Flattening in shear zones under constant volume: a theoretical evaluation

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Abstract

This paper presents a theoretical model based on strain energy and work rate calculations that evaluates the possible degrees of flattening in finite, ductile shear zones with rigid and deformable walls under constant volume conditions. The principal parameters governing the ratio of bulk flattening and shear rates ($S_r = \dot{\epsilon}_b/\dot{\gamma}_b$) in shear zones with rigid walls are found to be: (1) the length to width ratio (D_f; measured in the normal section parallel to the extrusion direction), and (2) the inclination of shear zone normal (α) with the bulk compression direction. Narrow and long shear zones ($D_f > 10$) are characterized by low S_r ratios, implying little flattening in the shear zone even when α is low (in the order of a few degrees). Accordingly, the kinematical vorticity number W_k is close to one when D_f is large (>10) or α is high (>20°), and is much less than one if D_f or α are low. The stretching rate of shear zone walls relative to the shear zone (R_f) is an additional parameter that controls the degree of flattening in shear zones with deformable walls. For given D_f and α values the flattening rate increases with increasing relative stretching rate R_f, and is significant at large values of R_f. Likewise the kinematical vorticity number W_k shows an inverse relation with the relative stretching rate of shear zone walls.

Keywords: Shear zones; Strain energy; Vorticity; Work rate

1. Introduction

Ductile shear zones are locales of intense deformation within a relatively less or undeformed country rock and are essentially characterized by non-coaxial deformation. However, in addition to the shear deformation, there may be a flattening component across the shear zone (Gapais et al., 1987; Ghosh and Sengupta, 1987; Jain, 1988; Mohanty and Ramsay, 1994) and in such cases the non-coaxiality of the bulk deformation is influenced by the ratio of flattening and shear rates ($\dot{\epsilon}/\dot{\gamma}$) and their relative orientations (Ramberg, 1975). Shear zone structures such as porphyroclast mantles, inclusion trails of synkinematic porphyroblasts, foliation drag, pressure shadows around rigid inclusions etc. are dependent on the kinematical vorticity number W_k (Ghosh and Ramberg, 1976; Passchier, 1987; Hanmer, 1990; Masuda et al., 1995; Beam, 1996; Jezek et al., 1999; Mandal et al., 2000), a function of the ratio of the shear and flattening strain rates (Truesdell, 1954; Means et al., 1980; Ghosh, 1987). W_k attains a value of one for simple shear deformation and decreases with increasing flattening component, to a minimum value of zero in the case of pure shear deformation. Estimation of the flattening component of progressive deformation is thus an essential part of the kinematic analysis of non-coaxial deformation. Flattening in shear zones has been taken into account in the kinematic analysis of transpression zones considering extrusion of ductile materials under the confinement of rigid blocks (Sanderson and Marchini, 1984; Fossen and Tikoff, 1993; Tikoff and Teyssier, 1994; Jones et al., 1997; Dutton, 1997). Weijermars (1992) has shown that in multilayers under a layer-oblique compression the deformation takes place dominantly by layer-parallel shear if the thickness and viscosity ratios of the stiff to soft layers are large. The flattening effect sets in only when these ratios are low to moderate. Similarly, the magnitude of flattening deformation in a shear zone is likely to be affected by the following parameters: (1) the viscosity contrast between the wall rock and the shear zone, (2) the orientation of the shear zone with respect to the principal axes of bulk stress, and (3) the length:width ratio of the shear zone. Mohanty and Ramsay (1994) have estimated the amount of flattening in a natural shear zone, and attributed it to synkinematic volume loss in the shear zone. In this paper, however, we intend to evaluate the relative magnitude of the flattening component in shear zones and determine the kinematical vorticity number under constant-volume condition by considering the three parameters mentioned above.

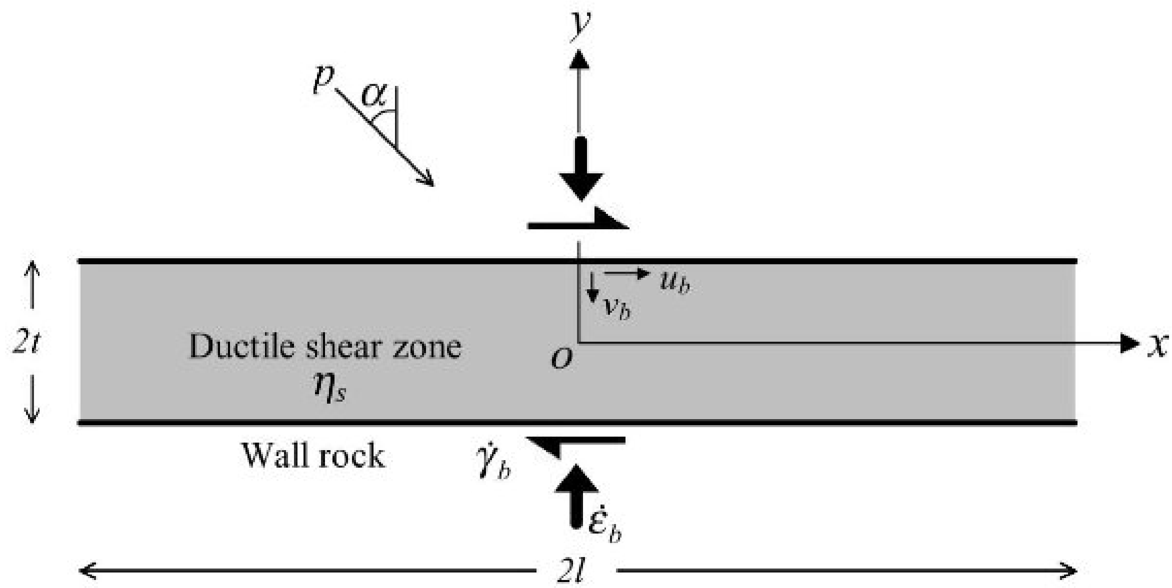

Fig. 1. Considerations of geometrical and kinematic parameters and Cartesian reference frame *oxy* in the theoretical analysis. 2*l* and 2*t* are the length and width of the shear zone, respectively. $\dot{\epsilon}_b$ and $\dot{\gamma}_b$ are the bulk flattening and shear rates of the shear zone. u_b and v_b are the normal and tangential velocity components of the shear zone wall. η_s is the viscosity of ductile rock within the shear zone. α is the inclination of shear zone normal to the direction of bulk differential compression (p).

