheme, Regression line, Transformed variable, Superpopulation model.

1. Introduction

Consider a finite population $U = (U_1, U_2, ..., U_N)$ consisting of N distinct and identifiable units. Let Y_i be the value of the study variable y on the unit U_i, $i = 1,2,..., N$. In practice we wish to estimate the population total $Y = \Sigma Y_i$ from the y values of the units drawn in a sample $u = (u_1, u_2,..., u_n)$ with maximum precision. The easiest of the probability sampling schemes for drawing a sample u is the Simple Random Sampling With Replacement (SRSWR) scheme for which an unbiased estimator of Y and its variance are given by

$$\hat{T}_{WR} = \frac{N}{n}\sum_{i=1}^{n} Y_i \tag{1.1}$$

$$V\left(\hat{T}_{WR}\right) = \frac{N}{n}\left[\sum_{i=1}^{n} Y_i^2 - \frac{Y^2}{N}\right] \tag{1.2}$$

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A more efficient sampling procedure than SRSWR scheme is Simple Random Sampling With Out Replacement (SRSWOR) scheme for which an unbiased estimator of Y remains the same as in (1.1) which henceforth shall be denoted by $\hat{T}_{WOR}$, but its variance expression is given by

$$V\left(\hat{T}_{WOR}\right)=\frac{N(N-n)}{n(N-1)}\left[\sum_{i=1}^{N} Y_i^2-\frac{Y^2}{N}\right] \tag{1.3}$$

In most of the surveys, we have readily available information on an auxiliary variable x, closely related to the study variate y taking values X_i on the units U_i, i=1,2,...,N. The efficient utilization of this information at the estimation stage i.e. in constructing estimators of Y is well known. However, we have to be careful about the sign of the correlation coefficient, say ρ, between y and x. For example for ratio estimators $\rho > 0$ is more suitable whereas product estimators are used in the complementary situation. Further, ratio (product) estimator will give a more precise result than conventional unbiased estimator based on SRS sampling, which does not use the information on x, when $\frac{\rho c_y}{c_x} > \frac{1}{2}\left(\frac{\rho c_y}{c_x} < -\frac{1}{2}\right)$ where c_y and c_x are respectively the coefficients of variation of study and auxiliary variables.

The use of auxiliary variable at the selection stage i.e. in determining the selection probabilities was initiated by Hansen and Hurwitz [7]. They recommended the selection of units from a finite population with Probability Proportional to Size With Replacement (PPSWR) scheme where the size measure is determined by the auxiliary variable x . An unbiased estimator of Y and its variance are given by

$$\hat{T}_{HH}=\frac{1}{n}\sum_{i=1}^{n}\frac{Y_i}{p_i} \tag{1.4}$$

$$V\left(\hat{T}_{HH}\right)=\frac{1}{n}\left[\sum_{i=1}^{N}\frac{Y_i^2}{p_i}-Y^2\right] \tag{1.5}$$

where $p_i=\frac{X_i}{X}$ with $X=\Sigma X_i$. However, a general theory of PPS sampling without replacement (PPSWOR) was suggested by Horvitz and Thompson [9] with

$$\hat{T}_{HT}=\sum_{i=1}^{n}\frac{Y_i}{\pi_i} \tag{1.6}$$

and variance expression, for fixed sample size, suggested by Yates and Grundy [24] as

$$V(\hat{T}_{HT}) = \sum_{i<j}^{N}\sum (\pi_i \pi_j - \pi_{ij})\left(\frac{Y_i}{\pi_i} - \frac{Y_j}{\pi_j}\right)^2 \tag{1.7}$$

where π_i and π_{ij} are first and second order inclusion probabilities of ith unit and i and jth unit respectively, $i \neq j$; $i, j = 1, 2, ..., N$. For inclusion probability proportional to size (IPPS) sampling scheme $\pi_i = np_i$, $\forall\, i$, $i = 1, 2, ..., N$ and in this design $\hat{T}_{HT}$ will be denoted by $\hat{T}'_{HT}$.

