On Some Measurement Problems In Economics

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Preface

In many branches of economic theory, quantitative relationships play a major role. Translating actions of individuals or groups of individuals, in the various societal processes of production, distribution or consumption, into measurable or quantifiable notions has been one of the most important problems. When we take the society as a whole, diverse economic activities of its members need to be collectively presented and it has given rise to interesting aggregation problems. Economists have tried to solve these problems by constructing social index numbers. Similarly, quantifying actions of individual units (for example, a firm / a consumer) has brought forth the notions such as production functions and demand functions. In this presentation, we discuss some such problems and introduce a few notions of quantification. The first three essays deal with the society as a whole. There, we discuss some income distributional problems. The next two essays are concerned with individual actions. One of these deals with theories of production and the other involves the individual as a consumer.

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Chapter 1

General Introduction

The purpose of this chapter is to prepare the background for presentation of the materials in different chapters of the dissertation.

1.1 Income Inequality

In income distribution theory sometimes it is required to rank alternative income profiles on the basis of some value judgements. The most common value judgements are the following : social preference for a more *equitable* profile and *higher* incomes, *celeris paribus*. These value judgements are referred to as *equity preference* and *efficiency preference* respectively.

To make these notions explicit, consider a community of n persons with income profile $y = (y_1, y_2, \ldots, y_n)$, where $y_i \geq 0$ is the income earned by individual i. For a fixed $n \geq 1$, the set of all income profiles is D^n, the non-negative orthant of the Euclidean n-space R^n with the origin deleted. The set of all possible income profiles is $D = \bigcup_{n \in N} D^n$, where N is the set of all positive integers. For any function $f : D \to R^1$ we denote the restriction of f on D^n by f^n, where $n \in N$. For all $n \in N$ and $y \in D^n$, we write $\lambda(y) = \frac{1}{n}\sum_{i=1}^{n} y_i$ for the mean of y. For all $n \in N$ and $y \in D^n$, $\bar{y}$ and $\overset{0}{y}$ represent respectively the illfare and welfare ranked permutations of y, that is, $\bar{y}_1 \leq \bar{y}_2 \leq \ldots \leq \bar{y}_n$ and $\overset{0}{y}_1 \geq \overset{0}{y}_2 \geq \ldots \geq \overset{0}{y}_n$. An $n-$coordinated vector of ones is denoted by 1^n, $n \in N$.

A reflexive, complete and transitive ranking of alternative social states is usually referred to as a *social welfare ordering (SWO)*. If an SWO is assumed to be continuous, then it can be represented by a *social welfare function (SWF)* that assigns a real number to each social state. Atkinson (1970) showed that many seemingly unrelated procedures for ranking income profiles are equivalent. His approach is based on the *Lorenz curve (LC)* which indicates the share of the total income enjoyed by the bottom t proportion ($0 \le t \le 1$) of the population. Formally, in terms of the illfare ranked permutation $\bar{x}$ of a given profile of income x over a population size n, the LC $L(x, \frac{i}{n})$ is a plot of $\frac{1}{n\lambda(x)}\sum_{j=1}^{i}\bar{x}_j$ against $\frac{i}{n}$, as i goes from 0 to n, where $L(x, 0) = 0$. For discussing the Atkinson result, we need to define the Lorenz domination relation. An income profile $x \in D^n$ is said to *Lorenz dominate* another profile $y \in D^n$ in the weak sense if the LC of x lies nowhere below the LC of y. Formally, x *weakly Lorenz dominates* y ($x \succeq_L y$ for short) if

$$\frac{1}{n\lambda(x)}\sum_{j=1}^{i}\bar{x}_j \ge \frac{1}{n\lambda(y)}\sum_{j=1}^{i}\bar{y}_j \tag{1.1}$$

for all $i = 0, 1, 2, \ldots, n$. We say that x *strictly dominates* y, what we write $x \succeq_{L^*} y$, if (1.1) holds with the additional restriction that there will be strict inequality for at least one $i < n$.

Atkinson proved the following remarkable consequence of the the strict Lorenz domination relation.

Theorem 1.1 *(Atkinson 1970) Suppose that the SWF is utilitarian symmetric*[1], *so* $W^n(x) = \sum_{i=1}^{n} U(x_i)$ *where* $U(x_i)$ *is the utility of person having income* x_i. *Then for* $x, y \in D^n$, *where* $\lambda(x) = \lambda(y)$, *the following conditions are equivalent :*
(i) $x \succeq_{L^*} y$
(ii) $W^n(x) > W^n(y)$ *for any strictly concave real valued utility function* U.

Dasgupta, Sen and Starrett (1973) and Rothschild and Stiglitz (1973) generalised the Atkinson theorem by weakening the concavity and the additivity assumptions sufficiently.

Theorem 1.2 *(Dasgupta, Sen and Starrett 1973) Let* x, y *be two profiles of income such that* $x, y \in D^n$ *and* $\lambda(x) = \lambda(y)$. *Then the following statements are equivalent:*
(i) $x \succeq_{L^*} y$

[1] A SWF is said to be symmetric if it is invariant with respect to a permutation of its arguments.

(ii) $W^n(x) > W^n(y)$ *for all strictly S-concave*[2] *SWFs* $W^n : D^n \rightarrow R^1$.

The condition of Lorenz domination (i.e., $x \succeq_{L^*} y$) is also equivalent to the condition that the profile $\bar{x}$ can be obtained from the profile $\bar{y}$ by a finite sequence of rank preserving transformations transferring income from the rich to the poor, i.e., $\bar{y}$ can be transformed into $\bar{x}$ by a finite sequence of transformations of the form

$$\bar{y}_i^{\alpha+1} = \bar{y}_i^\alpha + c^\alpha \leq \bar{y}_j^\alpha$$

$$\bar{y}_j^{\alpha+1} = \bar{y}_j^\alpha - c^\alpha \geq y_i^\alpha$$

for $i < j$ and $c^\alpha > 0$, with $\bar{y}_k^{\alpha+1} = \bar{y}_k^\alpha$ if $k \neq i, j$.

Note that the welfare implications of Lorenz domination given by the above theorems are for the identical mean income case[3]. Thus efficiency considerations are absent. The ability of the LCs to provide unambiguous ranking of income profiles with differing means is considerably improved by considering an extension of LC suggested by Shorrocks (1983) known as the *generalised Lorenz curve(GLC)*. The GLC is constructed by scaling up the LC by the mean income. Formally, in terms of the illfare ranked permutation $\bar{x}$ of a given profile $x \in D^n$, the GLC $GL(x, \frac{i}{n})$ is a plot of $\frac{1}{n}\sum_{j=1}^{i} \bar{x}_j$ against $\frac{i}{n}$ as i goes from 0 to n, where $GL(x, 0) = 0$. We say that an income profile $x \in D^n$ weakly dominates another profile $y \in D^n$ in the *generalised Lorenz sense* ($x \succeq_{GL} y$ for short)

$$\frac{1}{n}\sum_{j=1}^{i} \bar{x}_j \geq \frac{1}{n}\sum_{j=1}^{i} \bar{y}_j \tag{1.2}$$

for all $i = 0, 1, \ldots, n$. x strictly dominates y in the generalised Lorenz sense, what we write $x \succeq_{GL^*} y$, if (1.2) holds with strict inequality for at least one $i \leq n$.

[2]A function $f^n : D^n \rightarrow R^1$ is S-concave if $f^n(B\,x) \geq f^n(x)$ for all $x \in D^n$, where B is any bi-stochastic matrix of order n. A square matrix of order n is said to be bi-stochastic if all its entries are non-negative and each of its rows and columns sums to one. f^n is strictly S-concave if the weak inequality is replaced by a strict inequality whenever $B\,x$ is not a permutation of x. Two interesting examples of strictly S-concave SWFs are the average utilitarian rule $W_U^n(x) = \sum_{i=1}^{n} U(x_i)/n$ where U is increasing and strictly concave and the Gini SWF $W_G^n(x) = \sum_{i=1}^{n} [2(n-i)+1]\bar{x}_i/n^2$

[3]The Lorenz ordering is a quasiordering. That is, if two LCs intersect we cannot rank the profiles unambiguously by all strictly S-concave SWFs.

The following theorem relates GLCs with ordering of income profiles by SWFs.

Theorem 1.3 *(Shorrocks 1983) Let $x, y \in D^n$ be two arbitrarily given income distributions. Then the following statements are equivalent:*
(i) $x \succeq_{GL^*} y$
(ii) $W^n(x) > W^n(y)$ for all increasing, strictly S-concave SWFs $W^n : D^n \to R^1$.

While the Lorenz criterion provides a quasi ordering of income profiles, an alternative statistic that completely orders the set of income profiles is an inequality index. An inequality index I^n is a real valued function defined on D^n. I^n gives us a scalar representation of interpersonal income differences within a population of size n. I^n is the restriction of the function $I : D \to R^1$, which offers comparisons of inequality across populations. Clearly, for I^n to be well-defined we need $n \geq 2$. An inequality index I_R is said to be a *relative inequality index* if proportional changes in all incomes do not change inequality, that is, for all $n \in N$, $x \in D^n$,

$$I_R^n(c.x) = I_R^n(x) \tag{1.3}$$

where $c > 0$ is any scalar.

An alternative possibility to the invariance of inequality under equiproportionate income changes is that inequality should remain unaltered under equal absolute changes in incomes. This idea was proposed by Kolm (1976a, 1976b). Such indices are referred to as *absolute* indices of inequality. Formally, an inequality index I_A is said to be an *absolute inequality index* if for all $n \in N$, $x \in D^n$,

$$I_A^n(x + c.1^n) = I_A^n(x) \tag{1.4}$$

where c is any scalar such that $x + c.1^n \in D^n$.

The problem of choice between absolute and relative inequality is essentially a matter of value judgements[4].

[4]Bossert and Pfingsten (1990) (also Pfingsten (1986)) proposed an intermediate position between these two concepts by stipulating that a convex combination of relative and absolute variations in income should keep inequality unchanged. Formally, an inequality index I_μ is an *intermediate inequality* index if for all $n \in N, x \in D^n$,

$$I_\mu^n(x + c.(\mu.x + (1-\mu).1^n)) = I_\mu^n(x), \tag{1.5}$$

where $0 \leq \mu \leq 1$ and c is a scalar such that $x + c.(\mu.x + (1-\mu).1^n) \in D^n$. Here μ is a value judgement parameter. Equation (1.5) reduces to (1.3) ((1.4)) if μ is equal to one (zero).

The classes of all possible inequality indices satisfying (1.3), (1.4) and (1.5) respectively may be quite large. The following postulates have been suggested in the literature for an arbitrary inequality index $I : D \to R^1$ to reduce the number of alternative indices.

Normalisation (NM) : For all $n \in N$, $I^n(c.1^n) = 0$, where $c > 0$ is any scalar.

Condition NM says that when all incomes are equal, the inequality index takes the value zero.

Symmetry (SM) : For all $n \in N, x \in D^n$, $I^n(Px) = I^n(x)$, where P is any permutation[5] matrix of order n.

Condition SM means that the inequality index I remains unchanged when individuals exchange places. That is, the individuals are characterised only by their incomes.

Pigou-Dalton Transfers Principle (PD) : Suppose y is obtained from $x \in D^n$ by transferring income from a poorer person j to a richer person i, i.e., for some i, j (i) $x_j \leq x_i$, (ii) $y_i - x_i = x_j - y_j > 0$ and (iii) $y_k = x_k$ for all $k \neq i, j$. Then $I^n(y) > I^n(x)$.

Thus according to PD inequality should increase when income is transferred from one person to someone richer. Similarly I^n decreases under an equalising income transfer.

Dalton Population Principle (PP) : For all $n \in N, x \in D^n$, $I^{nk}(y) = I^n(x)$ where y is a k-fold replication of x, that is, $y = (y^{(1)}, y^{(2)}, \ldots, y^{(k)})$ and each $y^{(i)} = x$.

This postulate helps us to compare income profiles over different population sizes. Under PP, pooling of a particular income profile several times does not lead to any change in inequality.

Continuity (CON) : For all $n \in N$, I^n is a continuous function.

CON says that infinitesimal changes in incomes should lead to infinitesimal changes in inequality.

The following theorem of Foster (1985) identifies the class of inequality indices that implies and is implied by the Lorenz quasi-ordering.

Theorem 1.4 *(Foster 1985) Let $x^1, x^2 \in D$ be arbitrary. Then the following statements are equivalent :*

[5] A permutation matrix is a bi-stochastic matrix with exactly one positive entry in each row and each column.

(i) $I(x^1) > I(x^2)$ *for all relative inequality indices* $I : D \rightarrow R^1$ *that satisfy* **SM, PD** *and* **PP**.
(ii) $x^2 \succeq_{L^*} x^1$

This theorem shows that an unanimous ranking of income profiles by relative inequality indices can be obtained through the pairwise comparison of LCs of the profiles. Foster (1985) also has obtained particular cases of the above theorem for the fixed (variable) mean and variable (fixed) population size.

Moyes (1987) referred to the GLC of the profile $x - \lambda(x).1^n$ as the *absolute Lorenz curve* of x. The absolute LC $LA(x, i/n)$ coincides with the horizontal axis when incomes are equal. It is decreasing with i for $0 \leq i \leq i^*$ and increasing for $i^* < i \leq n$ where i^* is such that $\bar{x}_{i^*} < \lambda(x) \leq \bar{x}_{i^*+1}$.

Moyes showed that the unanimous ranking generated by the set of absolute inequality indices can be obtained through the pairwise comparisons of the absolute Lorenz curves of the alternative income distributions. For intermediate inequality indices a consistency result of this type was developed in Chakravarty (1989).

