BOSONISATION EXCERCISE IN THREE DIMENSIONS: GAUGED MASSIVE THIRRING MODEL

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Abstract

Bosonisation of the massive Thirring model, with a non-minimal and non-abelian gauging is studied in 2+1-dimensions. The static abelian model is solved completely in the large fermion mass limit and the spectrum is obtained. The non-abelian model is solved for a restricted class of gauge fields. In both cases explicit expressions for bosonic currents corresponding to the fermion currents are given.

Introduction

Ever since the explicit demonstration in 1+1-dimensions of the equivalence of the massive Thirring model and the sine-Gordon model order by order in perturbation theory in the charge zero sector [1] and the subsequent construction of the fermion operator by boson operators [2], the concept of bosonisation has proved to be an extremely useful one. However it was thought that this equivalence is an exclusive property of one dimensional space, where in reality there is no spin to distinguish fermions from bosons. Indeed, attempts to generalise bosonisation in two space dimensions met with limited success [3].

Renewed interest in 2+1-dimensional bosonisation has created a flurry of activity in recent years, where the problem is attacked from a different angle. The nonlocal fermion determinant generates local terms in the one loop perturbative evaluation, in the limit of large fermion mass. In the lowest orders of inverse fermion mass, the bosonised theory of the (abelian) massive Thirring model turns out to be Maxwell-Chern-Simons [4]. In fact the equivalence between massive Thirring and $CP(1)$ models in the large fermion mass limit was established much ago [5]. The situation is not that clear in the nonabelian models. For example, the $SU(2)$ Thirring model, in the limit where the Thirring coupling vanishes, can be identified with the $SU(2)$ Yang-Mills-Chern-Simons theory, in the limit where the Yang-Mills term vanishes [6].

In the present work, we consider a theory of non-minimally gauged Dirac fermions, with a Thirring [7] current-current self interaction. Both abelian and non-abelian gauge groups have been investigated. The model lives in 2+1-dimensions. We study the one loop bosonised version of the model in the large fermion mass (m) limit and keep only Chern-Simons (m independent) and Yang-Mills or Maxwell ($O(m^{-1})$ terms. The effect of still higher order terms in the inverse fermion mass is qualitatively discussed in the abelian context. The mapping between the fermion and the boson fields at the level of currents is obtained. The behaviour of the bosonic charge operator is studied in detail.

The paper is organised as follows: Section **II** deals with the nonabelian fermion model and its bosonisation. In Section **III** we discuss in detail the abelian theory. Section **IV** contains results for the nonabelian theory for a special class of gauge fields. The paper ends with a brief conclusion in Section **V**.

II. Bosonisation

The parent non-abelian fermion model that we wish to study is a system of Dirac fermions with a non-minimal gauging. There is a Thirring [7] type current-current self interaction term as well. The Lagrangian considered by us is

$$L = \bar{\psi} i\gamma^\mu D_\mu \psi - m\bar{\psi}\psi + \frac{g}{2}(\bar{\psi}\gamma^\mu T^a \psi)^2. \quad (1)$$

The covariant derivative is defined as

$$D_\mu = \partial_\mu - i\gamma A^a_\mu T^a - i\sigma K^a_\mu T^a \equiv \partial_\mu - i\bar{A}^a_\mu T^a, \quad K_\mu = \epsilon_{\mu\nu\lambda} A^{\nu\lambda}.$$

The anti-hermitian generators satisfy $[T^a, T^b] = f^{abc} T^c$ and the γ-matrices are defined via the Pauli matrices by

$$\gamma^0 = \sigma^3, \quad \gamma^1 = i\sigma^1, \quad \gamma^2 = i\sigma^2.$$

To keep track of the various combinations of vector fields that will appear, we introduce the notations,

$$(D_\mu)^{(W)ab} = \partial_\mu \delta^{ab} - \rho f^{abc} W^c_\mu; \quad W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \rho W^b_\mu W^c_\nu,$$

where W is some arbitrary vector field and ρ the associated coupling constant. The fermion current enjoys a conservation law,

$$(D^{\bar{A}}_\mu J^\mu)^a = o; \quad \bar{A}_\mu = \gamma A_\mu + \sigma K_\mu. \tag{2}$$

Note that no gauge field kinetic term such as the Yang-Mills or Chern-Simons term, is kept in the fermion model as they will be generated in the bosonisation process, along with other mixed terms. Hence even if such terms are kept, their coefficients will get renormalised by bosonisation.