A direct comparison of $\hat{T}'_{HT}$ $(\hat{T}_{HH})$ with $\hat{T}_{WOR}$ $(\hat{T}_{WR})$ is not easy unlike the comparison of ratio or product estimator with $\hat{T}_{WOR}$. But the form of the estimators $\hat{T}_{HH}$ and $\hat{T}'_{HT}$ indicate that they have smaller variance when Y_i is nearly proportional to p_i, $\forall\, i$, $i = 1,2, ..., N$ as the exact proportionality makes their variance zero. So recourse was taken to compare the expected variances under an assumed Super Population Model (SPM). In the literature, the most often used SPM, with its suitability based on the empirical findings of Mahalanobis [11], Smith [21] and Jessen [10] is

$$\left.\begin{array}{ll} y_i = \beta p_i + e_i & i = 1,2,\ldots,N \\ E(e_i \mid p_i) = 0 & \\ E(e_i^2 \mid p_i) = \sigma^2 p_i^g & \\ E(e_i e_j \mid p_i, p_j) = 0 & \sigma^2 > 0, g \geq 0 \end{array}\right] \tag{1.8}$$

where E (.) denotes the average over all finite populations that can be drawn from the super population. Henceforth this SPM will be denoted by model M1. There are a number of research papers – Godambe [6], Brewer [1], Rao [15], Hanurav [8], Rao [17] and Padmawar [12] amongst many others, in which this model M1 is successfully used for the purpose of comparing the different sampling strategies.

PPS sampling is expected to be more efficient than SRS sampling if the regression line of y on x passes through the origin (Raj [13]). When it is not so, a transformation on the auxiliary variable can be made so that the PPS sampling with modified sizes becomes more precise. Reddy and Rao [19] considered such modified sampling with transformed auxiliary variate viz. $X_i' = X_i + (1-k)\frac{\bar{X}}{k}$ for

$\rho > 0$ where as $X_i'' = -\left[X_i + (1-k)\frac{\overline{X}}{k}\right]$ when $\rho < 0$ and established its efficiency over SRSWR scheme empirically where $k = \rho\frac{c_y}{c_x}$. Further they proved that modified PPSWR scheme is better than the worst of the conventional PPSWR and SRSWR scheme.

Reddy and Rao's [19] study suggests that an appropriate SPM is like model M1 with x′ or x″ instead of x, as the case may be, but it can not be useful in practice as it requires a prior knowledge about a parameter k.

Rao [18], for 20 natural populations considered by Rao and Bayless [16] in which $\rho > 0$ observed that the value of k is near unity, so the amount of location shift in transformed variable x′ is negligible. Thus we can easily expect that regression line of y on x is slightly away from the origin and therefore the model M1 still remains appropriate. But when $\rho < 0$ though the transformed variable x″ has positive correlation with study variate, the amount of location shift in it is significant as k is negative in this situation. Thus model M1 is not appropriate when $\rho < 0$.

In this paper, for $\rho < 0$ a simple transformation on x is suggested which not only changes the sign of the correlation coefficient but also gives a positive value of the transformed variable and at the same time does not require a prior knowledge of k. Further, a suitable SPM is suggested using which the efficiency of different estimators of PPS sampling is studied. An empirical investigation into the performance of the estimators has also been made.

2. PPS Estimation with Negative Correlated Size Measure

Suppose that the auxiliary variable x (positive) has a negative correlation with study variate y. Then, though the estimators $\hat{T}_{HH}$ and $\hat{T}_{HT}$ remain to be unbiased, they have a larger variance when the regression line of y on x is far away from the origin. In this situation we suggest a transformation on x to x^* such that $X_i^* = (X-X_i)$, i = 1, 2,..., N. Naturally x^* is greater than zero. Further, we can easily see that correlation between y and x^* is always positive with magnitude equal to the correlation coefficient between y and x and $\Sigma X_i^* = (N-1)X$. So the modified probabilities of selection become

$$p_i^* = \frac{(1-p_i)}{(N-1)} \qquad i = 1,2,\ldots,N \tag{2.1}$$

We may call this Probability Proportional to Complementary Size method. Changing the sign of correlation by a transformation was also used by Srivenkataramana and Tracy [22] in an entirely different context i.e. for the use of product estimator in place of ratio estimator as its expressions for bias and mean square error can be exactly evaluated.

As the variable x^* or p^* has positive correlation with study variable y, an appropriate SPM will be

$$\left.\begin{array}{ll} y_i = \beta p_i^* + e_i & i = 1,2,\ldots,N \\ E\left(e_i \mid p_i^*\right) = 0 & \\ E\left(e_i^2 \mid p_i^*\right) = \sigma^2 \left(p_i^*\right)^g & \\ E\left(e_i e_j \mid p_i^*, p_j^*\right) = 0 & \sigma^2 > 0, g \geq 0 \end{array}\right] \tag{2.2}$$

as explained earlier. Henceforth this model will be called model M2. The appropriateness of the model M2 can further be strengthened from the discussion in section 1.