Before we conclude our discussion on inequality it may be worthwhile to provide some examples of different types of inequality indices. A general example of relative indices that meets NM, SM, PD, PP and CON is the Shorrocks (1980) *generalised entropy* family

$$S_c^n(x) = \frac{1}{nc(c-1)}\sum_{i=1}^{n}((\frac{x_i}{\lambda(x)})^c - 1), \quad c \neq 0, 1 \tag{1.6}$$

$$S_0^n(x) = \frac{1}{n}\sum_{i=1}^{n}\log\frac{\lambda(x)}{x_i} \tag{1.7}$$

$$S_1^n(x) = \frac{1}{n}\sum_{i=1}^{n}\frac{x_i}{\lambda(x)}\log\frac{x_i}{\lambda(x)} \tag{1.8}$$

where, $x \in D^n$. (It should be noted that if $x_i = 0$ for some i, then continuity of S_c^n will require that $c > 0$.) The parameter c reflects different perceptions of inequality. As c decreases S_c^n becomes more sensitive to transfers lower down the scale. S_0^n is the mean logarithmic deviation, S_1^n is the Theil (1967) entropy index and $2.S_2^n$ is the squared coefficient of variation. The Atkinson (1970) index $A^n(x)$ can be obtained from $S_c^n(x)$ via the increasing transform

$$A^n(x) = 1 - [c(c-1)S_c^n(x) + 1]^{\frac{1}{c}}, \quad for\ c \neq 0, \quad c < 1 \tag{1.9}$$

$$A^n(x) = \quad 1 - e^{-S_c^n(x)}, \; for \; c = 0. \tag{1.10}$$

Another example of a relative inequality index satisfying the above properties is the Gini coefficient

$$G^n(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mid x_i - x_j \mid /2n^2\lambda(x) \tag{1.11}$$

Among many interesting absolute indices which meet the above postulates are the variance $2(\lambda(x))^2 S_2^n(x)$, the absolute Gini index $\lambda(x)G^n(x)$ and Kolm's (1976a) index

$$K_\alpha(x) = \frac{1}{\alpha}\log \frac{1}{n}\sum_{i=1}^{n} e^{\alpha(\lambda(x)-x_i)}, \tag{1.12}$$

where $\alpha > 0$ is a transfer sensitivity parameter (For further discussions see Sen (1973), Cowell (1977), Blackorby and Donaldson (1978,1980,1984), Fields and Fei (1978), Donaldson and Weymark (1980,1983), Kakwani (1980), Weymark (1981), Basu (1987), Lambert (1989) and Chakravarty (1990)).

1.2 Tax progression and Income Inequality

The principle of taxation adopted by any government is an important economic decision affecting its people in various ways. An important job is therefore to look at the degree of progression of the tax structure. Usually, we call a fiscal system to be *progressive* if the average rate of taxation increases with income before tax. That is, a tax function with an increasing average rate results in a shift in the distribution of income toward equality. If the pre-tax income distribution is given, we can, therefore, use the Lorenz criterion to decide whether a tax system is egalitarian or not.

1.2.1 Further Notation and Definitions

Throughout the thesis, in the context of taxation results, we will assume that the set of feasible income distributions in an n-person society is $D_+^n = \{x \in R^n | x_i > 0 \; for \; all \; i\}$. A *taxation scheme* is a function $f : D^1 \rightarrow R^1$ that associates a pre-tax income u to a post-tax income $f(u)$; $t(u) = u - f(u)$ is the tax liability. The person with income level u will be called a *tax-payer, unaffected or subsidized* according as $t(u)$ is *positive, zero or negative*. For any $x \in D^n$, we will denote $(f(x_1), \ldots, f(x_n))$ by $f(x)$.

We now state some properties of income taxation in terms of an arbitrary $f : D^1 \rightarrow R^1$.

(a) **Weak Incentive Preservation (IP)** : f is non-decreasing in pre-tax incomes, that is, for all $u > v > 0, f(u) \geq f(v) > 0$.

(b) **Strong Average Progression (AP)** : Average tax rate is increasing in pre-tax incomes, that is, for all $u > v > 0, (u - f(u))/u > (v - f(v))/v$.

(c) **Strong Minimal Progression (MP)** : Tax liability is increasing in pre-tax incomes, that is, for all $u > v > 0, u - f(u) > v - f(v)$.

Properties IP and MP were introduced by Fei (1981). It should be noted that if tax liabilities are positive then AP implies MP. Strong version of IP can be obtained by replacing the weak inequality by a strict inequality. Similarly, weak versions of AP and MP require replacement of strict inequalities by weak ones.

1.2.2 The Existing Results

Kakwani (1977) showed that average progression will make the post-tax distribution more equitable than the pre-tax distribution according to the Lorenz criterion[6]. Jakobsson (1976) and later on Eichhorn, Funke and Richter (1984) and Thon (1987) showed that the implication is, in fact, an equivalence. Precisely Eichhorn, Funke and Richter proved the following result.

Theorem 1.5 *(Eichhorn, Funke and Richter 1984) For all $n \geq 2$, for all $x \in D^n_+$, $f(x) \succeq_{L^*} x$ if and only if the tax function satisfies IP and AP.*

This theorem shows that tax cuts according to a fiscal scheme satisfying IP and AP reduces relative inequality and vice versa.

The absolute counterpart to the Eichhorn-Funke-Richter (EFR) result is due to Moyes (1988). He showed that minimal progressivity along with IP is necessary and sufficient for the tax function to be uniformly equalising according to the absolute Lorenz domination. Formally, we have,

Theorem 1.6 *(Moyes 1988) For all $n \geq 2$, for all $x \in D^n_+$, $f(x) \succeq_{LA^*} x$ if and only if the taxation scheme satisfies IP and MP.*

[6] For demonstrating that an average progressive tax function reduces inequality, Kakwani implicitly assumed that it is weakly incentive preserving. In fact, in almost all such distributional comparisons, the preservation property is taken as an assumption. (See, for example, Jakobsson (1976), Lambert (1989) and Chakravarty (1990)).

It is important to note that both the EFR and Moyes results are proved for tax functions that are assumed to be independent of the population size. In **chapter 2** we propose two concepts of domination principles, building on an idea of Runciman (1966), which lead to identical taxation schemes as proposed by EFR and Moyes under the condition of population size independence. These dominance relations differ from the Lorenz criteria if the population size is given. This motivates us to look into the nature of the equivalent taxation schemes for the different inequality criteria when the population size is fixed.

1.3 Relative Deprivation and Welfare

For any person in a society the feeling of *relative deprivation* arises out of the comparison of his situation with the situations of persons better off than him. Runciman (1966) used the example of promotion to describe an individual's feeling of relative deprivation : 'The more people a man sees promoted when he is not promoted himself, the more people he may compare himself within a situation where the comparison will make him relatively deprived' (op. cit., p. 19). Thus, according to Runciman the extent of deprivation felt by an individual for not being promoted is an increasing function of the number of individuals who have been promoted. Yitzhaki (1979) considered relative deprivation in terms of income and quantified a particular case of Runciman's statement. He showed that one plausible index of average relative deprivation in a society is the absolute Gini index : the product of the Gini coefficient and the mean income of the society. Alternatives and variations of Yitzhaki's index have been suggested by many authors including Chakravarty and Chakraborty (1984), Kakwani (1984), Berrebi and Silber (1985) and Chakravarty (1990).

Hey and Lambert (1980) provided an alternative derivation of Yitzhaki's result. Essential to their alternaive characterization is Runciman's (1966) remark : 'The magnitude of relative deprivation is the extent of the difference between the desired situation and that of the person desiring it' (op. cit., p. 10). Kakwani (1984) used such differences, expressed as proportions of the total income, to generate the *Relative Deprivation Curve (RDC)* and showed that the area under this curve is the Gini coefficient for the society.

1.3.1 The Relative Deprivation Curve and the Associated Ordering

Following Runciman in an n-person society with income distribution x, the deprivation felt by an individual with income x_i relative to income x_j can be considered to be

$$\begin{aligned} d_{ij} &= (x_j - x_i)/n\lambda(x) \quad if\ x_j > x_i \\ &= 0 \quad if\ x_j \leq x_i \end{aligned} \tag{1.13}$$

Since the individual with income $\bar{x}_i$ is deprived of all incomes higher than $\bar{x}_i$, the total relative deprivation felt by this person is

$$d_i(x) = \sum_{j=i+1}^{n} (\bar{x}_j - \bar{x}_i)/n\lambda(x) \tag{1.14}$$

The RDC associated with the distribution x is defined as the plot of $d_i(x)$ against the cumulative proportion of population i/n, $i = 0, 1, \ldots, n$; where $d_0(x) = 1$ (see Kakwani (1984)). The extension $d_0(x) = 1$ ensures that the RDC is a closed graph. Clearly, $d_i(x)$ is a decreasing function of i/n.

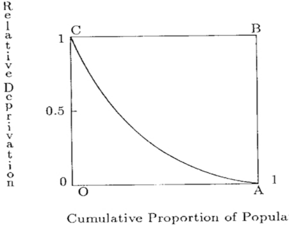

Figure 1.1 The Relative Deprivation Curve

If incomes are equally distributed there is no feeling of deprivation by any person. In this case the *RDC* coincides with the *no deprivation line* OA. On the other hand, if the entire income is monopolized by the richest person, the curve coincides with the line CB. This is the case of maximum relative deprivation.

We can rewrite $d_i(x)$ in (1.14) as follows

$$d_i(x) = 1 - L_i(x) - (n-i)\bar{x}_i/n\lambda(x) \tag{1.15}$$

where $L_i(x) = \sum_{j=1}^{i} \bar{x}_j/n\lambda(x)$ is the cumulative proportion of income enjoyed by the bottom i $(0 \leq i \leq n)$ proportion of the population. Yitzhaki (1979) and Hey and Lambert (1980) referred to the function $s_i(x) = L_i(x) + (n-i)\bar{x}_i/n\lambda(x)$ as the *relative satisfaction function* of the person with income $\bar{x}_i$. Since $d_i(x)$ is the complement (to 1) of $s_i(x)$, one can work with either $d_i(x)$ or $s_i(x)$. Evidently, by augmenting $L_i(x)$ by the proportion $(n-i)\bar{x}_i/n\lambda(x)$ we get a curve, which we can refer to as the *relative satisfaction curve (RSC)*.

Let us now suppose that the income distribution is represented by a distribution function $F : [0,\infty] \to [0,1]$. $F(z)$ is the cumulative proportion of persons with income less than or equal to z. $F(0) = 0$, $F(\infty) = 1$ and F is increasing. Then for any arbitrary income $z \in [0,\infty]$, the deprivation function $d_i(x)$ in (1.15) becomes

$$d_F(z) = 1 - F_1(z) - (1 - F(z))z/\lambda(F) \tag{1.16}$$

where $F_1(z) = \int_0^z t\ dF(t)/\lambda(F)$ is the ordinate of the LC for a continuum of population with $\lambda(F)$ being the mean income. We observe that $d(d_F(z))/dF(z) = (F(z) - 1)/f(z)$ where $f(z)$ is the income density function. This demonstrates rigorously that the RDC is monotonically decreasing. However, no definite conclusion can be drawn regarding the curvature of the RDC. To see this, note that $d^2(d_z(F))/d^2F(z) = [(f(z))^2 + (1-F(z))f'(z)]/(f(z))^3$ which may be positive, zero or negative depending on the form of F.

Definition 1.1 *Given two income distributions x, $y \in D^n$, we say that x weakly dominates y by the* relative deprivation criterion *($x \succeq_d y$ for short) if the RDC of x lies nowhere above that of y. Formally, $x \succeq_d y$ means that*

$$d_i(x) \leq d_i(y) \tag{1.17}$$

for all $i = 1,\ldots,n$.

Definition 1.2 *Given two income distributions $x,\ y \in D^n$, we say that x weakly dominates y by the* relative satisfaction criterion *($x \succeq_s y$ for short) if the RSC of y lies nowhere above that of x. Formally, $x \succeq_s y$ means that*

$$s_i(x) \geq s_i(y) \tag{1.18}$$

for all $i = 1, \ldots, n$.

Using (1.15) we rewrite $x \succeq_d y$ in (1.17) as

$$L_i(y) + (n-i)y_i/n\lambda(y) \leq L_i(x) + (n-i)x_i/n\lambda(x) \tag{1.19}$$

for all $i = 1, \ldots, n$. (1.19) means that $x \succeq_s y$. Thus, $x \succeq_d y$ and $x \succeq_s y$ are equivalent.

For strict relative deprivation dominance we require strict inequality in (1.17) for at least one i, $1 \leq i < n$. We denote this by $x \succeq_{d^*} y$. Similarly we can define strict relative satisfaction dominance.

In our *second essay* which is presented in **chapter 3** of the thesis we look for the condition that implies and is implied by the requirement that the RDC of x strictly dominates that of y, where x and y are two income distributions of a fixed total over a fixed population size. We also identify the class of average relative deprivation indices consistent with the relative deprivation ordering.

1.3.2 Relative Deprivation Indices

To discuss the Yitzhaki index of relative deprivation briefly, let us suppose that incomes are represented by a distribution function $F : [0,\infty] \rightarrow [0,1]$. For any income level t, $1 - F(t)$ is the proportion of population with income $> t$. Following Runciman, Yitzhaki defined $d^*_F(z) = \int_z^\infty (1 - F(t))dt$ as the extent of deprivation felt by an individual with income z. He then defined the aggregate relative deprivation index as

$$d^*(F) = \int_0^\infty d^*_F(z)dF(z) \tag{1.20}$$

He showed that $d^*(F)$ is equal to the absolute Gini index for the society. Yitzhaki obtained analogously the aggregate satisfaction function as the difference between the mean income and the absolute Gini index.

An interesting interpretation of the Gini index in the relative deprivation framework was given by Sen (1973). In any pairwise comparison, according to Sen, the individual with lower income may suffer from depression upon discovering that his income is lower. The average of all such depressions in all pairwise comparisons becomes the Gini index if the extent of depression is proportional to the differences in the incomes. Kakwani (1980) showed that if the individual's depression is proportional to the square of the income difference, we get the coefficient of variation as the average deprivation index. Berrebi and Silber (1985), by taking different forms of individual deprivation function, interpreted many commonly used indices of inequality (e.g., the Atkinson (1970) and Theil (1967) indices and the coefficient of variation) as relative deprivation indices. Their method is quite ad hoc and as a result the individual deprivation functions considered by them do not satisfy many desirable properties (see Chakravarty (1990)). Chakravarty and Chakraborty (1984) considered a normalised value of the sum of a continuous, increasing and convex transform of d_{ij}'s given by (1.13) as an aggregate index of relative deprivation. They showed that the proposed index is continuous, bounded, translation invariant, symmetric, quasi-convex and satisfies the Daltonian population principle.

It may be observed that all the indices discussed above have been derived without using any concept of welfare. The purpose of our *third essay*, which is presented in chapter 4, is to suggest an index of deprivation using welfare theoretic criteria.

1.4 Marginal and Total Production Cost Indices

An economic measure is ' a real-valued function whose domain is a set of vectors of economic figures and which satisfies a system of economically motivated conditions. The form and content of these conditions depend on what we want to measure' (Eichhorn (1978, p.5). Thus, an economic measure summarises some economic information into a single statistic. In Production Theory, two most commonly used summary statistics are the *total* and *marginal cost functions* . Baye, Deily and Jansen (1991) employed these two cost functions to construct *total* and *marginal cost indices* and examined their use in illuminating the course of an industry's evolution in terms of the impact of factor price changes. Their total cost index compares the total cost of producing an arbitrarily given output level under two factor price regimes. More precisely, their total cost index is defined as the ratio of the total cost of producing a given amount of output in the current period to the corresponding cost in the base period. Similarly, the marginal cost index considered by these authors compares the marginal costs of producing an additional unit of output under two different factor price vectors.