2. Theoretical analysis

2.1. Basic premises

The theoretical model is based on the assumption that the ductile rocks within the shear zone are Newtonian, and rheologically homogeneous. We also assume that there is no volume loss during the deformation, and the flow of material in response to flattening takes place along the shear direction. The analysis considers a non-slip condition at the shear zone boundary (cf. Sanderson and Marchini, 1984; Dutton, 1997).

The theory is developed by balancing the energy involved in the flow within the shear zone with the work to be done for the movement of the boundary walls. The flow field in the shear zone is formulated using the solution of Navier–Stoke's equations, as given by Jaeger (1969). Separate analyses for shear zones with rigid and deformable walls are presented in the following sections.

2.2. Shear zone with rigid walls

Consider the cross-section of a tabular shear zone parallel to the shear direction with a length and a width of *2l* and 2*t*, respectively. The shear zone normal is at angle α with the principal direction of bulk compressive stress (Fig. 1). We set a Cartesian reference frame *oxy* with the origin at the shear zone center and the *x* axis parallel to the shear zone boundary (Fig. 1). The shear zone is subjected to bulk shear at a rate $\dot{\gamma}_b$ in the *x* direction and bulk flattening at a rate $\dot{\epsilon}_b$ across the shear direction (i.e. along the *y* axis). The ratio of bulk flattening and shear rates ($\dot{\epsilon}_b/\dot{\gamma}_b$) is henceforth termed the *strain-rate ratio* S_r (cf. Ghosh and Ramberg, 1976).

We first consider the energy involved in the flow within the shear zone in response to flattening. The velocity components at any point within the shear zone due to the flattening deformation can be written as:

$$u = 3v_b x \frac{t^2 - y^2}{2t^3} \tag{1a}$$

$$v = v_b \frac{y(y^2 - 3t^2)}{2t^3} \tag{1b}$$

(Jaeger, 1969, Eq. 16, Section 40), where v_b is the velocity at which the rigid walls approach each other due to flattening deformation in the shear zone (Fig. 1). Using the velocity functions in Eqs. (1a) and (1b), the total energy required per unit time for viscous flow due to flattening is obtained as:

$$E_p = \frac{12\eta_s v_b^2}{t^3}\left[\frac{8}{5}t^2 + \frac{1}{3}l^2\right]l \tag{2}$$

Eq. (2) has been derived by integrating Eq. (A6) given in Appendix A. Balancing the energy in Eq. (2) with the work done required for flattening movement in the wall (Eq. (A8) in Appendix A), we have:

$$\frac{3\eta_s v_b^2}{t^3}\left[\frac{8}{5}t^2 + \frac{1}{3}l^2\right] = 2v_b p\cos 2\alpha \tag{3a}$$

This can be rewritten as:

$$3\eta_s \dot{\epsilon}_b\left[\frac{8}{5} + \frac{1}{3}\left(\frac{l}{t}\right)^2\right] = 2p\cos 2\alpha \tag{3b}$$