The analogue to Hansen and Hurwitz [7] and Horvitz and Thompson [9] estimators of Y in the proposed Modified PPSWR and PPSWOR schemes are

$$\hat{Y}_{HH} = \frac{1}{n} \sum_{i=1}^{n} \frac{Y_i}{p_i^*} \tag{2.3}$$

$$\hat{Y}_{HT} = \sum_{i=1}^{n} \frac{Y_i}{\pi_i^*} \tag{2.4}$$

respectively where π_i^* is the first order inclusion probability with probability set up p^*. The variance expressions of $\hat{Y}_{HH}$ and $\hat{Y}_{HT}$ can be obtained from (1.5) and (1.7) respectively by replacing p_i with p_i^*, π_i with π_i^* and π_{ij} by π_{ij}^*. Henceforth for IPPS-sampling design $\hat{Y}_{HT}$ will be denoted by $\hat{Y}'_{HT}$.

Now, for the estimators $\hat{Y}_{HH}$ and $\hat{Y}'_{HT}$, we have the following results, under the proposed model M2, on the lines of Raj [14], Godambe [6] and Rao [15] stated below without proof.

Theorem 2.1 : Under the proposed model specified by (2.2), $\hat{Y}_{HH}$ has smaller expected variance than the estimator $\hat{T}_{WR}$ for $g \geq 1$. However for $0 \leq g < 1$, it will be so if

$$\rho_{x^*(x^*)^{g-1}} > -\frac{N-1}{N}\frac{\beta^2}{\sigma^2}\frac{\sigma_{x^*}}{\sigma_{(x^*)^{g-1}}}$$

where σ_{x^*} and $\sigma_{(x^*)^{g-1}}$ are the standard deviations of variable x^* and $(x^*)^{g-1}$ respectively.

Theorem 2.2: Under the proposed model specified by (2.2), $\hat{Y}'_{HT}$ has smaller expected variance than the expected variance of any other linear unbiased estimator of Y.

Theorem 2.3 : Under the proposed model specified by (2.2), $\hat{Y}'_{HT}$ has smaller expected variance than the expected variance of $\hat{Y}_{HH}$ for all g.

Remark 2.1 : The other estimators in PPSWOR scheme can easily be defined by replacing p_i^* instead of p_i as in the $\hat{Y}_{HH}$ or $\hat{Y}_{HT}$ and the results regarding their comparison under the proposed model M2 specified by (2.2) can be obtained as in Rao [15], Chaudhuri and Arnab [2] and Padmawar [12].

Remark 2.2 : An IPPS sampling design for the situation considered can be obtained by each and every procedure of generating it as given in Chaudhuri and Vos [3] when p_i^* is used as an initial probability of selection instead of p_i, $i = 1,2,...,N$.

Remark 2.3 : Deshpande's [5] sampling procedure is an example of getting an IPPS sampling design for the situation considered though starting with set up p_i, $i = 1,2,...,N$.

Now in the following theorem we compare the intercept of the regression line of y on x^* with that of y on x.

Theorem 2.4 : For positive valued study and auxiliary variates the positive least square estimator of the intercept of regression line y on x^* i.e. $\hat{\alpha}_{yx^*}$ is smaller than that of least square estimator of regression line y on x i.e. $\hat{\alpha}_{yx}$ whereas in case of $\hat{\alpha}_{yx^*} < 0$ it is so if $|\beta_{yx}| < 2|\hat{\alpha}_{yx}|$.

Proof: The least square estimator of the intercept of regression line y on x^* is

$$\hat{\alpha}_{yx^*} = \bar{y} - \hat{\beta}^* \bar{x}^*$$

It can easily be seen that $\hat{\beta}^* = -\hat{\beta}_{yx}$ and $\bar{x}^* = X - \bar{x}$ so the above expression can be written as

$$\hat{\alpha}_{yx^*} = \hat{\alpha}_{yx} + \hat{\beta}_{yx} X$$

where $\hat{\alpha}_{yx}$ is the least square estimator of the intercept of regression line y on x. Clearly, for $\hat{\alpha}_{yx^*} > 0$, $\hat{\alpha}_{yx^*} < \hat{\alpha}_{yx}$ as $\hat{\beta}_{yx}$ is negative in the situation considered. But, for $\hat{\alpha}_{yx^*} < 0$, $|\hat{\alpha}_{yx^*}| < |\hat{\alpha}_{yx}|$ if $|\hat{\beta}_{yx}| X < 2|\hat{\alpha}_{yx}|$.