Formally, suppose that n inputs are used for producing a given output. The set of all possible input vectors used for producing different output levels is R^n_+, the non-negative orthant of R^n. $F : R^n_+ \to R^1_+$ is a production function. F is assumed to be continuous, increasing and quasiconcave. The cost function associated with F is

$$C(w, y) = \min_x (w.x \mid F(x) \geq y) \tag{1.21}$$

That is, $C(w, y)$ gives the minimum expenditure on factors $x \in R^n_+$ required to produce output level y when factor prices are given by $w \in R^n_{++}$, where $R^n_{++} = \{w \in R^n \mid w_i > 0 \ for \ all \ i, i = 1, \ldots, n\}$. $C(w, y)$ is non-decreasing, linearly homogeneous, concave and continuous in factor prices (see Diewert (1982)).

Definition 1.3 *The Baye, Deily and Jansen (BDJ) total cost index* $T_R : R^{2n}_{++} \times R^1_+ \to R^1_+$ *is defined by*

$$T_R(w^0, w^1, y) = \frac{C(w^1, y)}{C(w^0, y)} \tag{1.22}$$

where $w^1(w^0)$ *is the factor price vector in time period* $1(0)$.

Thus T_R makes a relative comparison between the total costs for producing the output level y in two time periods.

Definition 1.4 *The BDJ marginal cost index* $M_R : R^{2n}_{++} \times R^1_+ \to R^1_+$*, which compares the marginal costs associated with the total costs in (1.22), is defined by*

$$M_R(w^0, w^1, y) = \frac{C_y(w^1, y)}{C_y(w^0, y)} \tag{1.23}$$

where $C_y(w, y)$ *stands for the partial derivative of* $C(w, y)$ *wlth respect to* y.

The properties of T_R and M_R are stated in the following theorems.

Theorem 1.7 *(Baye, Deily and Jansen, 1991) The total cost index* $T_R(w^0, w^1, y)$ *satisfies the following properties :*

(i) **Homogeneity (HM)** *:* $T_R(w^0, \alpha\, w^1, y) = \alpha T_R(w^0, w^1, y)$ *where* α *is any positive scalar.*

(ii) Monotonicity (MN) : $T_R(w^0, w^1, y) \geq T_R(w^0, \bar{w}^1, y)$ *if* $w^1 \geq \bar{w}^1$,

$$T_R(w^0, w^1, y) \leq T_R(\bar{w}^0, w^1, y) \text{ if } w^0 \geq \bar{w}^0$$

(iii) Dimensionality (DM) : *For any positive scalar* α, $T_R(\alpha\, w^0, \alpha\, w^1, y) = T_R(w^0, w^1, y)$.

(iv) Identity (IN) : $T_R(w^0, w^0, y) = 1$.

Theorem 1.8 *(Baye, Deily and Jansen, 1991) The marginal cost index* $M_R(w^0, w^1, y)$ *satisfies the following properties :*

(i) HM : $M_R(w^0, \alpha\, w^1, y) = \alpha M_R(w^0, w^1, y)$ *where* α *is any positive scalar.*

(ii) Conditional Monotonicity (CM) : *If each factor is normal (that is,* $\partial x_i/\partial y > 0$ *for all* i*), then*

$$M_R(w^0, w^1, y) \geq M_R(w^0, \bar{w}^1, y) \text{ if } w^1 \geq \bar{w}^1,$$
$$M_R(w^0, w^1, y) \leq M_R(\bar{w}^0, w^1, y) \text{ if } w^0 \geq \bar{w}^0$$

(iii) DM : *For any positive scalar* α, $M_R(\alpha\, w^0, \alpha\, w^1, y) = M_R(w^0, w^1, y)$.

(iv) IN : $M_R(w^0, w^0, y) = 1$.

HM means that T_R and M_R are linearly homogeneous in current period factor prices, that is, an equiproportional variation in $w^1 = (w^1_1, \ldots, w^1_n)$ leads to the same kind of variation in T_R and M_R. Property MN of T_R is self explanatory. M_R, on the other hand, meets a restricted version of monotonicity - MN is satisfied if all the factors are normal. DM[7] means that T_R and M_R are homogeneous of degree zero in factor prices. Indices satisfying such property will be called *relative* indices.

Clearly, the BDJ total/marginal cost index gives us the proportionate change in the total/marginal cost of production between the base period and the current period. However, sometimes a producer may be interested in knowing the money amount by which the cost of production has gone up/down because of factor price changes. Evidently, in this case the appropriate total/marginal cost index is the difference between the total/marginal cost functions of the two periods. In the *fourth essay* of this dissertation, which is presented in **chapter 5**, we consider the difference form cost indices and study their properties in detail.

[7]The term '*Dimensionality*' has been borrowed from the literature on Index Number Theory (see Eichhorn and Voeller (1970)).

1.5 Demand Analysis with Zero Expenditure

Generally all household surveys show large fractions of households reporting zero consumption for some goods. This problem of modelling zero expenditures in applied demand analysis has received considerable attention in recent times. The literature (see Keen, 1986) recognises the following three broad sources of this phenomenon: (i) infrequent purchasing, i.e., the item was not bought over a relatively short enumeration period (durable goods will be a good example in this case), (ii) misreporting, i.e., data error and (iii) variation in preferences (and/or prices) across the sample : the consumer may have some liking for a particular brand of a differentiated product. Similarly, tastes or habits of consumers may also prevent them from buying the good at all, e.g., tobacco, alcohol.

The first two situations are mainly related to problems arising out of improper data collection and hence have been dealt with using statistical methods. For instance, Deaton and Irish (1984), Kay, Keen and Morris (1984), Keen (1986) have estimated demand models using censored regression approach. Pudney (1985) and Blundell and Mehgir (1987), among others, have also suggested different statistical approaches. Obviously, the main focus in these models are the econometric issues.

On a more economic-theoretic standpoint, Wales and Woodland (1983) have attempted to handle the problem using the formal equivalence of non-purchase with zero ration. Computational infeasibility has been the main drawback of this approach. In another approach, Blundell and Walker (1982) have used an idea due to Heckman (1979) in the context of assessing separability of labour supply from commodity demands. This method also is feasible only if the number of commodities showing zero purchases is small (see Deaton (1986) ; Blundell (1988)).

The third situation, i.e., heterogeneity of preference (and/or prices) has been modelled through formal utility maximisations. This has given rise to a vast literature. For instance, King (1980), Dubin and Mcfadden (1984) have taken this approach of quality choice in modelling demands for housing and electric appliances respectively. Hanemann (1984) has developed a unified framework for formulating demand models of discrete and/or continuous choices in which both the choices flow from the same utility maximisations. Novshek and Sonnenschein (1979) have extended neo-classical theory of demand to incorporate a situation where some consumers switch to a different brand when the price of the currently consumed brand rises.

We feel that the three sources of zero purchases do not exhaust all possible situations arising in household survey data, especially in the case of developing or underdeveloped countries. There, even if we allow corrections for short enumeration period (by considering average consumption over an extended period) and/or misreporting and ignore altogether the quality choices (for which anyway data are rarely available), we find a large number of items, perfectly divisible and commonly used, systematically missing from the basket of poor consumers. It is quite common to talk about items which are beyond one's means. In fact, Prais and Houthakker (1955), in their seminal study, mentions clearly about '*an initial income below which a commodity is not purchased*' in setting out the desirable properties for an algebraic formulation of Engel curves. Thus income differences play a vital role in explaining zero consumptions of many items. In order to do that formally, in the *fifth essay*, in chapter 6, we try to construct a preference structure where the commodities are divisible and where the number of commodities showing zero purchases may be large. The preference stucture considers a hierarchy of items and obtains a utility function which gives corner solutions at lower levels of income.

Chapter 2

On Tax progression and Inequality of Income Distribution

2.1 Introduction

Eichhorn, Funke and Richter (1984) proved that the Lorenz curve of income after tax will dominate the one before tax for all given pre-tax income distributions if and only if the tax function is strongly average progressive and weakly incentive preserving (see Eichhorn, Funke and Richter, 1984). Moyes (1988) showed that strong minimal progressivity along with IP is necessary and sufficient for the tax function to be uniformly equalising according to the absolute Lorenz domination. In this chapter we propose two dominance criteria ($R1$ and $R2$) that are based on a concept of Runciman (1966). Following EFR and Moyes, we then look for the population size independent tax functions that are equivalent to $R1$ and $R2$ respectively. Interestingly, it turns out that the tax function corresponding $R1$ ($R2$) is identical to the Moyes (EFR) function. This establishes, under the population size independence, the equivalence between $R2$ ($R1$) and the Lorenz (absolute Lorenz) criterion. But we find that if the population size is fixed, then $R1$ and $R2$ are different from the absolute Lorenz and the Lorenz dominations respectively. It will therefore be interesting to see what property a population size dependent tax function must have in order that it becomes inequality reducing. We show that a tax function satisfying IP (and depending on the population size) becomes equalising in the absolute Lorenz sense if and only if it meets *subgroup minimal progressivity,* a weaker concept than minimal progressivity. A similar result is derived for the

Lorenz domination case.

The chapter is organised as follows. The next section gives some definitions used in this chapter. Section 2.3 presents the main findings and section 2.4 makes some concluding remarks.

2.2 Some Definitions

This section defines two properties of a tax function that will be useful for our results presented in the next section.

Definition 2.1 Strong Subgroup Minimal Progression (SM) *: Let $n \geq 2$ be given. Then, given the pre-tax income distribution x, for any k, $1 \leq k \leq n-1$,*

$$\sum_{i=1}^{k}(\bar{x}_i - f(\bar{x}_i))/k \leq \sum_{i=k+1}^{n}(\bar{x}_i - f(\bar{x}_i))/(n-k) \tag{2.1}$$

with strict inequality holding for at least one k, $1 \leq k \leq n-1$.

Definition 2.2 Strong Subgroup Admissibility (SA) *: Let $n \geq 2$ be given. Then, given the pre-tax income distribution x, for any j, $1 \leq j \leq n$,*

$$\sum_{i=1}^{j}\bar{x}_i/n\lambda(x) \geq \sum_{i=1}^{j}(\bar{x}_i - f(\bar{x}_i))/n(\lambda(x) - \lambda(f(x))) \tag{2.2}$$

with strict inequality holding for at least one $j < n$.

SM means that for any partitioning of the population into two groups, the poor and the rich, the average tax liability of the latter should be at least as large as that of the former, with the additional restriction that for at least one partition the rich should have a larger average burden than the poor. SA means that the cumulative share of the total pre-tax income enjoyed by the bottom k $(0 \leq k \leq 1)$ proportion of the population is not smaller than their share in the total tax burden with the further requirement that the tax share should be smaller than the income share for at least one proportion.

It may be noted that MP implies SM, but the converse is not true. However, MP and SM are equivalent in a two-person society. If the tax function satisfies MP, then SA means that $x \succeq_{L^*} (x - f(x))$. That is, the tax vector is more unequal than the pre-tax distribution, which in turn means that a larger share of tax burden falls on the higher income groups. This is one notion of progressivity suggested by Jakobsson. It should be evident that SM and SA depend explicitly on n. Weak versions of SM and SA can be defined by withdrawing the requirement of at least one strict inequality in the above definitions.

We now consider two ranking relations building on an idea of Runciman (1966) that will be useful for our results to be presented in the next section. As pointed out in chapter 1 (section 1.3), Runciman argued that in any society the i^{th} person's deprivation (with income x_i) in comparison with j^{th} person's income x_j, where $x_i \le x_j$, can be taken as $(x_j - x_i)$. Since person i with income $\bar{x}_i$ is deprived of all incomes higher than $\bar{x}_i$, the total deprivation felt by this person in comparison with all higher incomes is $\sum_{j=i+1}^{n}(x_j - x_i)$. We then have

Definition 2.3 *Given $p, q \in D_+^n$, we say that p strongly dominates q by Runciman criterion 1 ($p \succeq_{R1} q$ for short) if*

$$\sum_{i=k+1}^{n}(q_i - q_k) \ge \sum_{i=k+1}^{n}(\bar{p}_i - \bar{p}_k) \tag{2.3}$$

for all k, $1 \le k \le n-1$, with strict inequality holding for at least one k.

Now, instead of considering simple income differences, as is done in (2.3), we can look at the utility differences of the form $U(x_j) - U(x_i)$, where U is increasing and concave. Assuming that $U(z) = \log z$, we have

Definition 2.4 *For $p, q \in D_+^n$, p strongly dominates q by Runciman criterion 2 ($p \succeq_{R2} q$ for short) if*

$$\sum_{i=k+1}^{n}(\log \bar{q}_i - \log \bar{q}_k) > \sum_{i=k+1}^{n}(\log \bar{p}_i - \log \bar{p}_k) \tag{2.4}$$

for all k, $1 \le k \le n-1$, with strict inequality holding for at least one k.

For weak dominance according to criterion $R1$ ($R2$) we drop the requirement that there should be at least one strict inequality in (2.3) ((2.4)).

Evidently, for arbitrary $x, y \in D^n$, $\frac{x}{n\lambda(x)} \succeq_{R1} \frac{y}{n\lambda(y)}$ is identical to $x \succeq_d y$ where $\succ_d$ is the relative deprivation dominance defined in chapter 1. Hence, the two relations are not the same in general. (But for a given mean income, the two relations are equivalent.) It should be clear that the relations $\succeq_d$ and $\succeq_{R2}$ are not also identical.

2.3 The Results

We will prove only strong versions of the different results. Weak versions can be proved under appropriate modifications. The following result identifies the tax structures that agree with the ranking criteria $R1$ and $R2$ respectively.

Theorem 2.1 *For all $n \geq 2$, for all $x \in D_+^n$,*
(a) The relation $f(x) \succeq_{R1} x$ holds if and only if the tax function satisfies IP and MP.
(b) The relation $f(x) \succeq_{R2} x$ holds if and only if the taxation scheme satisfies IP and AP.