The usual scheme of linearising the Thirring term in (1) is by introducing an auxiliary field B^a_μ, such that when B^a_μ is integrated out, the original model is reproduced. This gives us

$$\begin{aligned}\bar{L} &= \bar{\psi}\gamma^\mu(iD_\mu + B_\mu)\psi - m\bar{\psi}\psi - \frac{1}{2g}B^a_\mu B^{\mu a} \\ &= \bar{\psi}\gamma^\mu(i\partial_\mu + C_\mu)\psi - m\bar{\psi}\psi - \frac{1}{2g}B^a_\mu B^{\mu a},\end{aligned} \tag{3}$$

where $C_\mu \equiv B_\mu + \gamma A_\mu + \sigma K_\mu$. The quadratic term in B_μ constitutes a mass term for B_μ. This leads us to the evaluation of the fermion determinant, which is in general non-local, but yields local expressions under various approximation schemes. A gauge invariant Pauli-Villars regularisation has been invoked. We choose, in particular, the large fermion mass limit such that m^{-1} is a small term. This also restricts us to the low energy or long wavelength limit, where terms with smaller number of derivatives dominate. The Seeley coefficients in the fermion determinant are computed at the one loop level. With these restrictions, the bosonised Lagrangian is the following:

$$L_B = -\frac{a}{4}C^a_{\mu\nu}C^{\mu\nu a} - \frac{1}{2g}B^a_\mu B^{\mu a} + \alpha\epsilon^{\mu\nu\lambda}C^a_\mu(\partial_\nu C^a_\lambda + \frac{1}{3}f^{abc}C^b_\nu C^c_\lambda). \tag{4}$$

The coefficients $\alpha = 1/(4\pi)$ and $a = -1/(24\pi m)$ are known from bosonization rules.

Since there are a number of fields, coupling constants and parameters, a glossary of the dimensions of them in the $c = \hbar = 1$ system of units is provided below, with l denoting length,

$$[C_\mu] = [B_\mu] = [\psi] = [m] = \frac{1}{l}, \quad [A_\mu] = [\gamma] = \frac{1}{\sqrt{l}}, \quad [g] = [a] = l, \quad [\sigma] = \sqrt{l}.$$

The Lagrangian equations of motion following from (4) are,

$$2\sigma\epsilon^{\nu\mu\lambda}(D^{(A)}_\mu X_\lambda)^a + \gamma X^{\nu a} = 0, \tag{5}$$

where

$$X^{\nu a} = (aD^{(C)}_\mu C^{\mu\nu} + \alpha\epsilon^{\nu\mu\lambda}C_{\mu\lambda})^a.$$

$$X^a_\nu - \frac{1}{g}B^a_\nu = 0. \tag{6}$$

Putting (6) in (5) we get

$$2\sigma\epsilon^{\nu\mu\lambda}(D^{(A)}_\mu B_\lambda)^a + \gamma B^{\nu a} = 0, \tag{7}$$

This is essentially a generalised nonabelian self dual equation for B^a_μ. Our next task is to identify the operator that will correspond to the fermion current $J^a_\mu = \bar{\psi}\gamma_\mu T^a\psi$. The standard procedure is to introduce a source term $\sigma^a_\mu J^a_\mu$ in the fermion Lagrangian where σ^a_μ is an auxiliary field coupled to the operator J^a_μ in question. After bosonising this modified Lagrangian, $\frac{\delta L_B}{\delta\sigma^a_\mu}\mid_{\sigma=0}$ can be identified as the mapping of the fermion current. This shows us that the bosonised current j^a_μ is,

$$j^{a\nu} \equiv X^{\nu a} = (aD^{(C)}_\mu C^{\mu\nu} + \alpha\epsilon^{\nu\mu\lambda}C_{\mu\lambda})^a = \frac{1}{g}B^a_\nu. \tag{8}$$

The last equality follows from (6). It is reassuring to note that the whole structure is internally consistent since in the fermion model, the equation of motion for B^a_μ is

$$\frac{1}{g}B^a_\mu = J^a_\mu.$$

The above operator identity is preserved now as well,

$$\frac{1}{g}B^a_\mu = j^a_\mu.$$

The fermion current conservation equation in (2) in abelian theory reduces to

$$\partial_\mu J^\mu = 0.$$

From the expression of X^a_μ or from the nonabelian self-dual equation (7) it is clear that in the abelian theory the bosonic current conservation is valid as well,

$$\partial_\mu j^\mu = 0. \tag{9}$$

This makes the mapping between the currents J_μ and j_μ unambiguous. It is important to note that j_μ is a topological current, meaning that its conservation is assured by construction.