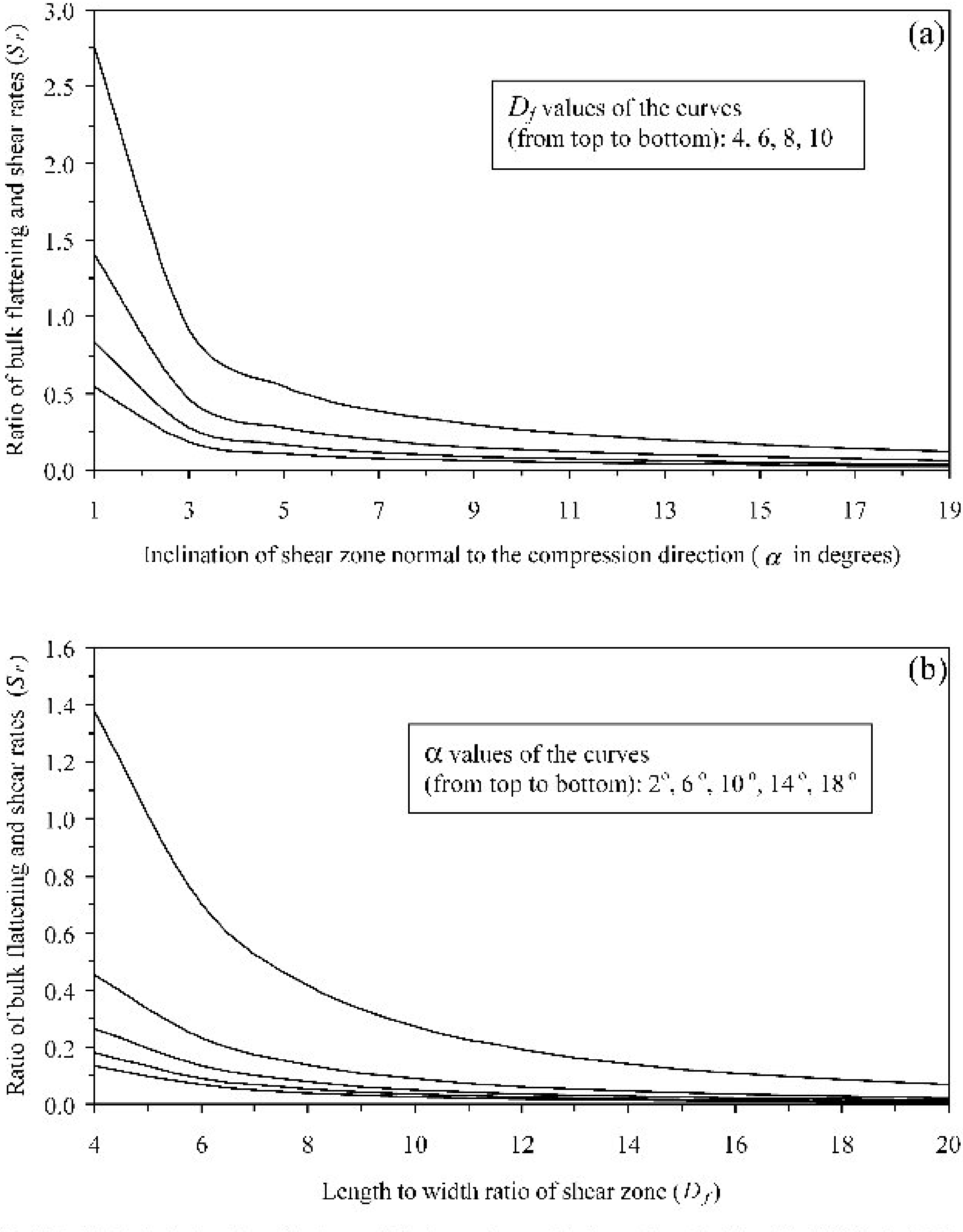

Fig. 2. Calculated plots showing the dependence of the degree of flattening upon the geometrical parameters — length to width ratio (D_f) and orientation (α) of the shear zone. (a) S_r versus α at different constant values of D_f. (b) S_r versus D_f at different constant values of α.

where p and α are the differential compressive stress and its orientation with respect to the shear zone normal (Fig. 1).

Similarly, we can calculate the energy and work associated with shearing, and derive the following equations (see Eqs. (A9) and (A10) in Appendix A).

$$\eta_s \frac{u_b}{t} = p\sin 2\alpha \tag{4a}$$

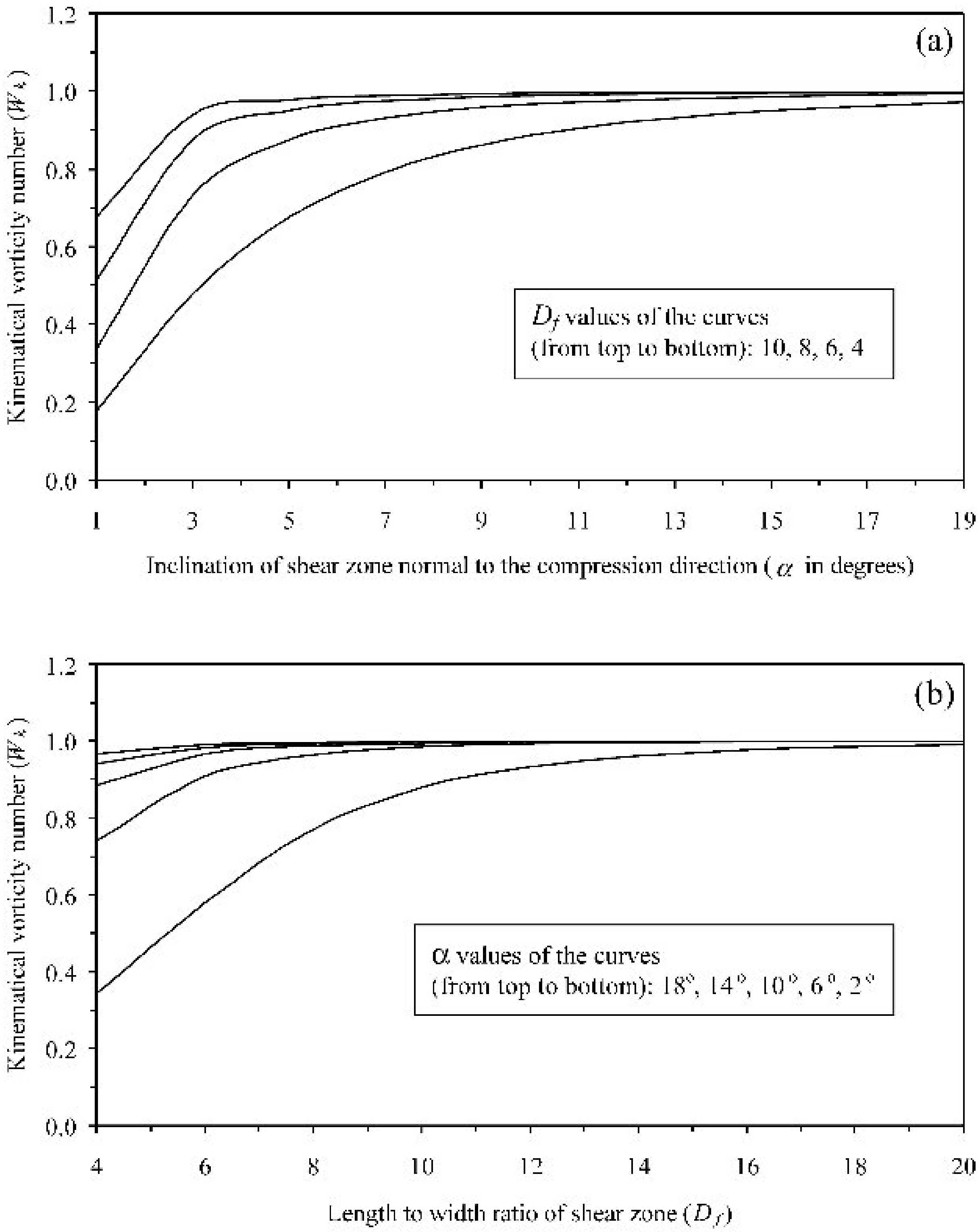

Fig. 3. Variations of bulk rotationality (W_k) with the geometrical parameters — length to width ratio (D_f) and orientation (α) of the shear zone. (a) W_k versus α at different constant values of D_f. (b) W_k versus D_f at different constant values of α.