Proof. (a) *Sufficiency* : Consider the illfare ranked permutation $\bar{x}$ of x. Then by IP, the resulting post-tax income vector is illfare ranked. Note that MP implies

$$\bar{x}_i - \bar{x}_{i-1} \geq f(\bar{x}_i) - f(\bar{x}_{i-1}) \tag{2.5}$$

for all $i = 2, 3, \ldots, n$ with strict inequality for at least one i. Now

$$\begin{aligned}
\sum_{i=k+1}^{n} (\bar{x}_i - \bar{x}_k) &= \bar{x}_{k+1} - \bar{x}_k + \bar{x}_{k+2} - \bar{x}_k + \ldots \\
&= \bar{x}_{k+1} - \bar{x}_k + \bar{x}_{k+2} - \bar{x}_{k+1} + \bar{x}_{k+1} - \bar{x}_k + \ldots \\
&\geq f(\bar{x}_{k+1}) - f(\bar{x}_k) + f(\bar{x}_{k+2}) - f(\bar{x}_{k+1}) + f(\bar{x}_{k+1}) - f(\bar{x}_k) + \ldots \\
&\quad (from\ (2.5)) \\
&= \sum_{i=k+1}^{n} (f(\bar{x}_i) - f(\bar{x}_k))
\end{aligned}$$

with strict inequality holding for at least one k. Hence we have $f(x) \succeq_{R1} x$.

Necessity : The structure of this part of the proof parallels that of EFR and Moyes. Suppose IP is violated. Thus, there exist $0 < u < v$ such that $f(u) > f(v)$. Consider $n \geq 2$ such that

$$(n-1) > (v-u)/(f(u) - f(v)) \tag{2.6}$$

Let $x = (u, u, ..., u, v)$. Then $f(x) = (f(u), f(u), ..., f(u), f(v))$. Consider the aggregate income shortfall of any one of the $(n-1)$ identical poor persons from the rich person in the pre-tax distribution x and that of the poor person from the rich ones in the post-tax distribution f(x). These are given respectively by $(v-u)$ and $(n-1)(f(u)-f(v))$. By the choice of n in (2.6), we have $(n-1)(f(u)-f(v)) > (v-u)$.

This contradicts the requirement $f(x) \succeq_{R1} x$. Suppose now that MP is violated. Then there exists $0 < u < v$ such that $v - f(v) < u - f(u)$. Let $x = (u, v)$. Then $f(x) = (f(u), f(v))$. Clearly, $f(v) > f(u)$. The income shortfall of the poor person from the rich one in the distributions x and $f(x)$ are given respectively by $(v - u)$ and $(f(v) - f(u))$. By assumption we have $v - u < f(v) - f(u)$, which contradicts the relation $f(x) \succeq_{R1} x$.

(b) *Sufficiency* : The proof is exactly similar to that of sufficiency part of $1(a)$, where $\bar{x}_i$ ($f(x_i)$) is replaced by $log \bar{x}_i$ ($\log f(\bar{x}_i)$).

Necessity : This is also similar to that of the necessity part of $1(a)$. When IP is violated, select $n (\geq 2)$ such that

$$(n-1) > (logv - logu)/(logf(u) - logf(v)) \tag{2.7}$$

where $v > u > 0$ and $f(u) > f(v)$. Then choose $x = (u, u, \ldots, u, v)$ and $f(x) = (f(u), f(u), \ldots, f(u), f(v))$ to get a contradiction. When AP is violated we have $logv - logu < logf(v) - logf(u)$ where $0 < u < v$. In this case choice of $x = (u, v)$ and $f(x) = (f(u), f(v))$ leads to a contradiction. ■

From this theorem we see that for all $n \geq 2$, $R1(R2)$ is equivalent to $LA^*(L^*)$. In our next result we show that the Lorenz and the absolute Lorenz dominations differ respectively from the ranking relations $R2$ and $R1$ defined in section 2.2 for a given population size.

Theorem 2.2 *Let $n \geq 2$ be given. Then for arbitrary $x, y \in D^n_+$,*
(a) $y \succeq_{L^*} x$ does not imply $y \succeq_{R2} x$.
(b) $y \succeq_{LA^*} x$ does not imply $y \succeq_{R1} x$.

Proof : This will be done by giving an example. Let $x = (10, 20, 30, 40)$ and $y = (14, 22, 24, 40)$. Then we have $y \succ_{L^*} x$ but not $y \succ_{R2} x$. Again for the same x, y the relation $y \succeq_{LA^*} x$ holds but not $y \succeq_{R1} x$. ■

We now try to find out what should be the population size dependent tax schemes which are equivalent for the different inequality criteria. In doing this, we assume, as is done in most such cases of distributional comparisons, that the tax functions satisfy IP.

Theorem 2.3 *Let $n \geq 2$ be given. Suppose that the tax function satisfies IP. Then for all $x \in D^n_+$, $f(x) \succeq_{LA^*} x$ if and only if SM holds.*

Proof: Consider the illfare ranked permutation $\bar{x}$ of x. Then by IP, $f(\bar{x})$ is illfare ranked. Let $t_i = \bar{x}_i - f(\bar{x}_i)$. Therefore, $\sum_{i=1}^{k}(f(\bar{x}_i) - \lambda(f(x))) \geq \sum_{i=1}^{k}(\bar{x}_i - \lambda(x))$ can be re-written as

$$\sum_{i=1}^{k}(\bar{x}_i - t_i - \lambda(x) + \lambda(t)) \geq \sum_{i=1}^{k}(\bar{x}_i - \lambda(x)) \tag{2.8}$$

where $\lambda(t)$ is the average tax liability. (2.8), on simplification, gives us

$$\sum_{i=1}^{k} t_i/k \leq \lambda(t) \tag{2.9}$$

Since $\lambda(t) = (\sum_{i=1}^{k} t_i + \sum_{i=k+1}^{n} t_i)/n$, from (2.9) it follows that

$$\sum_{i=1}^{k} t_i/k \leq \sum_{i=k+1}^{n} t_i/(n-k) \tag{2.10}$$

The absolute Lorenz domination implies that the inequality given by (2.8) is true for all $k, 1 \leq k \leq n-1$ with strict inequality holding for at least one k. Hence $f(x) \succeq_{LA^*} x$ implies that f satisfies SM. Similarly, it can be shown that (2.10) $\Longrightarrow$ $f(x) \succeq_{LA^*} x$. ■

Finally, we have

Theorem 2.4 *Let $n \geq 2$ be given. Suppose that the tax function satisfies IP. Then for all $x \in D_+^n$, $f(x) \succeq_{L^*} x$ if and only if SA holds.*

Proof: Consider the ordered pre-tax income distribution $\bar{x}$. Then, by IP, the post-tax distribution $f(\bar{x})$ is also non-decreasingly ordered. Let $t_i = x_i - f(\bar{x}_i)$. Then $f(x) \succeq_{L^*} x$ can be written as

$$\sum_{i=1}^{k}(\bar{x}_i - t_i)/(\lambda(x) - \lambda(t)) \geq \sum_{i=1}^{k} \bar{x}_i/\lambda(x) \tag{2.11}$$

where $1 \leq k \leq n$ and $\lambda(t)$ is the average tax liability. It should be noted that since both $x, f(x) \in D_+^n$ and $x \neq f(x)$, we have, $\lambda(f(x)) = \lambda(x) - \lambda(t) > 0$. From (2.11) we have

$$\sum_{i=1}^{k} \bar{x}_i\{1/(\lambda(x) - \lambda(t)) - 1/\lambda(x)\} \geq \sum_{i=1}^{k} t_i.\{1/(\lambda(x) - \lambda(t))\} \tag{2.12}$$

for all k, $1 \leq k \leq n$. From (2.12) it now follows that $\sum_{i=1}^{k} \bar{x}_i/\lambda(x) \geq \sum_{i=1}^{k} t_i/\lambda(t)$ for all k, $1 \leq k \leq n$ with $>$ for at least one k (since $f(x) \succeq_{L^*} x$). Hence SA holds. These steps can be retraced back to show the reverse implication. ■

2.4 Conclusions

Kakwani (1977) indicated that an average progressive taxation is uniformly equalising according to the Lorenz criterion. Later on, many authors, including Jakobsson (1976), Eichhorn et al. (1984) showed that under certain conditions the above implication turns out to be an equivalence. The absolute version of this result was proved by Moyes (1988). We have proposed two concepts of domination principles based on Runciman (1966) which, under the condition of population size independence, lead to identical tax functions as proposed by EFR and Moyes. These domination relations, however, differ from the Lorenz criteria if the population size is given. This motivates us to explore the nature of equivalent taxation schemes for the Lorenz criteria when the population size is fixed.

Chapter 3

On an Ordering Based on Relative Deprivation

3.1 Introduction

In this chapter we study the implications of the ranking relation generated by two non-intersecting relative deprivation curves (*RDCs*). More precisely, we show that given two income distributions x and y of a fixed total over a fixed population size, if the RDC of x strictly dominates that of y, then x strictly Lorenz dominates y. However, the converse is not true. As a next step we, therefore, look for the necessary and sufficient condition for the relative deprivation dominance to hold. These results are presented in Section 3.2. In Section 3.3 we identify the class of average relative deprivation indices that agrees with the above-mentioned dominance relation. That is, we isolate the class of all relative deprivation indices that will generate higher value for y than for x if the RDC of x dominates that of y and vice-versa. Section 3.4 provides a numerical illustration of the relative deprivation ordering using data for 23 countries. Finally, Section 3.5 makes some concluding remarks.

3.2 The Relative Deprivation Ordering

We begin the section with the social welfare implication of the relation $x \succeq_d y$.

Theorem 3.1 *Let $x, y \in D^n$, where $\lambda(x) = \lambda(y)$, be arbitrary. Then the relation $x \succeq_d y$ implies the following :*

(a) $\sum_{j=1}^{i} \bar{x}_j \geq \sum_{j=1}^{i} \bar{y}_j$ (3.1)

for all $i = 1, 2, \ldots, n$, with $>$ for at least one i, $1 \leq i < n$.

(b) *$W^n(x) > W^n(y)$ for all social welfare functions $W^n : D^n \longrightarrow R^1$ that satisfy strict S-concavity.*

Condition (a) in Theorem 3.1 means that the LC of x strictly dominates that of y. Condition (b) says that x is regarded as better than y by any equity oriented (in the sense of strictly S-concave) Social Welfare Function (SWF).

Proof of Theorem 3.1: The inequality system $d_i(x) \leq d_i(y)$ gives us

$$\sum_{j=i+1}^{n} (\bar{x}_j - \bar{x}_i) \leq \sum_{j=i+1}^{n} (\bar{y}_j - \bar{y}_i) \tag{3.2}$$

for all $i = 1, 2, \ldots, n$, with $<$ for at least one i, $1 \leq i < n$. For $i = 1$, (3.2) becomes

$$\sum_{j=1}^{n} \bar{x}_j - n\bar{x}_1 \leq \sum_{j=1}^{n} \bar{y}_j - n\bar{y}_1 \tag{3.3}$$

which in view of the fact that $\lambda(x) = \lambda(y)$ gives

$$\bar{x}_1 \geq \bar{y}_1 \tag{3.4}$$

This in turn implies that

$$\sum_{j=2}^{n} \bar{x}_j \leq \sum_{j=2}^{n} \bar{y}_j \tag{3.5}$$

Putting $i = 2$ in (3.2) we have

$$\sum_{j=3}^{n} \bar{x}_j - (n-2)x_2 \leq \sum_{j=3}^{n} \bar{y}_j - (n-2)\bar{y}_2 \tag{3.6}$$

Now multiply both sides of (3.5) by $(n-2)$ to get

$$(n-2)\sum_{j=3}^{n} \bar{x}_j + (n-2)\bar{x}_2 \leq (n-2)\sum_{j=3}^{n} \bar{y}_j + (n-2)\bar{y}_2 \tag{3.7}$$

The left (right) hand side of (3.6) when added with the corresponding side of (3.7) yields

$$\sum_{j=3}^{n} \bar{x}_j \leq \sum_{j=3}^{n} \bar{y}_j \tag{3.8}$$

Thus, in general we have

$$\sum_{j=i}^{n} \bar{x}_j \le \sum_{j=i}^{n} \bar{y}_j \tag{3.9}$$

for all $i = 1, 2, \ldots, n$, with $<$ for at least one i, $1 \le i < n$. It should be noted that if for some i, say i_0, the inequality in (3.9) becomes strict, then all the following inequalities will also be strict. We, in particular, have $\bar{x}_n < \bar{y}_n$, which is a necessary and testable condition for the relation $x \succeq_d y$ to hold.

Given that $\sum_{j=1}^{n} \bar{x}_j = \sum_{j=1}^{n} y_j$, it now follows from (3.9) that

$$\sum_{j=1}^{i} \bar{y}_j \le \sum_{j=1}^{i} \bar{x}_j \tag{3.10}$$

for all $i = 1, 2, \ldots, n$, with $<$ for at least one i, $1 \le i < n$. But (3.10) means that $x \succeq_{L^*} y$. This demonstrates part (a) of the theorem.

By Dasgupta-Sen-Starrett's theorem (Theorem 1.2 in Chapter 1) we know that the relation $x \succeq_{L^*} y$, where $\lambda(x) = \lambda(y)$, is equivalent to the condition that $W^n(x) > W^n(y)$ for all strictly S-concave $W^n : D^n \longrightarrow R^1$. Using part (a) of this theorem, we can now conclude that $x \succeq_d y$ implies $W^n(x) > W^n(y)$.

This completes the proof of the theorem. ■

Theorem 3.1 thus gives us the consequences of the relative deprivation dominance relation in terms of social welfare and the Lorenz criterion. Two other interesting implications of the above dominance relation are stated in the following corollary.

Corollary 3.1 *Let $x, y \in D^n$, where $\lambda(x) = \lambda(y)$, be arbitrary. Then the relation $x \succeq_d y$ implies that*
(a) x is regarded as at least as good as y by Rawlsian maximin criterion[1]*, that is, $\min_i\{x_i\} \ge \min_i\{y_i\}$*
(b) y is regarded as better than x by the maximax criterion, that is, $\max_i\{y_i\} > \max_i\{x_i\}$

The proof of this corollary directly follows from the proof of Theorem 3.1 and hence omitted. ■

It is important to note that the converse of Theorem 3.1 is not true. To see this, let $y = (10, 20, 30, 40)$ and $x = (10, 24, 26, 40)$. Then we have $x \succeq_{L^*} y$ but

[1] See Rawls (1971). It should be noted that the Rawlsian criterion is S-concave but not strictly so.

not $x \succeq_d y$. The reason behind this is as follows. In chapter 1, we have seen that given $\lambda(x) = \lambda(y)$, the relation $x \succeq_{L^*} y$ is equivalent to the condition that $\bar{x}$ can be obtained from $\bar{y}$ through a finite sequence of transformations transferring income from the rich to the poor. Now, consider a transfer of this type from the person with income $\bar{y}_i$ to the person with income $\bar{y}_j$, where $y_j < \bar{y}_i < \bar{y}_n$. Clearly, with such a transfer while the aggregate deprivation of the poorer person goes down, it increases for the richer person, thus, making the net effect ambiguous. However, Theorem 3.1 is quite appealing since it gives us a sufficient condition for the Lorenz ordering to hold.