The Hamiltonian in the static limit simply reduces to the Lagrangian with a negative sign,

$$H_B = -L_B. \tag{10}$$

In the next section, we will show that the abelian bosonised model has a local gauge invariance. This gauge symmetry along with the set of time independent equations of motion helps us to solve the abelian model completely.

III. Abelian theory

In the abelian case, one can replace the covariant derivatives by simple derivatives and (7) is reduced to

$$2\sigma\epsilon^{\nu\mu\lambda}\partial_\mu B_\lambda + \gamma B^\nu = 0, \tag{11}$$

We are interested in the behaviour of the matter density B_0 in the static limit. Hence all time derivatives are dropped. The above equation is broken up in component form,

$$\gamma B_0 + 2\sigma B_{12} = 0$$

$$-\gamma B_1 + 2\sigma \partial_2 B_0 = 0, \quad \gamma B_2 + 2\sigma \partial_1 B_0 = 0. \tag{12}$$

From the last two equations we get,

$$\gamma B_{12} = 2\sigma \nabla^2 B_0 = -\frac{\gamma^2}{2\sigma} B_0.$$

Combining with the first of (12), it is found that B_0 satisfies the time independent Helmholtz equation

$$\nabla^2 B_0 + (\frac{\gamma}{2\sigma})^2 B_0 = 0. \tag{13}$$

In fact the above equation is true for B_μ.

Now we show the gauge invariance in the model. The Lagrangian is

$$L_B = -\frac{a}{4} C_{\mu\nu} C^{\mu\nu} - \frac{1}{2g} B_\mu B^\mu + \frac{\alpha}{2} \epsilon^{\mu\nu\lambda} C_\mu C_{\nu\lambda}. \tag{14}$$

Rewriting

$$C_\mu = B_\mu + \gamma A_\mu + \sigma K_\mu = B_\mu + \bar{A}_\mu$$

the field tensor breaks in to two decoupled parts,

$$C_{\mu\nu} = B_{\mu\nu} + \bar{A}_{\mu\nu}.$$

In terms of these redefinitions, (14) becomes

$$L_B = -\frac{a}{4} (B + \bar{A})_{\mu\nu} (B + \bar{A})^{\mu\nu} - \frac{1}{2g} B_\mu B^\mu + \frac{\alpha}{2} \epsilon^{\mu\nu\lambda} (B + \bar{A})_\mu (B + \bar{A})_{\nu\lambda}. \tag{15}$$

Clearly the action is invariant up to a total derivative under the local gauge transformation,

$$\bar{A}_\mu \rightarrow \bar{A}_\mu + \partial_\mu \phi$$

where ϕ is some arbitrary function.

This allows us to choose a gauge

$$\bar{A}_0 \equiv \gamma A_0 + 2\sigma A_{12} = 0. \tag{16}$$

which makes $\bar{A}_{0i} = 0$ and $B_{0i} = -\partial_i B_0$ in the static case. Using this gauge and static expressions, we simplify the components related to K_μ field,

$$K_\mu = 2\epsilon_{\mu\nu\lambda} \partial^\nu A^\lambda,$$

$$K_0 = 2A_{12}, \quad K_1 = -2\partial_2 A_0, \quad K_2 = 2\partial_1 A_0,$$
$$K^{i0} = 2\partial^i A_{12}, \quad K_{12} = -2\nabla^2 A_0.$$

Now, from (6), for $\nu = 0$ we get,

$$(\frac{a\gamma^2}{4\sigma^2} + \frac{\alpha\gamma}{\sigma} + \frac{1}{g})B_0 + 2\sigma(\nabla^2 A_0 + (\frac{\gamma}{2\sigma})^2 A_0) = 0,$$

(where (13) has been used), and for $\nu = 1$ and $\nu = 2$ we get,

$$-a\partial_2 C_{12} + 2\alpha\partial_2 B_0 + \frac{B_1}{g} = 0, \quad -a\partial_1 C_{12} + 2\alpha\partial_1 B_0 - \frac{B_2}{g} = 0,$$

which are combined to give,

$$(\frac{\gamma}{2\sigma})(\frac{a\gamma^2}{4\sigma^2} + \frac{\alpha\gamma}{\sigma} + \frac{1}{g})B_0 - 2\sigma a\nabla^2(\nabla^2 A_0 + (\frac{\gamma}{2\sigma})^2 A_0) = 0.$$