or

$$\eta_s \dot{\gamma}_b = p\sin 2\alpha \tag{4b}$$

In Eq. (3b), the bulk flattening rate, $\dot{\epsilon}_b$ is inversely proportional to the square of the length to width ratio (l/t) of the shear zone. For given p and α values, the flattening rate thus decreases strongly with an increase in the l/t ratio. On the other hand, the bulk shear rate, $\dot{\gamma}_b$ is independent of the l/t

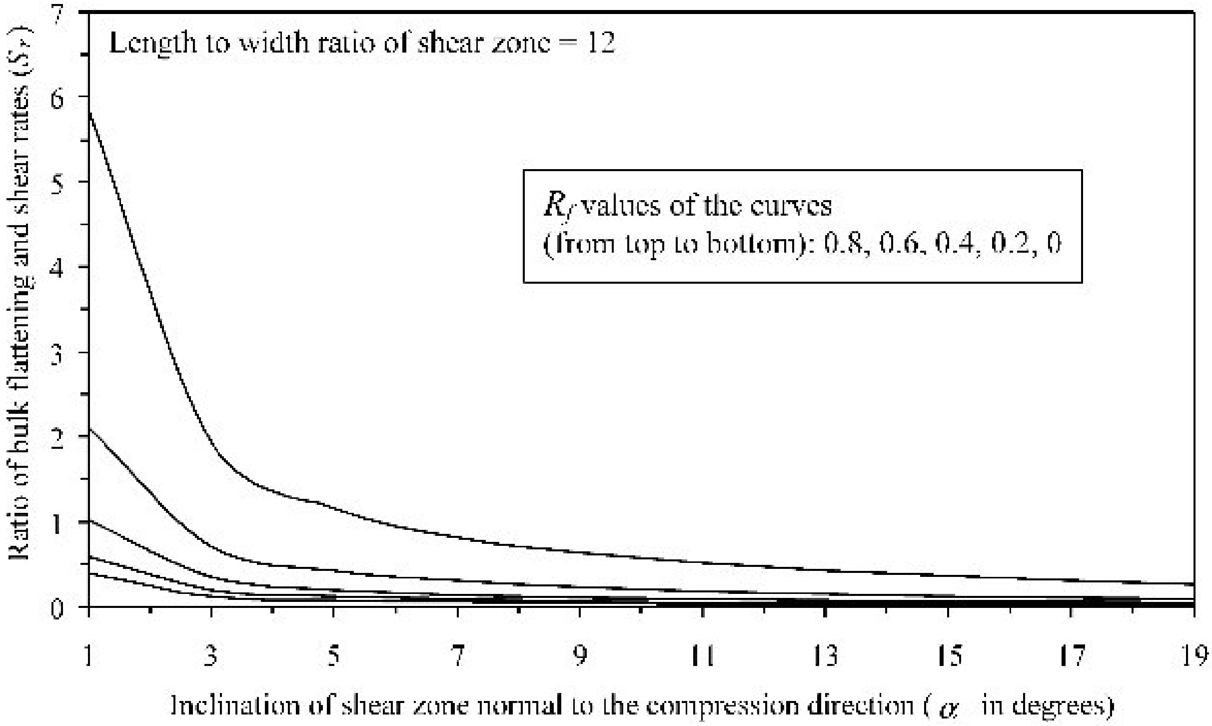

Fig. 4. Variations of strain ratio S_r with the inclination of shear zone normal α at different values of relative stretching rates (R_f) of deformable shear zone walls.

ratio (Eq. (4b)). It is therefore evident from these relations that the shear component of the bulk deformation is insensitive to the l/t ratio, and would dominate over the flattening component in shear zones with large l/t ratios. From Eqs. (3a), (3b), (4a) and (4b), the ratio of bulk flattening and shear rates are obtained as:

$$\frac{\dot{\epsilon}_b}{\dot{\gamma}_b} = \frac{2\cot 2\alpha}{3\left(\frac{8}{5} + \frac{1}{3}\frac{l^2}{t^2}\right)} \Rightarrow S_r = \frac{10}{24 + 5D_f^2}\cot 2\alpha \tag{5}$$

where D_f is the l/t ratio of the shear zone.

Eq. (5) reveals the relationship of the strain ratio S_r with two geometrical parameters—the orientation of the shear zone with respect to the bulk compression direction (α) and its length to width ratio ($D_f = l/t$). At given D_f, the strain ratio S_r decreases steeply with increasing α, and is nearly zero when α is greater than 20° (Fig. 2a). Similarly, S_r decreases with increasing D_f, and lies below 0.5 at $D_f > 8$ if $\alpha > 2°$ (Fig. 2b). For large values of D_f (>20), S_r is virtually zero for any non-zero value of α.