We will now look for the transformations so that the relation $x \succeq_d y$ becomes equivalent to the condition that x can be obtained from y by means of these transformations. Given $x, y \in D^n$ with $\lambda(x) = \lambda(y)$, we say that x is obtained from y through an adjustment program $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ if $x = y - \alpha$ where the vector α is not identically zero. Since $x \in D^n$, α satisfies the feasibility condition $\alpha \leq y$. We call the i^{th} person a *contributor*, *beneficiary* or *unaffected* according as $\alpha_i > 0$, $\alpha_i < 0$ or $\alpha_i = 0$. Given $\lambda(x) = \lambda(y)$, we have $\sum_{i=1}^{n} \alpha_i = 0$. Since α is different from the zero vector, there is at least one pair (i, j) such that $\alpha_i < 0$ (> 0) and $\alpha_j > 0$ (< 0). An adjustment program can be explained in many ways. For instance, if we say that x is obtained from y through a fiscal program that does not modify the aggregate income, then α is the corresponding tax-subsidy vector (see Fei (1981)).

We call an adjustment program to be *just* if for each i, $1 \leq i < n$, $\alpha_i \leq \sum_{j=i+1}^{n} \alpha_j/(n-i)$, with $<$ for at least one i, where it is assumed that the set of individuals that are richer than person i in the pre-adjusted distribution is $\{i+1, \ldots, n\}$. To explain this, let $\sum_{j=i+1}^{n} \alpha_j > 0$. Then *justness* demands that person i's sacrifice should not exceed the average sacrifice of the persons who are richer than him. A similar explanation can be given for the case $\sum_{j=i+1}^{n} \alpha_j \leq 0$. We then have :

Theorem 3.2 *Let $x, y \in D^n$, where $\lambda(x) = \lambda(y)$, be arbitrary. Then the following statements are equivalent :*

(a) $x \succeq_d y$

(b) The adjustment program $\bar{y} - \bar{x}$ is just.

Proof: By justness we have $\bar{\alpha}_i \leq \sum_{j=i+1}^{n} \bar{\alpha}_j/(n-i)$ for all $i = 1, 2, \ldots, n$ with $<$ for at least one i, $1 \leq i < n$, where $\bar{\alpha} = \bar{y} - \bar{x}$. That is, $(n-i)(\bar{y}_i - \bar{x}_i) \leq \sum_{j=i+1}^{n} (\bar{y}_j - \bar{x}_j)$, which on rearrangement gives

$$\sum_{j=i+1}^{n} (\bar{x}_j - \bar{x}_i) \leq \sum_{j=i+1}^{n} (\bar{y}_j - \bar{y}_i) \tag{3.11}$$

for all i, $1 \leq i < n$ with $<$ for at least one i. Thus, (b) $\Longrightarrow$ (a). Similarly we can show that (a) $\Longrightarrow$ (b). ■

What Theorem 3.2 says is the following : If the RDCs of the distributions x and y (with the same mean) do not cross, then the distribution associated with the RDC closer to the no-deprivation line can be obtained from the other distribution through a just adjustment program and vice-versa. The intuitive reasoning behind this result is quite clear. The relative deprivation $d_i(x)$ of the person with income $\bar{x}_i$ is proportional to the gap between the average income of the persons richer than him and his own income. Any redistribution that decreases the average income of the richer persons by $\delta > 0$, will decrease $d_i(x)$ by δ. Consequently, he can at most sacrifice an amount less than or equal to δ. It should be noted that the ordering developed above is a quasi-ordering : it is transitive but not complete. When the two curves cross, we cannot get one from the other through a just adjustment program. Clearly, the reason for which the Lorenz ordering does not imply the relative deprivation ordering is that in the Lorenz case a sacrifice from a rich person to a poor person need not be just.

Again, since relative deprivation dominance implies Lorenz domination and not vice versa, the sequence of rank preserving progressive transfers by which one can obtain the Lorenz dominating distribution from the dominated distribution should be different from the just adjustment program. To understand this more explicitly, we have the following theorem.

Theorem 3.3 *Let $x, y \in D^n$, where $\lambda(x) = \lambda(y)$, be arbitrary. Then $x \succeq_d y$ implies $\bar{y}$ can be transformed into $\bar{x}$ by the following sequence of transformations*

$$\bar{y}^i_{n-i} = \quad \bar{y}^{i-1}_{n-i} + c^i \quad \leq \bar{y}^{i-1}_{n-i+1}$$

$$\bar{x}_{n-i+1} = \bar{y}^i_{n-i+1} = \quad \bar{y}^{i-1}_{n-i+1} - c^i \quad \geq \bar{y}^{i-1}_{n-i}$$

$$\text{with } \bar{y}^i_k = \bar{y}^{i-1}_k \quad \text{if } k \neq n-i,\ n-i+1$$

$$c^i = \bar{y}^{i-1}_{n-i+1} - \bar{x}_{n-i+1} \geq 0, \quad \bar{y}^0_i = \bar{y}_i,$$

$c^i = 0$ at any stage implies that the transformation is complete, $i = 1, \ldots, n$.

To prove this result we need the following lemma.

Lemma 3.1 *Let α be a just adjustment program. Then if for some i, $1 \leq i < n$, $\sum_{j=i+1}^{n} \alpha_j = 0$, we have $\alpha_j = 0$ for all $j = 1, \ldots, i$.*

Proof : By just adjustment we have for all $i = 1, \ldots, n-1$, $\alpha_i \leq \frac{1}{n-i} \sum_{j=i+1}^{n} \alpha_j$. Now, if $\sum_{j=i+1}^{n} \alpha_j = 0$ for some i, we have $\alpha_i \leq 0$. Then either $\alpha_i = 0$, or $\alpha_i < 0$. Suppose $\alpha_i < 0$. Then $\sum_{j=i}^{n} \alpha_j < 0$ and $\alpha_{i-1} \leq \frac{1}{n-i+1} \sum_{j=i}^{n} \alpha_j < 0$. Thus $\alpha_i < 0$ implies $\alpha_{i-1} < 0$. Repeating the above argument we have $\alpha_{i-2} < 0, \ldots, \alpha_1 < 0$. This implies $\sum_{j=1}^{n} \alpha_j = \sum_{j=1}^{i} \alpha_j + \sum_{j=i+1}^{n} \alpha_j < 0$, which is a contradiction. Hence $\alpha_i = 0$ and $\sum_{j=i}^{n} \alpha_j = 0$ which in turn implies that $\alpha_{i-1} \leq 0$. Thus arguing similarly we have the desired result. ■

Proof of Theorem 3.3 : Note that since $x \succeq_d y$ holds, the adjustment program $\bar{y} - \bar{x}$ is just and hence we can invoke lemma 3.1 in our proof. By corollary (3.1) we have $\bar{y}_n > \bar{x}_n$. Now, $\bar{x}_n = \bar{y}_n - (\bar{y}_n - \bar{x}_n) = \bar{y}^0_n - c^1$. Also,

$$\begin{aligned} \bar{y}^1_{n-1} &= \bar{y}^0_{n-1} + c^1 \\ &= \bar{y}_{n-1} + (\bar{y}_n - \bar{x}_n) \geq \bar{x}_{n-1} \end{aligned} \qquad (3.12)$$

(from 3.9 we know that $\bar{y}_{n-1} + \bar{y}_n \geq \bar{x}_{n-1} + \bar{x}_n$)

Now, if (3.12) holds with equality, we have $\sum_{j=n-1}^{n} \alpha_j = 0$. Hence from lemma 3.1 it follows that $\bar{y}_{n-i} = \bar{x}_{n-i}$ for $i = 2, \ldots, n-1$. If (3.12) holds with strict inequality,

$$\begin{aligned} \bar{x}_{n-1} &= \bar{y}^1_{n-1} - (\bar{y}^1_{n-1} - \bar{x}_{n-1}) \\ &= \bar{y}^1_{n-1} - c^2 \end{aligned}$$

$$\begin{aligned} \text{and } \bar{y}^2_{n-2} &= \bar{y}^1_{n-2} + c^2 \\ &= \bar{y}_{n-2} + (\bar{y}^1_{n-1} - \bar{x}_{n-1}) \\ &= \bar{y}_{n-2} + (\bar{y}_{n-1} + (\bar{y}_n - \bar{x}_n) - \bar{x}_{n-1}) \geq \bar{x}_{n-2} \end{aligned}$$

(by (3.9) again)

and we can repeat the above argument to complete the demonstration. ■

The difference between the above sequence of transformations and the type which holds for the Lorenz situation is that in case of deprivation dominance, we can start transferring income from the richest person and this sequence stops only if the remaining lower parts of the distributions are identical. This follows from the fact that corollary 3.1(b) need not hold for the Lorenz case in general. This is demonstrated by the example given on page 27.

3.3 Numerical Measures

Using a numerical measure of relative deprivation it is possible to provide a complete ranking of alternative income distributions. Since the ordering based on the RDC is a quasi-ordering, the numerical measures typically involve stronger assumptions about the nature of deprivation than in the case of RDCs. Given the diversity of numerical measures, it is reasonable to identify the class of indices that will yield a ranking of distributions consistent with the ordering provided by two non-intersecting RDCs.

We now propose the following properties for a general deprivation index $I : D \to R^1$.

Homogeneity (HOM) : For all $n \in N$, $x \in D^n$, $I^n(x) = I^n(cx)$, where $c > 0$ is arbitrary.

Just adjustment principle (JAP) : If for any arbitrary $n \in N$, $x \in D^n$, x is obtained from y by a just adjustment program, then $I^n(x) < I^n(y)$.

Symmetry (SYM) : For all $n \in N$, $x \in D^n$, $I^n(x) = I^n(u)$, where u is any permutation of x.

Population principle (POP) : For all $n \in N$, $x \in D^n$, $I^n(x) = I^{mn}(v)$, where $v = (x^1, \ldots, x^m)$ and $x^i = x$ for all $i = 1, \ldots, m$.

Properties HOM, SYM and POP have already been explained in the context of inequality. Property JAP requires overall deprivation to decrease under a just sacrifice.

We then have :

Theorem 3.4 *Let $x, y \in D$ be arbitrary. Then the following statements are equivalent :*
(a) $x \succeq_d y$
(b) $I(x) < I(y)$ *for all relative deprivation indices* $I : D \longrightarrow R^1$ *that satisfy* **HOM, SYM, JAP** *and* **POP.**

Proof: The proof of this theorem is similar to that of Proposition 1 of Foster (1985) and hence omitted. ■

Theorem 3.4 shows that an unambiguous ranking of income distributions generated by a very large class of overall deprivation indices can be obtained through the pairwise comparisons of the RDCs of the distributions.

If the mean income is fixed and the population size is a variable, the set of all possible income distributions is an appropriate subset D_c of D, where $D_c = \{x \in D : M(x) = c\}$. For indices that are consistent with the relative deprivation ordering in this case we have the following :

Theorem 3.5 *Let $x, y \in D_c$ be arbitrary. Then the following statements are equivalent :*
(a) $x \succeq_d y$
(b) $I(x) < I(y)$ *for all relative deprivation indices* $I : D_c \longrightarrow R^1$ *that satisfy* **SYM, JAP** *and* **POP.**

We can also focus our attention on fixed population, arbitrary mean income case. In such a case, the domain of definition of the deprivation index is D^n, where $n \in N$ is fixed.

Theorem 3.6 *Let $x, y \in D^n$ be arbitrary. Then the following statements are equivalent.*
(a) $x \succeq_d y$
(b) $I^n(x) < I^n(y)$ *for all relative deprivation indices* $I^n : D^n \longrightarrow R^1$ *that satisfy* **HOM, SYM** *and* **JAP.**

The proofs of theorems 3.5 and 3.6 parallel respectively those of propositions 2 and 3 of Foster (1985) and hence omitted. It is easy to see that if both mean incomes and population sizes are fixed, then ranking by the class of all deprivation indices satisfying SYM and JAP is consistent with that generated by RDCs [2].

We know that for arbitrary $x, y \in D$, the relation $x \succeq_{L^*} y$ holds if and only if $J(x) < J(y)$ for all inequality indices $J : D \to R^1$ that are homogeneous of degree zero, symmetric, population replication invariant and reduced by an income transfer from a rich person to a poor person (see Foster (1985) and Shorrocks and Foster (1988) and Chakravarty(1990)). Since $x \succeq_{L^*} y$ is implied by $x \succeq_d y$ on D, we have $J(x) < J(y)$ whenever $x \succeq_d y$ holds. This in turn shows that the relative deprivation ordering on the general domain D implies an ordering of income distributions in terms of inequality.

3.4 Numerical Illustration

This section provides an international comparison of relative deprivation using the dominance criterion developed in this paper. As pointed out by Kakwani (1984), there are several problems associated with the comparability of income distribution data from different countries owing to the differences in income units, population coverage and year of survey. In this section, we, therefore, use the data on 23 countries, selected by Kakwani (1984), that are comparable in terms of the above three criteria.

Table 3.1 presents the index of GDP per capita and the values of the relative deprivation for nine decile groups for each of the 23 countries. The RDCs cross in 68 out of 253 pairwise comparisons (that is, in 26.8 per cent cases). Out of these 68 cases, in 14 cases Lorenz domination was observed. This result thus confirms that the Lorenz dominance does not imply the relative deprivation dominance.

As shown earlier, a necessary condition for the deprivation dominance is not only the Lorenz dominance, but also that the richest person in the distribution corresponding to the higher RDC must be richer than the richest person in the other distribution. Table 3.2 presents (i) the pair of countries for which the first

[2] If the set of income distributions is given by D_c, we can also have a ranking of distributions in terms of SWFs. That is, if $x \succeq_d y$ holds, then we have $W(x) > W(y)$ for all SWFs $W : D_c \to R^1$ that are strictly S-concave and population replication invariant. The proof of this follows from Theorem 2 of Dasgupta, Sen and Starrett (1973) and Theorem 3.1 of this chapter. It may be noted that the SWFs given in footnote 2 of chapter 1 remain invariant under replications of populations.

country Lorenz dominates the second, but the RDCs intersect, and (ii) the mean income of the top decile group (treated as the income of the richest person) for each pair of countries. In twelve cases (except for Hong Kong - Costa Rica and Australia - U.K.) the condition on the richest person's income is satisfied, but the RDCs still cross. This indicates that this condition, although necessary, is far from being sufficient. In the two cases mentioned above the necessary condition itself has been violated.