Remark 2.4 : The result of theorem 2.4 holds good even if the sample value in the least square estimate of intercept is replaced by its parameter i.e. $\bar{y} - \hat{\beta}\bar{x}$ by $\bar{Y} - \beta\bar{X}$.

3. *Robustness of Estimators*

In this section, we first give two lemmas which will be useful for comparison of the estimators $\hat{Y}_{HH}\left(\hat{Y}'_{HT}\right)$ and $\hat{T}_{HH}\left(\hat{T}'_{HT}\right)$ under models M1 and M2.

Lemma 3.1 : (Royall [20]) : Let $0 \le b_1 \le b_2 \le \ldots \le b_m$ and $c_1 \le c_2 \le \ldots \le c_m$ satisfying $\sum_{i=1}^{m} c_i \ge 0$. Then $\sum_{i=1}^{m} b_i c_i \ge 0$.

Lemma 3.2 : Let $b_1 \ge b_2 \ge \ldots \ge b_m \ge 0$ and $c_1 \ge c_2 \ge \ldots \ge c_m$ satisfying $\sum_{i=1}^{m} c_i \ge 0$. Then $\sum_{i=1}^{m} b_i c_i \ge 0$.

Proof : Omitted.

Now in the following theorems we compare the expected variances of $\hat{T}_{HH}$ and $\hat{Y}_{HH}$ under model M1 and M2.

Theorem 3.1 : Under the model M1 specified by (1.8), the sufficient condition that $\hat{T}_{HH}$ has smaller expected variance than $\hat{Y}_{HH}$ is

$$g > 1 - \left\{\frac{p_{max}}{(1 - p_{max})}\right\} \tag{3.1}$$

Proof : Under model M1, the expected variances of $\hat{T}_{HH}$ and $\hat{Y}_{HH}$ are

$$nEV\left(\hat{T}_{HH}\right)=\sigma^2\sum_{i=1}^{N} p_i^{g-1}(1-p_i)$$

and $$nEV\left(\hat{Y}_{HH}\right)=\beta^2\sum_{i=1}^{N}\left(\frac{p_i}{\sqrt{p_i^*}}-\sqrt{p_i^*}\right)^2+\sigma^2\left[\sum_{i=1}^{N} p_i^g\left(\frac{1}{p_i^*}-1\right)\right]$$

respectively and the difference between them can be written as

$$n\left[EV\left(\hat{Y}_{HH}\right)-EV\left(\hat{T}_{HH}\right)\right]=\beta^2\sum_{i=1}^{N}\left(\frac{p_i}{\sqrt{p_i^*}}-\sqrt{p_i^*}\right)^2+\sigma^2\sum_{i=1}^{N} b_i c_i$$

where $c_i = Np_i - 1$ and $b_i = \dfrac{p_i^{g-1}}{\left\{(N-1)p_i^*\right\}}$. The first term of the above expression is always positive. Now for the second term we observe that $\Sigma c_i = 0$ and c_i is an increasing function of p_i. So in view of Royall's lemma 3.1 it can be shown that $\Sigma b_i c_i > 0$ provided b_i is also an increasing function of p_i. A sufficient condition for this is that first derivative of b_i with respect to p_i is greater than zero which gives

$$g>1-\left\{\frac{p_i}{(1-p_i)}\right\}$$

In the above expression the lowest of the upper limit for g will be obtained when $p_i = p_{max}$ for i and hence the Theorem.

Theorem 3.2 : Under the model M2 specified by (2.2), the sufficient condition that $\hat{Y}_{HH}$ has smaller expected variance than $\hat{T}_{HH}$ is

$$g>2-\left(\frac{1}{p_{max}}\right) \tag{3.2}$$

Proof : Under model M2, the expected variances of $\hat{Y}_{HH}$ and $\hat{T}_{HH}$ are

$$nEV\left(\hat{Y}_{HH}\right)=\sigma^2\sum_{i=1}^{N}\left(p_i^*\right)^{g-1}\left(1-p_i^*\right)$$

and

$$nEV\left(\hat{T}_{HH}\right)=\beta^2\sum_{i=1}^{N}\left(\frac{p_i^*}{\sqrt{p_i}}-\sqrt{p_i}\right)^2+\sigma^2\left[\sum_{i=1}^{N}\left(p_i^*\right)^g\left(\frac{1}{p_i}-1\right)\right]$$