The relation between the strain ratio S_r and kinematical vorticity number is:

$$W_k = \frac{1}{\sqrt{1 + 4S_r^2}}, \tag{6}$$

(Ghosh, 1987). Eqs. (5) and (6) reveal that W_k increases steeply, and then asymptotically tends to be one, as the inclination of shear zone normal (α) increases (Fig. 3a). For large D_f values, W_k becomes nearly one at a small value of α. For example, $W_k = 0.9$ at $\alpha = 5°$ when $D_f = 12$ (Fig. 3b). The kinematical vorticity number is virtually one for any non-zero value of α when D_f is very large (>20) (Fig. 3b).

The theoretical results imply that in the case of rigid walls flattening component is likely to be negligibly small in shear zones with a large length to width ratio, even when the shear zone is at a high angle to the principal compression direction (i.e. for very low α). Flattening becomes significant (>50%) when α is less than 6° and D_f is less than seven (Figs. 2 and 3).

2.3. Shear zone with deformable walls

The velocity functions in Eqs. (1a) and (1b) are valid for shear zones with rigid walls. It is commonly noticed in natural shear zones that the wall rocks have undergone deformation during the movement in the shear zones. The wall deformation may take place both by pure shear or a combination of pure and simple shear. In the present analysis, however, we need to consider only the stretching component parallel to the shear zone boundary. Let the shear zone boundaries experience stretching at a rate $\dot{\epsilon}_w$ in the shear direction, which is assumed to occur in response to the normal stress component σ_n (Eq. (A7a)). Flow within shear zones with stretching walls can be described by the following velocity functions (see Appendix A).

$$u = \frac{3}{2}(v_b - \dot{\epsilon}_w t)x\frac{t^2 - y^2}{t^3} + \dot{\epsilon}_w x \quad (7a)$$

$$v = \frac{1}{2}(v_b - \dot{\epsilon}_w t)y\frac{y^2 - 3t^2}{t^3} - \dot{\epsilon}_w y \quad (7b)$$

It may be noted that Eqs. (7a) and (7b) simplify to Eqs. (1a) and (1b) for $\dot{\epsilon}_w = 0$, representing the case of shear zone with rigid walls, as modeled in the previous section. Using the velocity functions in Eqs. (7a) and (7b), we can find the total strain energy required for flattening and shear deformations in the shear zone following the same method as in the previous section (details in Appendix A). The total energies involved in flattening and shear deformations, respectively, are:

$$E_p = 4\eta_s\left[\frac{3(v_b - \dot{\epsilon}_w t)^2}{t^3}\left(\frac{8}{5}t^2 + \frac{1}{3}l^2\right)l + 4\dot{\epsilon}_w l(2v_b - \dot{\epsilon}_w t)\right] \quad (8)$$

$$E_s = 4\eta_s u_b^2\frac{l}{t} \quad (9)$$

Substituting the expressions of E_p and E_s in Eqs. (8) and (9) and after some algebra (see Appendix A), the ratio (S_r) of bulk flattening and shear rates is obtained as:

$$S_r = \frac{2\cot 2\alpha}{3(1 - R_f)^2\left(\frac{8}{5} + \frac{1}{3}D_f^2\right) + 4(2 - R_f)R_f} \quad (10)$$

where $R_f = \dot{\epsilon}_w/\dot{\epsilon}_b$, i.e. the ratio of stretching rate of the shear zone boundary to the bulk shortening rates across the shear zone and D_f is the length to width ratio of the shear zone. The parameter R_f is actually a measure of competence contrast between the shear zone and its wall rocks (see Discussion). Using Eq. (10) we can analyze the additional effects of wall rock deformation on shear zone kinematics. For a given value of D_f (say 12), when R_f is low (<0.5), S_r increases a little with decrease in the inclination of shear zone normal α, and is always less than one (Fig. 4). In contrast, for large values of R_f (e.g. $R_f = 0.8$), S_r increases steeply to attain a large value (>1) when the shear zone normal makes lower (<5°) inclinations.

Using Eqs. (6) and (10), the bulk non-coaxiality of shear zones can be analyzed by varying the length to width ratio (D_f) of the shear zone and the ratio of wall stretching rate and bulk flattening rate across the shear zone (R_f) for different shear zone orientations (α) (Fig. 5). For $\alpha = 2°$, W_k increases more or less linearly with D_f at large values of R_f (>0.5). The variations show gentle gradients, and W_k remains low (generally less than one) at large values of D_f (Fig. 5a). The theoretical result indicates that shear zones with a large length to width ratio can have a low non-coaxiality of flow only if the relative stretching rate in the wall rock is significant. With a decrease in R_f, W_k versus D_f variations increase their gradient and become progressively non-linear. For very low values of R_f (<0.2), W_k increases steeply and tends to be one at moderate values of D_f (>15). Similar W_k–D_f variations are obtained for $\alpha = 18°$, but the W_k values are characteristically in the higher range (>0.6) so that the curves representing W_k–D_f variations are close to the line $W_k = 1$, implying a simple shear type of deformation (Fig. 5b).

3. Discussion

The present analysis reveals the principal parameters determining the ratio of bulk flattening to shear rates or the bulk kinematical vorticity number W_k in rigid-walled shear zones. These are: (1) the length to width ratio (D_f) of the shear zone in the normal section parallel to the extrusion direction, and (2) the orientation of the shear-zone normal with respect to the bulk compression direction (α). W_k increases with increasing α, and is close to one when the length to width ratio D_f is large. Our model thus suggests that narrow shear zones hosted in a rigid ambience should be ideally dominated by simple shear deformation.