From the above set of equations, we finally obtain an equation involving A_0 only,

$$\nabla^2(\nabla^2 A_0 + (\frac{\gamma}{2\sigma})^2 A_0) + \frac{\gamma}{2a\sigma}(\nabla^2 A_0 + (\frac{\gamma}{2\sigma})^2 A_0) = 0. \quad (17)$$

Note that for small a we have approximately

$$\nabla^2 A_0 + (\frac{\gamma}{2\sigma})^2 A_0 = 0,$$

which is identical to (13).

We now consider two special cases: **(i)** $\gamma = 0, \quad \sigma = 0$, the Thirring model [7] and **(ii)** $g = 0, \quad \sigma = 0$, the Deser Redlich model [5]. Note that the third option, i.e., bosonisation of the free fermion theory with $\gamma = \sigma = g = 0$ is not permissible in this scheme as g^{-1} is present.

In Case **(i)**,the set of equations of motion in (5,6,7) now reduce to the single equation.

$$a\partial_\mu B^{\mu\nu} + \alpha\epsilon^{\nu\mu\lambda}B_{\mu\lambda} - \frac{B^\nu}{g} = 0. \quad (18)$$

Breaking it into components, we end up with the equations,

$$2\alpha B_{12} + a\nabla^2 B_0 - \frac{B_0}{g} = 0; \quad \frac{B_{12}}{g} - a\nabla^2 B_{12} + 2\alpha\nabla^2 B_0 = 0. \quad (19)$$

This reproduces a static equation of motion involving only B_0,

$$a^2(\nabla^2)^2 B_0 - (\frac{2a}{g} - 4\alpha^2)\nabla^2 B_0 + \frac{B_0}{g^2} = 0. \quad (20)$$

Rewriting the above equation in the form, where a^2 has been dropped,

$$\nabla^2 B_0 + \frac{1}{(2g\alpha)^2}(1 - \frac{a}{2g\alpha^2})^{-1} B_0 = 0,$$

we make an expansion in a,

$$\nabla^2 B_0 + \frac{1}{(2g\alpha)^2}(1 + \frac{a}{2g\alpha^2} + ...)B_0 = 0.$$

Note that the B_0 mass term is renormalised by fermion mass corrections. With $C_\mu = B_\mu$, the bosonised Lagrangian and current reduce to the well known forms,

$$L_B = -\frac{a}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2g} B_\mu B^\mu + \alpha \epsilon^{\mu\nu\lambda} B_\mu \partial_\nu B_\lambda.$$

$$X^\nu = a \partial_\mu B^{\mu\nu} + \alpha \epsilon^{\nu\mu\lambda} B_{\mu\lambda}.$$

In Case **(ii)**, for $\sigma = 0$, $C_\mu = B_\mu + \gamma A_\mu$ and the Lagrangian becomes,

$$L_{DR} = -\frac{a}{4} (B + \gamma A)_{\mu\nu} (B + \gamma A)^{\mu\nu} - \frac{m}{6\pi} B_\mu B^\mu + \alpha \epsilon^{\mu\nu\lambda} (B + \gamma A)_\mu \partial_\nu (B + \gamma A)_\lambda. \tag{21}$$

The above Lagrangian breaks up into two pieces, a B_μ independent one,

$$L_{DR}(A) = -\frac{a\gamma^2}{4} A_{\mu\nu} A^{\mu\nu} + \alpha\gamma \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda, \tag{22}$$

and a B_μ dependent one,

$$L_{DR}(A, B) = -\frac{a}{4} B_{\mu\nu} B^{\mu\nu} + \alpha \epsilon^{\mu\nu\lambda} B_\mu \partial_\nu B_\lambda$$

$$-\frac{1}{2g} B_\mu B^\mu - \frac{a\gamma}{2} B_{\mu\nu} A^{\mu\nu} + 2\alpha\gamma \epsilon^{\mu\nu\lambda} B_\mu \partial_\nu A_\lambda. \tag{23}$$