3.5 Conclusions

A quantification of the extent of deprivation felt by an individual in a society is the normalized value of the sum of his income shortfalls from richer individuals, where the coefficient of normalization is the reciprocal of the society's aggregate income [Runciman (1966)]. Kakwani (1984) plotted such individual deprivations against the cumulative proportions of persons in the society to generate the relative deprivation curve. In this paper we looked at the consequences of the relative deprivation ordering, an ordering generated by two non-intersecting relative deprivation curves. It is shown that the Lorenz ordering drops out as an implication of the deprivation ordering. We then developed the condition that implies and is implied by this ordering. Such a condition becomes helpful for isolating the class of all relative deprivation indices consistent with the above ordering. A numerical illustration of the deprivation ordering has also been provided.

Table 3.1 : Index of GDP per capita and the values of the relative deprivation for nine decile groups

Sl. No.	Country	Index of GDP per capita 1970	Decile group								
			1	2	3	4	5	6	7	8	9
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	Bangladesh	4.29	0.65	0.57	0.49	0.43	0.37	0.30	0.24	0.18	0.11
2	Malawi	4.58	0.78	0.66	0.62	0.53	0.50	0.43	0.38	0.32	0.25
3	India	5.48	0.82	0.72	0.66	0.59	0.52	0.45	0.38	0.30	0.20
4	Tanzania	6.04	0.92	0.86	0.79	0.73	0.66	0.59	0.51	0.41	0.30
5	Pakistan	9.93	0.68	0.57	0.4	0.40	0.3	0.29	0.24	0.18	0.12
6	Sri Lanka	10.10	0.72	0.60	0.5	0.46	0.4	0.34	0.27	0.21	0.13
7	Philippines	10.40	0.88	0.74	0.6	0.61	0.5	0.46	0.38	0.30	0.20
8	Korea	13.70	0.69	0.61	0.5	0.48	0.4	0.34	0.27	0.20	0.12
9	Honduras	14.80	0.95	0.90	0.82	0.77	0.70	0.62	0.53	0.43	0.30
10	Turkey	18.10	0.90	0.82	0.75	0.68	0.62	0.55	0.47	0.39	0.29
11	Malaysia	18.90	0.88	0.75	0.68	0.61	0.55	0.48	0.41	0.34	0.26
12	Brazil	22.80	0.91	0.86	0.80	0.74	0.67	0.60	0.52	0.43	0.33
13	Costa Rica	25.30	0.79	0.68	0.62	0.55	0.48	0.41	0.34	0.27	0.18
14	Mexico	28.30	0.80	0.78	0.74	0.70	0.64	0.58	0.51	0.44	0.34
15	Chile	29.90	0.81	0.72	0.66	0.60	0.58	0.45	0.40	0.35	0.27
16	Hongkong	31.70	0.79	0.66	0.58	0.51	0.45	0.39	0.32	0.26	0.18
17	Japan	58.20	0.66	0.48	0.41	0.34	0.28	0.23	0.18	0.13	0.09
18	U.K.	63.50	0.77	0.59	0.50	0.42	0.35	0.28	0.20	0.14	0.08
19	New Zealand	64.30	0.66	0.52	0.45	0.38	0.32	0.26	0.21	0.15	0.10
20	Australia	75.90	0.76	0.55	0.46	0.38	0.31	0.25	0.19	0.13	0.08
21	Germany	77.80	0.78	0.64	0.56	0.49	0.42	0.35	0.28	0.21	0.13
22	Canada	89.00	0.89	0.74	0.60	0.48	0.38	0.29	0.22	0.15	0.10
23	U.S.A.	100.00	0.86	0.69	0.59	0.50	0.32	0.34	0.27	0.20	0.12

Table 3.2 : Pairs of countries for which the relative deprivation curves intersected but there was Lorenz dominance and the mean income of the top decile group for each pair

Country 1	$\succeq_{L^*}$	Country 2	Mean incomes of the top decile group	
			Country 1	Country 2
(1)		(2)	(3)	(4)
Bangladesh	...	U.S.A	1.150	29.100
India	...	Philippines	2.201	3.858
Tanzania	...	Honduras	2.821	7.045
Pakistan	...	Korea	2.691	3.836
Pakistan	...	U.S.A.	2.691	29.100
Sri Lanka	...	Germany	2.939	22.640
Philippines	...	Malaysia	3.858	7.768
Korea	...	U.S.A.	3.836	29.100
Hongkong	...	Costa Rica	10.683	8.653
Japan	...	U.K.	13.444	15.177
Japan	...	Australia	13.444	17.837
Australia	...	U.K.	17.837	15.177
U.K.	...	U.S.A.	15.177	29.100
New Zealand	...	U.S.A.	15.818	29.100

Chapter 4

An Ethical Index of Relative Deprivation

4.1 Introduction

In chapter 1, we have made a brief review of the existing indices of relative deprivation. It has been observed that these indices have been derived without using any concept of welfare. The purpose of this chapter is to suggest an index of deprivation using welfare theoretic criteria. More precisely, we use an ethical judgement, which is known as *fairness,* to construct the proposed index. (See Feldman and Kirman (1974), Varian (1974) and Kolm (1991) for detailed discussion on fairness.) In a particular case, this index is shown to be related to the Yitzhaki (1979,1982) index of relative deprivation. Needless to say, the ethical index proposed here is not meant to supplant the existing descriptive indices. Rather it is designed with a different objective - it is a measure of the extent of unfairness existing in the income distribution.

This chapter is organised as follows. In the next section we discuss fairness and construct individual indices of relative deprivation. The index for the society as a whole is presented in section 4.3. Finally, we make some concluding remarks in section 4.4

4.2 Fairness and Individual Deprivation

Fairness is basically a criterion of non-envy (see Feldman (1980)). A situation is called *fair* if nobody prefers any others' position to his own. While fairness can be applied to many allocational problems, we use a simple structure here. For this let U_i denote the utility function of individual i, $i = 1, \ldots, n$. U_i is assumed to depend only on individual i's income y_i. Furthermore, U_i is continuous, increasing and strictly concave. Throughout this chapter it will be assumed that incomes are arranged in non-decreasing order. Furthermore, the mean income $\lambda(y)$ is assumed to be positive.

Definition 4.1 *An income allocation* $y = (y_1, y_2, \ldots, y_n)$ *is* fair *if for every pair* (i,j), $U_i(y_i) \geq U_i(y_j)$.

Definition 4.2 *An income allocation* y *is called* unfair *if for at least one pair* (i,j), $U_i(y_i) < U_i(y_j)$.

For the rest of the chapter it will be assumed that all the individual possess the same utility function U. In this special case of identical utility function, from definition 4.2 it follows that an income allocation y is unfair if and only if y is not an equitable profile, i.e., y is not of the form $\lambda.1^n$ for some $\lambda > 0$. Thus inequity in income distribution gives rise to unfairness. Clearly, in this case, for any inequitable income profile, a person is going to envy all persons richer than him.

We now define the i^{th} person's deprivation d_{ij} in comparison with j^{th} person's income y_j, where $y_j \geq y_i$, as the extent of utility shortfall that arises because of unfairness :

$$d_{ij} = U(y_j) - U(y_i) \tag{4.1}$$

Clearly, d_{ij} is consistent with Runciman's remark mentioned in section 1.3 of chapter 1. Note that increasingness of U ensures that $d_{ij} \geq 0$. We then define the total deprivation d_i felt by person i in comparison with all higher incomes as the sum of the gaps of the form d_{ij} given by (4.1). Formally,

$$d_i(y) = \sum_{j=i+1}^{n} (U(y_j) - U(y_i)) \tag{4.2}$$

Clearly, d_i gives us the degree of unfairness arising out of the comparison of y_i with higher incomes.

The individual deprivation index d_i possesses the following properties :

(i) $d_i \geq d_j$ whenever $y_j \geq y_i$.

(ii) d_i is independent of the incomes of the persons poorer than individual i.

(iii) d_i decreases when rank preserving income transfers take place from a person richer than i to someone poorer than i.

(iv) An increase in the income of a person richer than i increases d_i.

(v) d_i is a continuous function.

(vi) An increase in the incomes of all persons by a constant absolute amount decreases d_i. This is due to the fact that U is increasing and strictly concave. For equal amount of increases, U increases more at lower levels of income so that the gaps are narrowed down.

(vii) d_i increases if income equalising transfers take place among persons, who are all richer than individual i, assuming that ranks are not altered as a result of the transfer. The intuitive reasoning behind this is as follows. Due to strict concavity of U, the equalising transfer increases the well-being of the richer individuals, which in turn makes the i^{th} person more deprived in terms of welfare. However if the individual deprivation function is defined directly on income gaps, this property need not hold (see Yitzhaki (1979), Chakravarty (1990)).

4.3 The Average Index

In this section we derive the deprivation index for the society as a whole and discuss its properties.

Definition 4.3 *For any income configuration y, the average relative deprivation index d is defined as the normalised value of the individual deprivation indices given in (4.2). Formally,*

$$\begin{aligned} d(y) &= A(n,y)\sum_{i=1}^{n-1} d_i(y) \\ &= A(n,y)\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(U(y_j)-U(y_i)) \qquad (4.3) \end{aligned}$$

where $A(n,y) > 0$ is the coefficient of normalisation.

Since the overall index given by (4.3) is the normalised value of the sum of all utility shortfalls of the poor persons from the richer ones, it can be regarded as an index of unfairness existing in the income distribution.

To derive a specific functional form of d we will use the homogeneity axiom (HOM) and the normalisation axiom stated below. All other axioms discussed in chapter 3 will be shown to be verified by the specific index. Champernowne (1974) indicated that in the case of maximum inequality (that is, when the entire income in the society is monopolized by a single person) the value of an inequality index should depend only on n, the number of persons in the society. Furthermore, it should be increasing in n and in the limit as the number of persons increases, its value should tend to one. Likewise, we also stipulate the following regarding the maximal value of our relative deprivation index.

Normalisation (NOM): In an n-person society, if one individual gets the entire income and all others receive zero income, then $d = f(n)$ where $0 < f(n) < 1$, $f(n)$ is increasing in n and $f(n)$ tends to 1 as n goes to infinity[1].

We then have

Theorem 4.1 *The only average relative deprivation index of the form (4.3) that satisfies NOM and HOM is*

$$\frac{f(n)}{n-1}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(s_j^{\alpha} - s_i^{\alpha}) \tag{4.4}$$

where $s_i = \dfrac{y_i}{n\lambda(y)}$ *is the income share of individual* i.

Proof : d in (4.3) satisfying NOM gives

$$A(n, y) = \frac{f(n)}{(n-1)(U(n\lambda) - U(0))}$$

which on substitution in (4.3) yields

$$d = \frac{f(n)\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(U(y_j) - U(y_i))}{(n-1)(U(n\lambda) - U(0))} \tag{4.5}$$

Now, assume for simplicity that n = 2. Then d in (4.5) becomes

$$\begin{aligned} d &= \frac{f(2)(U(y_2) - U(y_1))}{(U(y_1 + y_2) - U(0))} \\ &= \frac{f(2)(g(y_2) - g(y_1))}{g(y_1 + y_2)} \\ where,\ g(z) &= U(z) - U(0) \end{aligned} \tag{4.6}$$

[1] An interesting example of $f(n)$ is $1 - \frac{1}{n}$. Kakwani (1984) also indicated that aggregate relative deprivation should be maximum in this extreme situation. He, however, did not specify anything regarding the functional form of the deprivation index in this case.

By HOM, d is homogeneous of degree zero in (y_1, y_2). Hence we have,

$$\begin{aligned} d &= f(2)\frac{(g(1) - g(y_1/y_2))}{g(1 + y_1/y_2)} \\ &= f(2)\frac{g(1)}{g(1 + y_1/y_2)} - f(2)\frac{g(y_1/y_2)}{g(1 + y_1/y_2)} \\ &= g_1(y_1, y_2) - g_2(y_1, y_2). \ (say) \end{aligned}$$

Observe that both $g_1(.)$ and $g_2(.)$ are homogeneous of degree zero in (y_1, y_2). Now we have $g_2(y_1, y_2) = f(2)\frac{g(y_1)}{g(y_1 + y_2)}$ and $g_1(y_1, y_2) = f(2)\frac{g(y_2)}{g(y_1 + y_2)}$. Dividing g_1 by g_2, we then have

$$\frac{g_1(y_1, y_2)}{g_2(y_1, y_2)} = \frac{g(y_2)}{g(y_1)} \tag{4.7}$$

The left hand side of (4.7) is homogeneous of degree zero (since both g_1 and g_2 are homogeneous of degree zero). Consequently, the right hand side of (4.7) also satisfies zero degree homogeneity. This implies that $\frac{g(y_2)}{g(y_1)}$ will depend only on $\frac{y_2}{y_1}$. Hence $\frac{g(y_2)}{g(y_1)}$ must be of the form $h(\frac{y_2}{y_1})$ for some continuous function h. Therefore, $g(z) = c.z^{\alpha}$ [see Aczel (1966), p.144]. Increasingness and strict concavity of $U(.)$ show that $c > 0$ and $0 < \alpha < 1$. Substituting the form of $g(.)$ in (4.6) we get $U(z) = U(0) + g(z) = \theta + c.z^{\alpha}$, where $\theta = U(0)$. Substituting the derived form of U in (4.5) we get the desired form of d. This establishes the necessity part of the theorem.

The sufficiciency part can be verified by checking that d in (4.4) satisfies HOM and NOM.

We now state/prove some properties satisfied by the index given by (4.4).

(i) For a given n, d lies in the interval $[0, f(n)]$, the lower limit is attained in the case when the income distribution is egalitarian. It achieves the upper limit in the extreme case described in NOM.

(ii) d remains invariant under affine transformations of U.

(iii) d satisfies symmetry.

(iv) d is increasing in α. α can, therefore, be interpreted as the deprivation aversion parameter. Larger the value of α, larger the deprivation index, given other things. In the case when $\alpha = 1$, d becomes a multiple of the Gini-coefficient. (It should be noted that for $\alpha = 1$, the utility function is concave but not strictly concave.) This gives us an ethical interpretation of the Gini-index in terms of aggregate unfairness existing in the society. Obviously, in this case d also becomes a multiple of the Yitzhaki index. It may be pointed out that for any two income distributions x, y, if $\lambda(x) = \lambda(y)$, then the ranking of distributions by d (where $\alpha = 1$) is same as that generated by the Yitzhaki index.

(v) d satisfies just adjustment principle (JAP).