respectively, and the difference between them can be written as

$$n\left[EV\left(\hat{T}_{HH}\right)-EV\left(\hat{Y}_{HH}\right)\right]=\beta^2\sum_{i=1}^{N}\left(\frac{p_i^*}{\sqrt{p_i}}-\sqrt{p_i}\right)^2+\sigma^2\sum_{i=1}^{N}b_i' c_i'$$

where $c_i' = 1-Np_i$ and $b_i' = \dfrac{\left(p_i^*\right)^{g-1}}{\left\{(N-1)p_i\right\}}$. The first term of the above expression is always positive. Now, for the second term, we observe that $\Sigma\, c_i' = 0$ and c_i' is a decreasing function of p_i. So in view of lemma 3.2 it can be shown that $\Sigma\, b_i' c_i' > 0$ provided b_i' is also a decreasing function of p_i. A sufficient condition for this is that the first derivative of b_i' with respect to p_i is less than zero yielding

$$g > 2-\left(\frac{1}{p_i}\right)$$

In the above expression the lowest of the upper limit for g will be obtained when $p_i = p_{max}$ for i and hence the Theorem.

Remark 3.1 : Similar results for comparing $\hat{Y}_{HT}'$ with $\hat{T}_{HT}'$ can be obtained on the lines of Theorems 3.1 and 3.2.

Remark 3.2 : It is possible to envisage the use of the strategy consisting of SRS scheme together with a ratio estimator based on the transformed x-variable. A comparison between this and the PPS sampling could easily be made on the lines of Cochran [19] and hence is not repeated here. However, one can think of a product estimator when the auxiliary variable is negatively correlated with the study variable. But in this paper, our method of transformation and subsequent PPS selection yield a very simple and unbiased estimator whereas with product estimation one ends up in biased estimators. Also a comparison between the product strategy and PPS strategy would be quite similar to the one between ratio strategy and PPS strategy mentioned above.

4. Empirical Illustration

To study the behaviour of the estimators $\hat{Y}_{HH}$ and $\hat{Y}'_{HT}$ with respect to the conventional estimators of equal and unequal probability schemes, we consider the five populations A, B, C, D and E, details of which are given in Table 1. The populations A, B and C are the same as the three populations of Yates and Grundy's [24] whereas population D is of Stuart's [23] with size measure in a reverse order of magnitude as compared to the original one cited in reference so that the correlation coefficient becomes negative. The population E is of Stuart's [23].

Table 1

Populations								
Unit		A	B	C		D		E
Number	x	Y	Y	Y	x	Y	x	Y
1	0.4	0.5	0.8	0.2	0.49	4	0.4	4
2	0.3	1.2	1.4	0.6	0.25	9	0.2	9
3	0.2	2.1	1.8	0.9	0.16	16	0.2	16
4	0.1	3.2	2.0	0.8	0.09	25	0.1	25
5					0.01	36	0.1	36

Table 2 gives the percentage efficiency of the proposed estimators $\hat{Y}_{HH}$ and $\hat{Y}'_{HT}$ with the conventional estimators $\hat{T}_{WR}$, $\hat{T}_{WOR}$, $\hat{T}_{HH}$, and $\hat{T}'_{HT}$ for n = 2 where Brewer's [1] IPPS sampling scheme has been used for the estimators $\hat{Y}'_{HT}$ and $\hat{T}'_{HT}$.

It is clear from Table 2 that the proposed estimators $\hat{Y}_{HH}$ and $\hat{Y}'_{HT}$ performed better than the conventional equal and unequal probability estimators. Within the

Table 2. Percentage efficiency of the proposed estimators

	Percentage efficiency				
Population	$\hat{Y}_{HH}$ Vs $\hat{T}_{WR}$	$\hat{Y}'_{HT}$ Vs $\hat{T}_{WOR}$	$\hat{Y}_{HH}$ Vs $\hat{T}_{HH}$	$\hat{Y}'_{HT}$ Vs $\hat{T}'_{HT}$	$\hat{Y}'_{HT}$ Vs $\hat{Y}_{HH}$
A	179.93	183.02	889.49	1061.20	152.57
B	310.15	283.28	2615.39	2606.48	137.00
C	173.27	157.48	828.70	748.92	136.33
D	196.97	200.56	7854.75	10152.80	271.53
E	146.67	147.36	575.70	571.96	267.91

bouquet of proposed estimators, $\hat{Y}'_{HT}$ based on Brewer's [1] IPPS scheme with modified sizes also performs better than the corresponding PPSWR scheme.

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