In natural and experimental shear zones, the material of the walls may also undergo ductile deformation, albeit at much slower rates than that within the shear zone. Our analysis indicates that the degree of wall rock deformation along the shear zone boundary is an additional factor in controlling the bulk kinematics within such shear zones. In order to analyze this, we have considered a parameter R_f, which represents the ratio of stretching rate of the shear zone walls to bulk-shortening rate across the shear zone. The parameter is actually a measure of rheological contrast between the shear zone and the wall rocks. Putting $E_p = 8\sigma_n l v_b$ in Eq. (8) and then substituting σ_n by $2\eta_w\dot{\epsilon}_w$ and after algebraic manipulation we obtain the relation:

$$\frac{\eta_s}{\eta_w} = \frac{4R_f}{3G_f(1 - R_f)^2 + 4R_f(2 - R_f)} \quad (11)$$

where η_s and η_w are the viscosities of the shear zone and the wall rocks, respectively. G_f is a geometrical factor, which has a relation with D_f as:

$$G_f = \frac{8}{5} + \frac{1}{3}D_f^2 \quad (12)$$

It may be noted in Eq. (11) that $R_f = 1$ when $\eta_s = \eta_w$, as in the case of homogeneous media and η_w tends to infinity as $R_f = 0$ in the case of shear zones with rigid walls. Our theoretical results suggest that flattening deformation can be important in shear zones only when the relative stretching rate R_f is not too low (Fig. 4). This condition can prevail if the viscosity contrast between the shear zone and its walls is not too large, as revealed from Eq. (11). Shear zones with a large viscosity contrast will have low R_f values and are unlikely to experience a large amount of shortening, even

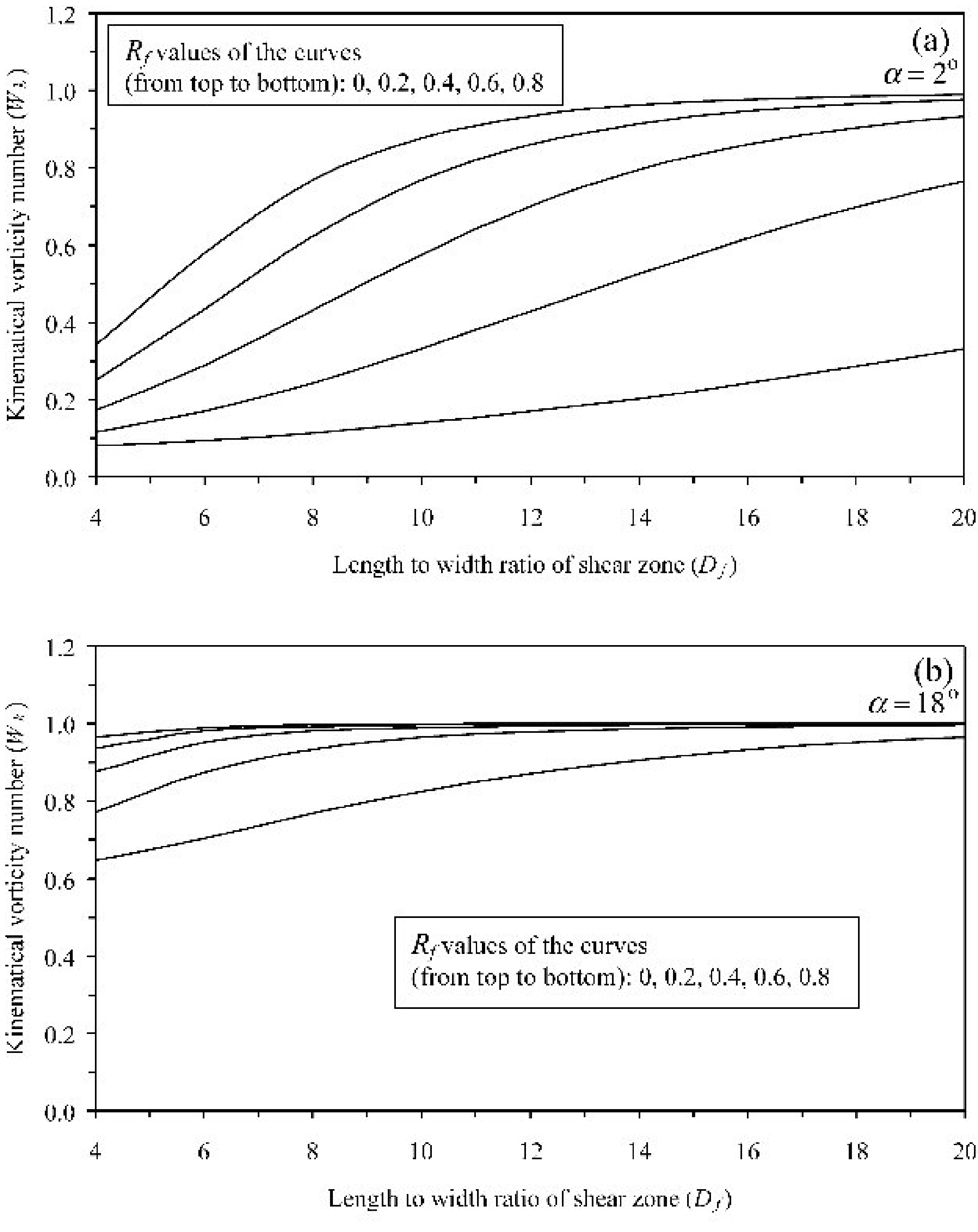

Fig. 5. Variations of kinematical vorticity number (W_k) with length to width ratio (D_f) of shear zone. R_f is the relative stretching rate of the deformable shear zone walls. (a) $\alpha = 2°$ and (b) $\alpha = 18°$, α is the inclination of shear zone normal to the bulk compression direction.

if the shear zone normal is at a low angle to the principal compression direction.