To see this, consider the vector x obtained from y by a just adjustment program. Then $x = y - \beta$ where $\beta_i \leq \frac{1}{n-i}\sum_{j=i+1}^{n} \beta_j, i = 1, \ldots, n-1$, with $<$ for at least one i and $\sum_{j=1}^{n} \beta_j = 0$. Denote the share vectors of x and y by s and l respectively. Thus $s_i = l_i - \frac{\beta_i}{n\lambda}$ where λ is the common mean of x and y. Note that $d(x) = d(y)$ if $\alpha = 0$. For $\alpha = 1$,

$$d(x) = d(y) - \frac{f(n)}{n(n-1)\lambda}\sum_{i=1}^{n-1}\left[\sum_{j=i+1}^{n} \beta_j - (n-i)\beta_i\right] \tag{4.8}$$

$$< d(y)$$

since by just adjustment the second term on the right hand side of (4.8) is positive. Now d is continuous and increasing in α. Therefore, $d(x) < d(y)$ for all α, $0 < \alpha \leq 1$. This shows that d satisfies just adjustment principle.

(vi) If the form of $f(n)$ is given explicitly, we can verify whether the resulting index satisfies Population Principle (POP). For instance if $f(n) = 1 - 1/n$, then $d = \frac{1}{n}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(s_j^\alpha - s_i^\alpha)$. In this case, d meets POP.

Clearly, for all $f(n)$'s for which d meets POP, d agrees with the relative deprivation dominance ordering discussed earlier.

(vii) d is a continuous function in its arguments.

4.4 Conclusions

By quantifying a particular case of a statement of Runciman (1966), Yitzhaki (1979) showed that one reasonable index of relative deprivation in a society is the product of the mean income and the Gini coefficient in the society. Alternatives and variations of Yitzhaki's index have been suggested by many authors. (See Chakravarty (1990) for a discussion.)

But all these indices are descriptive indices - they have been generated without using any ethical notion. In this essay we have suggested an index of deprivation using a moral criterion which is known as *fairness.* In a particular case, the index is shown to be related to the Yitzhaki index.

Clearly, the problem of choice between the descriptive indices and the new index will depend on the purpose. If we want to look at the extent of deprivation felt by different persons directly in terms of income, then the descriptive indices are appropriate. On the other hand, the ethical index is relevant for determining the extent of *unfairness* existing in the income distribution.

Chapter 5

On Production Cost Indices

5.1 Introduction

In chapter 1 we have discussed the Baye-Deily-Jansen (1991) total and marginal production cost indices in detail. These indices remain unaltered if all factor prices change by an equiproportionate amount and we have called them relative indices. It may be noted that if a firm uses only one input for its productive activity then the cost function is given by the product of the input price and some function of the output level[1]. Consequently, in this particular case, if the current and base period input prices increase or decrease by the same amount, then a cost index expressed as the difference between the costs of the two periods will remain invariant. A cost index that satisfies this invariance property in the general case of arbitrary number of inputs will be called an *absolute* index.

In this chapter we consider cost indices defined as the difference between the total/marginal cost functions of the current and the base periods and discuss their properties. Needless to say, they are not meant to supplant the Baye-Deily-Jansen (BDJ) indices. As mentioned in Chapter 1, these indices are constructed with a different aim. In the next section we state some variants of the BDJ indices. Section 5.3 presents the new indices. Finally, section 5.4 makes some concluding remarks.

[1]This follows from linear homogeneity of the cost function in factor prices.

5.2 Variants of the BDJ Indices

As an alternative to T_R we may use

$$t_r(w^0, w^1, y) = \frac{C(w^1, y)}{C(w^0, y)} - 1 \tag{5.1}$$

t_r gives us the proportionate change in the total cost of producing the output level y in the current period 1 over what it was in the base period 0. t_r meets MN and DM but not HM. It does not satisfy IN also, instead it satisfies :

Normalisation (NORM) : $t_r(w^0, w^0, y) = 0$

A positive (negative) value of t_r indicates that the cost of producing y has increased (decreased) in the current period in comparison with that in the base period. t_r is bounded below by minus one. T_R, discussed in chapter 1, is bounded below by zero. Both T_R and t_r are unbounded above. Since the two indices differ by a constant value of 1, it does not matter whether we use T_R or t_r for empirical purpose. Probably, the only advantage of t_r lies in its simple interpretation.

Similarly, an alternative to M_R is

$$m_r(w^0, w^1, y) = \frac{C_y(w^1, y)}{C_y(w^0, y)} - 1 \tag{5.2}$$

m_r determines the proportionate change in marginal cost for producing an additional unit of output in the current period over what it was in the base period. m_r meets DM, CM and $NORM$. m_r has similar properties as t_r with respect to boundedness.

5.3 New Indices of Marginal and Total Costs

As an index of total cost we now consider

$$T_A(w^0, w^1, y) = C(w^1, y) - C(w^0, y) \tag{5.3}$$

T_A is a measure of the change in the total cost for producimg the output level y in the current period over what it was in the base period. Analogously, our marginal cost index, which gives us the change in marginal cost for producing one more unit of output in period 1 over the corresponding cost in period 0 is defined by

$$M_A(w^0, w^1, y) = C_y(w^1, y) - C_y(w^0, y) \tag{5.4}$$

T_A (M_A) is positive or negative according as the total cost (marginal cost) is greater/smaller in the current period than in the base period. Both T_A and M_A are unbounded.

Note that, given other things, a decrease in T_A registers an increase in profit differential between the periods under consideration. Furthermore, T_A will increase in y if $M_A > 0$. This is intuitively reasonable : if there is a positive change in the unit cost of production between the two periods, then one can expect that the total cost has gone up in the current period.

In the case of T_R and M_R, property DM is ensured by the fact that the cost function is linearly homogeneous in factor prices. But linear homogeneity of $C(w, y)$ implies that T_A and M_A are also linearly homogeneous in factor prices. In fact, no property of cost function guarantees that T_A and M_A are absolute indices. Now, suppose that $C(w, y)$ is *translatable* in factor prices. According to translatability, for all $w \in R^n_{++}$,

$$C(w + \alpha.1^n, y) = C(w, y) + \alpha\, b(y) \tag{5.5}$$

where α is any scalar such that $(w + \alpha.1^n) \in R^n_{++}$ and $b(y)$ is a real-valued function of y. Thus, translatability means that an equal augmentation of all factor prices by a constant will augment the cost function by a multiple of the constant. The following theorem, which is easy to demonstrate, gives a sufficient condition for T_A and M_A to remain invariant under equal absolute changes in w^1 and w^0.

Theorem 5.1 *If the cost function is translatable, then the indices $T_A(w^0, w^1, y)$ and $M_A(w^0, w^1, y)$ are absolute, that is, translatability of the cost function implies that for all $(w^0, w^1) \in R^{2n}_{++}$,*

$$T_A(w^0 + \alpha.1^n, w^1 + \alpha.1^n, y) = T_A(w^0, w^1, y),$$
$$M_A(w^0 + \alpha.1^n, w^1 + \alpha.1^n, y) = M_A(w^0, w^1, y)$$

where α is a scalar such that $(w^0 + \alpha.1^n, w^1 + \alpha.1^n) \in R^{2n}_{++}$.

The translatability condition in (5.5) has the follwing interesting implication involving different inputs used in the productive activity.

Theorem 5.2 *For a given output level y, translatability of a cost function $C(w, y)$ defined in (5.5) implies that*

$$\sum_{i=1}^{n} x_i(w, y) = b(y) \tag{5.6}$$

Proof : Differentiate both sides of (5.5) partially with respect to α, and set $\alpha = 0$, to get :

$$\sum_{i=1}^{n} C_i(w, y) = b(y) \tag{5.7}$$

where $C_i(w, y) = \partial C(w, y)/\partial w_i$. Using Shephard's lemma (see Varian (1984, p. 54)) in (5.7), we get $\sum_{i=1}^{n} x_i(w, y) = b(y)$ which is the desired result. ■

We know that the vector $[x_i(w, y)]$ traces out the economically relevant part of the isoquant $F(x) = y$ as w varies. Therefore, we can say that translatability implies that the economically relevant part of the isoquant of the production function is contained in the hyperplane $\sum_{i=1}^{n} x_i(w, y) = b(y)$.

In the following two propositions we state some properties of T_A and M_A.

Theorem 5.3 *For all cost functions* $C(w, y)$ *the total cost index* $T_A(w^0, w^1, y)$ *satisfies MN and NORM.*

Theorem 5.4 *For all cost functions* $C(w, y)$ *the marginal cost index* $M_A(w^0, w^1, y)$ *satisfies CM and NORM.*

It is quite clear that T_A and M_A meet NORM. The remaining parts of these theorems can be established using arguments similar to that adopted in the proofs of theorems 1.7 and 1.8 of chapter 1. T_A and M_A do not satisfy HM. Using linear homogeneity and positivity of C and C_y it is easy to verify that T_A and M_A will go up under equiproportionate increases in current period factor prices.

Generally, the total index T_A will differ from the marginal index M_A. BDJ demonstrated that homotheticity of $C(w, y)$ is necessary and sufficient for T_R to be equal to M_R. Homotheticity of $C(w, y)$ demands that it should of the form $h(w)f(y)$. One implication of homotheticity is that all expansion paths are linear and pass through the origin[2]. The following theorem shows that equality of T_A and M_A is guaranteed by a particular homothetic cost function.

Theorem 5.5 $T_A(w^0, w^1, y) = M_A(w^0, w^1, y)$ *for all* $w^0, w^1 \in R^n_{++}$ *if and only if the associated cost function is of the form*

$$C(w, y) = K(w)e^y \tag{5.8}$$

[2]An application of a homothetic cost function to generate an index of efficiency of merger of firms can be found in Chakravarty (1992).

Proof : Equality of T_A and M_A requires

$$C(w^1, y) - C(w^0, y) = C_y(w^1, y) - C_y(w^0, y)$$

from which we have

$$C(w^1, y) - C_y(w^1, y) = C(w^0, y) - C_y(w^0, y) \tag{5.9}$$

From (5.9) it follows that

$$C(w, y) - C_y(w, y) = constant \tag{5.10}$$

for all $y \in R^1_+$ and for all $w \in R^n_{++}$.

Now, linear homogeneity of C in factor prices implies that C_y is also linearly homogeneous in factor prices. But from (5.9) it is evident that the right hand side of (5.10) does not involve w. Hence the right hand side of (5.10) is zero. Rearranging (5.10) we then have

$$\frac{C_y(w, y)}{C(w, y)} = 1 \tag{5.11}$$

Integrating both sides of (5.11) with respect to y, we get $\log C(w, y) = y + q_1(w)$, where $q_1(w)$ is the constant of integration. Hence $C(w, y) = K(w)e^y$, where $K(w) = e^{q_1(w)}$. This completes the necessity part of the proof. The sufficiency is easy to verify. ■

It is evident that the total cost index T_A will always depend on y. But there is a possibility that the marginal cost index will be independent of the output level. The following theorem shows this.

Theorem 5.6 *$M_A(w^0, w^1, y)$ is independent of y if and only if the associated cost function is of the type*

$$C(w, y) = a(w) + b(w)\, y \tag{5.12}$$

Proof : If M_A is independent of y, then

$$C_y(w^1, y_1) - C_y(w^0, y_1) = C_y(w^1, y_2) - C_y(w^0, y_2)$$

for all $w^0, w^1 \in R^n_{++}$ and $y_1, y_2 \in R^1_+$. Therefore,

$$C_y(w^1, y) - C_y(w^0, y) = constant \tag{5.13}$$

for all $y \in R^1_+$ and fixed $w^0, w^1 \in R^n_{++}$. Differentiating both sides of (5.13) partially with respect to y we get

$$C_{yy}(w^1, y) - C_{yy}(w^0, y) = 0 \tag{5.14}$$

where $C_{yy}(w,y) = \partial C_y(w,y)/\partial y$. Therefore, $C_{yy}(w^1,y) = C_{yy}(w^0,y)$. Hence $C_{yy}(w,y)$ is independent of w. But linear homogeneity of C in w implies that C_{yy} is also linearly homogeneous in w. This means that $C_{yy}(w,y) = 0$, which on integration with respect to y yields

$$C_y(w,y) = b(w) \tag{5.15}$$

where $b(w)$ is integration constant. Integrating (5.15) with respect to y we get $C(w,y) = a(w) + b(w)y$ where $a(w)$ is the constant of integration. This establishes the necessity part of the theorem. The sufficiency is easy to verify. ■

The cost function given by (5.15) is quasi-homothetic. In general, quasi-homotheticity requires that $C(w,y)$ should be of the form $U(w) + V(w)\ G(y)$. If $G(0) = 0$, then $C(w,0) = U(w)$. $U(w)$ can be interpreted as the fixed cost of production in the short-run. For the cost function given by (5.12), $C_y(w,y) = b(w)$. That is, in this case $b(w)$ is the constant marginal cost of production. With a quasi-homothetic cost function all the expansion paths are linear and need not pass through the origin. All homothetic functions are quasi-homothetic, but the converse is not true. (See Färe and Lovell (1984) for further discussions.)

We may now illustrate different cost indices using some examples. Suppose the cost function is of the following type.

$$C(w,y) = A^{-1/(a+b)}[(\frac{a}{b})^{b/(a+b)} + (\frac{a}{b})^{-a/(a+b)}]w_1^{a/(a+b)}w_2^{b/(a+b)}y^{1/(a+b)} \tag{5.16}$$

This cost function corresponds to the *Cobb – Douglas* production function (see Varian (1984, pp. 28-29)) given by

$$y = F(x) = Ax_1^a x_2^b \tag{5.17}$$

The Cobb-Douglas cost function is not translatable and the cost indices corresponding to it, T_A and M_A are not absolute.

Next, consider the following cost function

$$C(w,y) = (\frac{w_1}{a} + \frac{w_2}{b})y \tag{5.18}$$

which is associated with the *Leontief* technology

$$y = F(x) = \min\{ax_1, bx_2\} \tag{5.19}$$

Since this function is translatable, the corresponding cost indices T_A and M_A are absolute.

Alternatives of interest arises from the cost function

$$[\sum_{i=1}^{n} w_i - (\sum_{i=1}^{n}(w_i - \bar{w})^2)^{1/2}]f(y) \tag{5.20}$$

where $\bar{w} = \sum_{i=1}^{n} w_i/n$. This homothetic cost function satisfies translatability. Consequently, the associated cost indices T_A and M_A are absolute.

Thus, given any cost function we can construct the corresponding total and marginal production cost indices given by (5.3) and (5.4) respectively. These indices will be absolute if the cost function is translatable. The functional forms of these indices will differ depending upon the technology.

5.4 Conclusions

Baye, Deily and Jansen (1991) suggested marginal and total cost of production indices which 'may be used used to illuminate the course of an industry's evolution'. Their total cost index is the ratio between the costs of producing a given volume of output in the current period and the base period. The marginal cost index is defined analogously.