Rewriting the latter equation in the following form,

$$L_{DR}(A, B) = B_\mu P^{\mu\nu} B_\nu + B_\mu Q^\mu, \tag{24}$$

where,

$$P^{\mu\nu} = -\frac{1}{2g} g^{\mu\nu} + \frac{a}{2} (g^{\mu\nu} \partial^\mu \partial_\mu - \partial^\mu \partial^\nu) - \alpha \epsilon^{\mu\nu\lambda} \partial_\lambda,$$

$$Q^\mu = \gamma (a (g^{\mu\nu} \partial^\mu \partial_\mu - \partial^\mu \partial^\nu) - 2\alpha \epsilon^{\mu\nu\lambda} \partial_\lambda) A_\nu.$$

Performing the gaussian integration for B_μ leads to the formal result,

$$L_{DR}(A, B) \approx -\frac{1}{4} Q^\mu (P^{-1})_{\mu\nu} Q^\nu. \tag{25}$$

At present we are only interested in getting local terms with smaller number of derivatives, and hence we take the inverse of $P_{\mu\nu}$ as simply

$$(P^{-1})_{\mu\nu} \approx -2g g_{\mu\nu}.$$

No gauge fixing is cosidered so far. Substituting this back in (25) yields,

$$L_{DR}(A, B) = \frac{g\gamma^2}{2} [(a (g^{\mu\nu} \partial^\lambda \partial_\lambda - \partial^\mu \partial^\nu) - 2\alpha \epsilon^{\mu\nu\lambda} \partial_\lambda) A_\nu]$$

$$[(a (g^{\mu\eta} \partial^\rho \partial_\rho - \partial^\mu \partial^\eta) - 2\alpha \epsilon^{\mu\eta\rho} \partial_\rho) A_\eta]. \tag{26}$$

Keeping in mind the condition of lowest number of derivatives, we take only the following contribution in the effective action,

$$L_{DR}(A,B) = g(\alpha\gamma)^2 A_{\mu\nu}A^{\mu\nu} = g(\frac{\gamma}{4\pi})^2\frac{6\pi}{m}A_{\mu\nu}A^{\mu\nu}. \tag{27}$$

Thus we notice that in this order the coefficient of the Maxwell term in A_μ gets modified whereas the Chern-Simons term in A_μ remains unaltered. The final form of the action to the order stated is,

$$L_{DR} = -\gamma^2(\frac{g}{\pi^2} - \frac{a}{4})A_{\mu\nu}A^{\mu\nu} + \alpha\gamma\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda, \tag{28}$$

This is exactly the model studied in [5], if following [5] the Thirring coupling g is taken to be proportional to m^{-1}.

Let us now discuss the effects of higher order (in m^{-1}) Seelay terms in the fermion determinant in the full theory. Considering the next Seelay term our Lagrangian will be,

$$\bar{L}_B = L_B + y\epsilon^{\mu\nu\lambda}C_\mu\partial^\rho\partial_\rho\partial_\nu C_\lambda, \tag{29}$$

where L_B is given in (14) and y is of order $O(m^{-2})$. Clearly the equations of motion in (5), (6) and (7) will remain unchanged structurally, with X_ν changing to $\bar{X}_\nu$,

$$\bar{X}^\nu = X^\nu + y\epsilon^{\nu\mu\lambda}\partial^\rho\partial_\rho C_{\mu\lambda}. \tag{30}$$

However, in the full theory this will not change the behaviour of B_0. On the other hand, in the pure Thirring model, the B_μ-equation in (7) is modified to,

$$a\partial_\mu B^{\mu\nu} + \epsilon^{\nu\mu\lambda}(\alpha + y\partial^2)B_{\mu\lambda} - \frac{B^\nu}{g} = 0. \tag{31}$$

The resulting time-independent equations are,

$$\frac{B_{12}}{g} - a\nabla^2 B_{12} + 2(\alpha + y\nabla^2)\nabla^2 B_0 = 0,$$

$$a\nabla^2 B_0 - \frac{B_0}{g} + 2(\alpha + y\nabla^2)B_{12} = 0.$$

Neglecting terms of $O(ay)$ we get,

$$B_{12} = \frac{1}{2\alpha}(1 + \frac{y}{\alpha}\nabla^2)^{-1}(\frac{B_0}{g} - a\nabla^2 B_0)$$

$$\approx \frac{1}{2\alpha}(\frac{B_0}{g} - (a - \frac{y}{\alpha g})\nabla^2 B_0).$$

Hence the B_0-equation becomes,

$$2\alpha a^2(\nabla^2)^2 B_0 + (4\alpha^2 - \frac{2a}{g} + \frac{y}{\alpha g^2})\nabla^2 B_0 + \frac{B_0}{g^2} = 0. \tag{32}$$

Comparing with (20) we note that now the first term can not be ignored. The next Seelay term, $(\epsilon^{\mu\nu\lambda}B_\mu B_{\nu\lambda})^2$ obviously causes more complications in the pure Thirring model and obtaining an equation involving B_0 only is non-trivial. In the full theory, the generic feature is that these type of changes leaves the B_0-equation intact.