The present analysis is based on a normal section of the shear zone parallel to the shear direction along which lateral flow of material in response to flattening is allowed. Several workers have shown that the direction of material flow in response to flattening can be at any orientation with respect to the shear direction (e.g. Dutton, 1997). It may be noted that our analysis is based on independent estimations of energy required for flattening and shear deformations in the shear zone, and the orientation of material flow direction with respect to the shear direction thus does not influence the results obtained. However, the parameter l of our analysis is to be considered along the direction of flow that takes place in response to flattening. For example, in vertical shear zones where the shear direction is horizontal and the extrusion of material due to flattening has taken place in the vertical direction, the length and width of the shear zone are to be measured on a vertical cross-section normal to the shear direction.

4. Conclusions

1. The principal parameters controlling the degree of flattening in shear zones under constant volume conditions are: (a) the length:width ratio of the shear zone in a normal section parallel to the extrusion direction, (b) the orientation of the shear zone with respect to the bulk compression direction, and (c) the viscosity contrast between the shear zone and wall rocks.
2. In shear zones with rigid walls, the flattening component is likely to be negligibly small if their length to width ratio is high, even if the shear zone is at a high angle to the principal compression direction. Flattening becomes significant in shear zones with a low length to width ratio and the orientation of the shear zone normal (α) is at a low angle to the bulk compression direction.
3. In shear zones with deformable walls, the viscosity contrast between the wall rocks and shear zone is an additional parameter controlling the degree of flattening. The deformation in a shear zone is likely to be dominated by simple shear if the competence contrast is high. Flattening can be significant when this contrast is low.
4. There are three principal limitations that adhere to our analysis. (a) It does not take into account the effect of volume loss in the shear zone deformation. However, Mohanty and Ramsay (1994) have shown that shear zones may experience a large amount of volume loss leading to significant flattening across the shear zone. Thus, the theoretical results presented here may not strictly conform to those from natural shear zones that have evidently undergone a synkinematic volume change. (b) The analysis considers Newtonian, homogeneous material and assumes a non-slip condition at the shear zone boundary. (c) The analysis predicts that little flattening is possible if D_f is large. However, many natural shear zones such as the Ossa–Morena zone, the Ibero–American zone, and the San-Andreas fault, have a large length to width ratio (D_f) on the outcrop sections, but record significant flattening across the shear zones. It is possible that in these shear zones there has been upward extrusion of material and their dimensions when measured on a normal section parallel to the extrusion direction would yield a D_f value within a range for which flattening is possible. However, some other factors, e.g. slip/non-slip conditions at these transpression zones might have also played roles in dictating transpression.

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Appendix A

A.1. Energy calculations for shear zones with rigid walls

The rate of energy required for viscous flow within an infinitesimal volume in the shear zone is:

$$dE_p = 2\eta_s(\epsilon_{xx}^2 + \epsilon_{yy}^2 + 2\epsilon_{xy}^2)dxdy \tag{A1}$$

(cf. Jeffery, 1922). The total energy (E_p) required per unit time for flattening deformation in the shear zone can be obtained by integrating Eqs. (A1) as:

$$E_p = 4\left[2\eta_s \int_0^l \int_0^t (\epsilon_{xx}^2 + \epsilon_{yy}^2 + 2\epsilon_{xy}^2)dxdy\right], \tag{A2}$$

where ϵ_{xx},ϵ_{yy} and ϵ_{xy} are the strain-rate components and η_s is the viscosity of rocks within the shear zone. Now imposing the condition $\epsilon_{xx} + \epsilon_{yy} = 0$, Eq. (A2) simplifies to:

$$E_p = 16\eta_s \int_0^l \int_0^t (\epsilon_{xx}^2 + \epsilon_{xy}^2)dxdy \tag{A3}$$

From Eqs. (1a) and (1b), we have:

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = 3v_b \frac{t^2 - y^2}{2t^3} \tag{A4}$$

$$\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = -\frac{3}{2}v_b \frac{xy}{t^3} \tag{A5}$$

Substituting the strain components (Eqs. (A4) and (A5)) in Eq. (A3), we have:

$$E_p = 36\eta_s \frac{v_b^2}{t^6} \int_0^t \int_0^l \left\{\left(t^2 - y^2\right)^2 + (xy)^2\right\}dxdy \tag{A6}$$

Now, we calculate the work to be done for movement of the shear zone walls involving the stresses acting on them. As the shear zone normal is at angle α, the deviatoric normal and shear stress components on the shear zone boundary are:

$$\sigma_n = p\cos 2\alpha \tag{A7a}$$

$$\tau = -p\sin 2\alpha \tag{A7b}$$

where $p = \sigma_1 - \sigma_2/2$ and σ_1 and σ_2 are the principal bulk stresses. Work done per unit time for moving the shear zone walls during flattening deformation is:

$$E_p = 8lv_b p\cos 2\alpha \quad \text{(A8)}$$

The shear stress component on the shear zone boundary develops a homogeneous simple shear at a rate $\dot{\gamma}_b$. Substituting $\epsilon_{xy} = \dot{\gamma}_b/2$, $\epsilon_{xx} = 0$ and $\epsilon_{yy} = 0$ in Eq. (A3), we get the total energy required for bulk shear in the shear zone as:

$$E_s = 4\eta_s lt\dot{\gamma}_b^2 \Rightarrow E_s = 4\eta_s \frac{l}{t} u_b^2, \quad \text{(A9)}$$

where u_b is the rate of displacement of shear zone wall in the shear direction occurring in response to the shear stress component τ (Eq. (A7b)), which requires the following rate of work:

$$E_s = 4pu_b l\sin 2\alpha \quad \text{(A10)}$$

A.2. Formulation of velocity fields in shear zones with deformable walls

The velocity field in Eqs. (1a) and (1b) is valid for shear zones with rigid walls. Following the same method as Jaeger (1969), we can determine the velocity field for shear zones with deformable walls in the following way. According to Navier–Stroke's equation, the viscous flow can be represented by:

$$-\frac{1}{\eta_s}\frac{\partial p_o}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{(A11a)}$$

$$-\frac{1}{\eta_s}\frac{\partial p_o}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \quad \text{(A11b)}$$

Applying the condition of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{(A12)}$$

Eqs. (A11a) and (A11b) give rise to:

$$\frac{\partial^2 p_o}{\partial x^2} + \frac{\partial^2 p_o}{\partial y^2} = 0 \quad \text{(A13)}$$

The pressure (p_o) distribution due to flattening deformation are symmetrical with respect to the x and y axes. Thus, the solution of p_o in Eq. (A13) can be expressed in terms of a polynomial function as (Jaeger, 1969, p. 141):

$$p_o = \frac{1}{2}c_1 y^2 - \frac{1}{2}c_1 x^2 + c_2 \quad \text{(A14)}$$

where c_1 and c_2 are constants, which need to be determined from given boundary conditions. In the present case, v is independent of x, and the velocity function of u must not have any terms of second or higher degrees of x, as revealed from Eq. (A13). Therefore, $\delta^2 u/\delta x^2 = 0$. After substituting the expression of Eq. (A14), Eq. (11) gives:

$$\frac{\partial^2 u}{\partial y^2} = \frac{c_1 x}{\eta_s}$$

the solution of which is:

$$u = -\frac{1}{2}\frac{c_1}{\eta_s}xy^2 + yf_1(x) + f_2(x)$$

As the velocity component in this case is symmetrical with respect to the x axis, this must be an even function of y. Then, $f_1(x)$ must be zero. Now imposing the boundary condition $u = \dot{\epsilon}_w x$ at $y = t$, we find:

$$f_2(x) = \frac{1}{2}\frac{c_1}{\eta_s}xt^2 + \dot{\epsilon}_w x.$$

The function for the velocity component u is then:

$$u = \frac{1}{2}\frac{c_1}{\eta_s}x\left(t^2 - y^2\right) + \dot{\epsilon}_w x \quad \text{(A15)}$$

From Eq. (A12) of continuity and Eq. (A15), it follows:

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} \Rightarrow \frac{\partial v}{\partial y} = -\left[\frac{1}{2}\frac{c_1}{\eta_s}\left(t^2 - y^2\right) + \dot{\epsilon}_w\right] \Rightarrow v = -\frac{c_1}{6\eta_s}y\left(y^2 - 3t^2\right) - \dot{\epsilon}_w y + f_3(x) \quad \text{(A16)}$$

If the velocity component v has to assume a constant velocity at $y = \pm t$, $f_3(x)$ has to be essentially zero. Imposing $v = -v_b$ at $y = t$, we get $c_1 = 3\eta_s/t^3(v_b - \dot{\epsilon}_w t)$. Substituting the expression of constant c_1 in Eqs. (A15) and (A16), we have:

$$u = \frac{3}{2}(v_b - \dot{\epsilon}_w t)x\frac{t^2 - y^2}{t^3} + \dot{\epsilon}_w x \quad \text{(A17a)}$$

$$v = \frac{1}{2}(v_b - \dot{\epsilon}_w t)y\frac{y^2 - 3t^2}{t^3} - \dot{\epsilon}_w y \quad \text{(A17b)}$$

The strain components can be obtained by differentiating Eqs. (A17a) and (A17b) as:

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = \frac{3}{2}(v_b - \dot{\epsilon}_w t)\frac{t^2 - y^2}{t^3} + \dot{\epsilon}_w \quad \text{(A18a)}$$

$$\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = -\frac{3}{2}(v_b - \dot{\epsilon}_w t)\frac{xy}{t^3} \quad \text{(A18b)}$$

Substituting the strain components from Eqs. (A18a) and (A18b) in Eq. (A3), the total strain energy required per unit time in the shear zone is:

$$E_p = 16\eta_s \int_0^l \int_0^t \left[\left\{\frac{3}{2}\left(v_b - \dot{\epsilon}_w t\right)\frac{t^2 - y^2}{t^3} + \dot{\epsilon}_w\right\}^2 + \left\{\frac{3}{2}\frac{v_b - \dot{\epsilon}_w t}{t^3}xy\right\}^2\right]dxdy \quad \text{(A19)}$$

We now determine the energy budget for bulk shear in the shear zone. The tangential velocity of the shear zone boundary is u_b, which develops shear at a rate, $\dot{\gamma}_b = u_b/t$ in the shear zone. The total energy required for shearing motion is then:

$$E_s = 4\eta_s \dot{\gamma}_b^2 l t \tag{A20}$$

Substituting E_p and E_s (Eqs. (A7a) and (A7b)) in Eqs. (A19) and (A20), and dividing the derivative expressions, we have

$$\frac{3\frac{(v_b - \dot{\epsilon}_w t)^2}{t^3}\left(\frac{8}{5}t^2 + \frac{1}{3}l^2\right) + 4\dot{\epsilon}_w(2v_b - \dot{\epsilon}_w t)}{\frac{u_b^2}{t}} = \frac{2v_b \cos 2\alpha}{u_b \sin 2\alpha}$$

Replacing $v_b = \dot{\epsilon}_b t$ and $u_b = \dot{\gamma}_b t$, and after some algebra the equation can be reorganized as:

$$\frac{3(\dot{\epsilon}_b - \dot{\epsilon}_w)^2\left(\frac{8}{5} + \frac{1}{3}\frac{l^2}{t^2}\right) + 4\dot{\epsilon}_w(2\dot{\epsilon}_b - \dot{\epsilon}_w)}{\dot{\gamma}_b^2} = 2\cot 2\alpha \frac{\dot{\epsilon}_b}{\dot{\gamma}_b} \tag{A21}$$

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