IV. Non-abelian Theory:

As has been emphasised before [6], the results are far more complicated in the non-abelian scenario. For arbitrary non-abelian gauge fields A^a_μ, identification between the fermi and bose currents is problematic. From the equations of motion given in (5), (6) and (7), the following covariant conservation equation emerges,

$$(D^{(C)}_\mu j^\mu)^a = 0.) \tag{33}$$

But this is different from (2). Also there is no local gauge invariance in the nonabelian bosonised version due to the nature of cross terms between B^a_μ and $\bar{A}^a_\mu$ present in the theory.

However, these problems can be completely removed for a restricted class of gauge fields, formerly used in [8], that are proportional to the generators of the Cartan subalgebra only,

$$[h^\alpha, h^\beta] = 0, \quad A_- = \sum_{\alpha=1}^{r} A_\alpha h^\alpha, \quad A_+ = -\sum_{\alpha=1}^{r} A^*_\alpha h^\alpha, \tag{34}$$

where

$$A_\pm = A_1 \pm iA_2.$$

In the fermion problem [8] it was assumed that the fermion fields ψ are proportional to the ladder generators e^α only,

$$\psi = \psi_\alpha e^\alpha, \tag{35}$$

where,

$$[e^\alpha, e^{-\beta}] = \delta_{\alpha\beta} h^\alpha, \quad [h^\alpha, e^\beta] = K_{\beta\alpha} e^\beta, \quad [h^\alpha, e^{-\beta}] = -K_{\beta\alpha} e^{-\beta}.$$

Note that $(e^\beta)^+ = -e^{-\beta}$, $(h^\beta)^+ = h^\beta$ and the Cartan Matrix $K_{\alpha\beta}$ is real and for $SU(N)$ symmetric. Thus in the fermion model the charge is

$$J_0 \approx [\psi^+, \psi] \approx h^\alpha$$

this shows that the charge is also in the Cartan subalgebra. This ansatz prompts us to restrict B^a_μ in the Cartan subalgebra. But with A^a_μ already in the Cartan subalgebra the entire system is reduced to essentially an abelian one, with just a non-interacting index tagged along each of the fields, reminding us of the nonabelian nature. Hence, in the lowest order of inverse fermion mass, we get a number of decoupled static Helmoltz equations for the non-abelian charge B^a_0,

$$\nabla^2 B^a_0 + (\frac{\gamma}{2\sigma})^2 B^a_0 = 0. \tag{36}$$

V. Conclusion

As an application of 2+1-dimensional bosonisation, we have studied thoroughly the non-minimally gauged massive Thirring model. Computing the fermion determinant up to first order in inverse fermion mass, the charge in the abelian model is shown to obey the (static) massive Helmholtz equation. Special cases leading to known results in the Thirring and Deser-Redlich models are derived. For abelian gauge group, effects of higher order terms are also discussed. In case of non-abelian gauge fields, a restricted class of gauge fields reduces the system to essentially a group of decoulped abelian ones and the charges behave in an identical fashion to the abelian one.

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References

[1] S. Coleman, Phys. Rev. D11 (1975)2088.

[2] S. Mandelstam, *ibid* 3026.

[3] M. Luscher, Nucl. Phys. B326 (1989)557; E. C. Marino, Phys. Lett. B263 (1991)63.

[4] E. Fradkin and F. Schaposnik, Phys. Lett. B338 (1994)253; R. Banerjee, *ibid.* 358 (1995)297.

[5] S. Deser and A. N. Redlich, Phys. Rev. Lett. 61 (1988)1541.

[6] N. Banerjee, R. Banerjee and S. Ghosh, Nucl. Phys. B481 (1996)421.

[7] See for example, E. Abdalla, M. C. B. Abdalla and K. D. Rothe in *Two Dimensional Quantum Field Theory*, World Sc., 1991.

[8] R. Jackiw and S.-Y. Pi, Prog. Theor. Phys. Suppl. 107 (1992)1; B. Grossman, Phys. Rev. Lett. 65 (1990)3230.