In this essay we have suggested alternative indices of total and marginal cost of production. Our total cost index gives the amount by which the total cost of production of a given volume of output in the current period deviates from the corresponding cost in the base period. The marginal cost index is defined in a similar manner. Clearly, the problem of choice between the BDJ indices and the new indices will depend on the purpose. If we want to look at the proportionate changes in the costs then BDJ indices are appropriate. On the other hand, the new indices are relevant for looking at increase/ decrease in costs resulting from factor price changes. Some properties of the new indices have also been investigated.

By construction all these indices do not depend on output demand and output prices. Therefore, they can be employed to study how profitability would change because of changes in factor prices.

Chapter 6

A Model Of Consumer Behaviour With Zero Expenditure

6.1 Introduction

We have discussed the problem of modelling zero expenditure in chapter 1. In this chapter we construct a preference structure where the number of commodities showing zero purchases may be large and the goods are divisible. Consumers, who have identical utility functions, have some priorities on the space of commodities and this ordering in turn generates corner solutions.

The chapter is organised as follows. The next section outlines the basis of our proposed preference structure. Section 6.3 formalises the structure in terms of a utility function. Section 6.4 gives some evidence in support of our model and section 6.5 gives some concluding remarks.

6.2 The Preference Structure

To focus our attention on income (or total expenditure) as the sole explanatory variable we consider consumers with identical preference structures. Now, consumption, as a function of income, can exhibit various patterns. We consider only the case where at low expenditure levels the optimal consumption basket consists of many zeroes and where the zeroes gradually disappear at higher total expenditure levels. In our case the marginal utility of a good is conditioned by consumption of some other items and hence an additively separable preference structure is ruled out.

We have noted that goods in our case enter into the consumption basket sequentially as total expenditure rises. Thus, we can order the commodities by their first arrival in someone's basket.

In an interesting paper, Encarnación (1990) has considered a hierarchical lexicographic preference of discrete choice based on *attributes*[1]. There, initially the choice is based on the attribute having rank 1. Once a *minimum level* ('satisfying level') of this is achieved, consumers' choice switches to depend on the next one and so on. Obviously, this preference has no notion of indifference and the demand points are discontinuous.

The idea of *minimum level* also appears in Debreu (1959). In describing the consumption sets he talks of '*dated commodities*', where there is a '*...minimum quantity of the first commodity which consumer must have available in order to survive ... If his input of the first commodity is less than or equal to this minimum, ... his input of the second commodity must be zero ...*' (op. cit., p. 52). He then, using free disposability of goods, describes the preference structure corresponding to this situation. The indifference curves are initially vertical half-lines upto a point and then become smooth curves (see Debreu (1959), p. 54-55).

We try to exploit these ideas of 'minimum/satisfying level' in our preference structure and incorporate substitution. We assume that the consumer has some ordering of the commodities and unless he has some minimum of the rank 1 items, the expenditure on other items will be zero. Note here that by rank 1 items we mean a group of commodities corresponding to rank 1. Rank 1 items can be thought of as items corresponding to the attribute *subsistence* [2]. We assume implicitly that we can

[1]The term attribute (also referred to as characteristic) is borrowed from Lancaster (1966,1971,1979). In the Lancasterian approach, there is a characteristic space of finite dimension and a consumer derives utility from the characteristic bundle $z = (z_1, z_2, \ldots, z_s)$ that he consumes. Each argument corresponds to a specific attribute, e.g., nutrition, shelter, social standing etc. In this approach consumption can be either combinable or noncombinable. In the combinable case one unit of good i is described by a vector $c_i = (c_{i1}, c_{i2}, \ldots, c_{is})$ of characteristics, where one or more arguments may be zero. A person consuming commodity bundle q would consume $\sum_{k=1}^{n} q_k c_{ki}$ of characteristic i. Formally, $z = \sum_{i=1}^{n} q_i c_i$.

In the non-combinable case, consumption is related to indivisibilities. (see Friedman (1983), Lancaster (1966,1971,1979)

[2]Clearly, in our case the term attribute bears a different meaning from Lancaster's set up. In the framework considered by us, items corresponding to different attributes enter consumer's basket

(contd.)

order the commodities into distinct groups, where each group provides a particular attribute, e.g., *subsistence, comfort, societal status* and so on. Once the satisfying level of a particular attribute is reached, there will be substitution between items belonging to this attribute and the preceding attributes and those belonging to the next attribute. We rule out dramatic price changes that would reverse the hierarchy of attributes.

Let us call the attributes described above *composite characteristics*, since these are associated in a mutually exclusive and exhaustive manner to a set of items widely differing in nature. Thus *subsistence* may consist of some food, some fuel and some clothing. It is therefore clear that each set of items corresponding to a composite characteristic can be further divided into homogeneous groups of items such as food, clothing etc, where each homogeneous group provides an attribute which could be called a *simple characteristic*. Figure 6.1 explains the structure.

We assume that whenever positive consumptions of items belonging to a particular composite characteristic take place, there will be positive consumption on at least one item belonging to each simple characteristic. This strict positivity of each simple characteristic items will enable us to define the sub-utility function on the space of simple characteristics in a standard neo-classical framework. The nature of the ordering on the space of composite characteristic tells us that the utility function must be asymmetrically separable (recursive) in the composite characteristics so that commodities corresponding to rank one composite characteristic are separable from all other items. Similarly, commodities associated with rank one and rank two composite characteristics are jointly separable from the rest of the items and so on (see Blackorby, Primont and Russell (1978)).

6.3 The Model

In view of the discussions above, the goods space is assumed to be completely partitioned into ranked classes (with several items in each class and each class representing a particular composite characteristic). The poorest man in the society will generally be consuming the highest ranked-class items and with increase in total expenditure the items of lower ranked-class enter successively his consumption

on the basis of his needs whereas in the Lancasterian set up a particular attribute may be derived from all items in the commodity space.

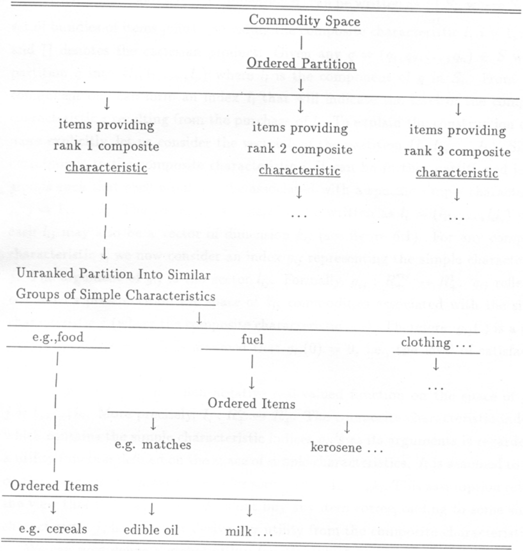

Figure 6.1: The Partitioned Commodity Space

basket. Hence, the hierarchy in composite characteristics implies a corresponding hierarchy of frequency of reporting of items[3]. We have already seen that the structure of consumer's choice described this way can be represented diagrammatically as in figure 6.1.

Formally, the goods space S (dimension n) can be written as $\prod_{i=1}^{k} S_i$, where S_i is the set of bundles of items jointly providing the composite characteristic i, $i = 1, \ldots, k$. and $\prod$ denotes the cartesian product. Given any $q = (q_1, q_2, \ldots, q_n) \in S$ we can partition q into $\{l_1, l_2, \ldots, l_k\}$ where l_i is the component of q in S_i. From the l_i component one can form an index I_i that will indicate the level of the composite characteristic i resulting from the purchase of l_i. To explain the construction of I_i's more explicitly, let us consider the vector l_i of the partition $\{l_1, l_2, \ldots, l_k\}$. Since l_i corresponds to the composite characteristic i, it can be further partitioned into k_i groups such that each group can be associated with a specific simple characteristic j, $j = 1, \ldots, k_i$. The vector l_i can therefore be written as $l_i = (l_{i1}, \ldots, l_{ik_i})$ where each l_{ij} may also be a vector of dimension k_{ij} (see figure 6.1). For any composite characteristic i, we now consider an index g_{ij} representing the simple characteristic j. The argument of g_{ij} is the vector l_{ij}. Formally, $g_{ij} : R_+^{k_{ij}} \rightarrow R_+^1$. g_{ij} reflects a consumer's preference on the space of k_{ij} commodities associated with the simple characteristic j (where the composite characteristic is i). Therefore, $g_{ij}(.)$ is a standard utility function. It is assumed that $g_{ij}(0) = 0$, i.e., the level of satisfaction from zero consumption is null.

We now define I_i as a non-negative real-valued function on the space of g_{ij}'s, $j = 1, \ldots, k_i$. More precisely, $I_i : R_+^{k_i} \rightarrow R_+^1$. The composite characteristic index I_i which contains the simple characteristic indices g_{ij}'s as its arguments is regarded as a utility function defined on the space of simple characteristics. I_i is assumed to take on the value zero whenever $g_{ij} = 0$ for some j, $j = 1, \ldots, k_i$. This assumption reflects the view that if the consumer does not buy any item corresponding to some simple characteristic j, he does not derive any utility from the composite characteristic i.

We can now define a global utility function G whose arguments are the indices $I_i, i = 1, \ldots, k$.

[3]Priority patterns have been analysed by Pyatt (1964) and Paroush (1965) in a discrete choice framework using frequency distribution of household durables by order of purchase.

We then have

$$\begin{aligned} G(I_1, \ldots, I_k) &= G(I_1(l_1), \ldots, I_k(l_k)) \\ &= U(l_1, \ldots, l_k) \\ &= U(q_1, \ldots, q_n) \end{aligned}$$

From now on, by i^{th}-ranked items we mean the items corresponding to the composite characteristic i. To derive a form of utility function, for the time being, we will restrict our attention to a two characteristic set up. Furthermore, assume that for any $I_2 \geq 0$,

$$G(I_1, I_2) = 0 \ , \tag{6.1}$$

whenever $I_1 = 0$.

This condition means that the first ranked items are essential for a consumer's utility maximisation.

We then have

Theorem 6.1 *Suppose that the utility function $G(I_1, I_2)$ is not bounded above and satisfies continuity, weak monotonicity and condition (6.1). Then $G(I_1, I_2)$ can be written as*

$$\begin{aligned} G(I_1, I_2) &= G_1(I_1) \quad if\ I_1 < \bar{I}_1 \\ &= G_2(I_1, I_2) \quad if\ I_1 \geq \bar{I}_1 \end{aligned} \tag{6.2}$$

where $G_1 : R^1_+ \to R^1_+$ and $G_2 : R^2_+ \to R^1_+$, $\bar{I}_1$ is some constant and

$$G_1(\bar{I}_1) = G_2(\bar{I}_1, I_2)\ for\ any\ I_2 \geq 0 \tag{6.3}$$

Proof: Let $T_1 = \{I_1 : G(I_1, I_2) = G(I_1, 0)$ for all $I_2 \geq 0\}$ and $T_2 = \{I_1 : G(I_1, I_2) \neq G(I_1, 0)$ for all $I_2 \geq 0\}$. By construction, the sets T_1 and T_2 are disjoint. We will now show that these two sets are intervals in R^1_+. That is, if $t_1 \notin T_1$ and $t_2 > t_1$, $t_2 \notin T_1$. Suppose $t_2 \in T_1$. Then $G(t_2, z) = G(t_2, 0)$ for all $z \geq 0$. Since, $t_1 \notin T_1$, $t_1 \in T_2$. Now, weak monotonocity of the utility function means its non-decreasingness in individual arguments. Therefore, we must have $G(t_1, z) > G(t_1, 0)$. Given unboundedness of the utility function, we can find a z^* such that

$$G(t_1, z^*) > G(t_2, 0) = G(t_2, z^*) \tag{6.4}$$

Since $t_2 > t_1$, (6.4) violates weak monotonicity of the utility function. Hence $t_2 \in T_1$. Therefore, T_1 and T_2 are intervals. Note that $0 \in T_1$. Thus $T_1 = [0, \sup T_1]$ and $T_2 = [\inf T_2, \infty)$. Continuity of the utility function requires that $\sup T_1 = \inf T_2$. This is the required value of $\bar{I}_1$. Hence $T_1 = [0, \bar{I}_1)$, $T_2 = [\bar{I}_1, \infty)$. Thus we have,

$$\begin{aligned} G(I_1, I_2) = &\quad G(I_1, 0) \quad if\ I_1 < \bar{I}_1 \\ = &\quad G_1(I_1) \quad (say) \\ and\quad G(I_1, I_2) = &\quad G_2(I_1, I_2) \quad if\ I_1 \geq \bar{I}_1\ (say) \end{aligned}$$

Clearly, continuity demands that $G_2(\bar{I}_1, I_2) = G_1(\bar{I}_1)$. This completes the proof of our theorem. ■

The set up in (6.2) can be extended to the case of k characteristics as follows. Since I_1 is separable from $(I_2, I_3, \ldots, I_k)$ we can define a sub-utility function $H(I_2, I_3, \ldots, I_k)$ such that

$$G(I_1, I_2, \ldots, I_k) = G_3(I_1, H(I_2, I_3, \ldots, I_k))\ for\ some\ G_3(.) \tag{6.5}$$

Now, I_2 has the property that whenever $I_2 = 0$, $H(I_2, I_3, \ldots, I_k) = 0$ for all $(I_3, I_4, \ldots, I_k) \geq 0.1^{k-2}$. Thus characteristic 2 items are essential for positive contribution of items corresponding to characteristics $2, \ldots, k$ in the utility function. Evidently, this structure can be extended for the remaining characteristics.

Hence the general form of the utility function $G(I_1, I_2, \ldots, I_k)$ can be written as

$$\begin{aligned} U(q) = G(I_1, I_2, \ldots, I_k) = &\quad G_1(I_1) &\quad if\ I_1 < \bar{I}_1 \\ = &\quad G_2(I_1, I_2) &\quad if\ I_1 > \bar{I}_1,\ I_2 < \bar{I}_2 \\ \vdots &\quad \ldots & \\ &\quad G_k(I_1, \ldots, I_k) &\quad if\ I_1 \geq \bar{I}_1,\ \ldots,\ I_{k-1} > \bar{I}_{k-1} \end{aligned} \tag{6.6}$$

In Encarnación's (1990) model, the *satisfying levels* of attributes are determined by biological and individual factors or socially determined norms. He speaks of *enough* food, a large *enough* variety of dishes for the guests at a feast etc. In our case, $\bar{I}_1$ represents the *satisfying* level of utility derived from rank 1 items. There will be different combinations of these items, each giving the same level of $\bar{I}_1$ so that, in general, setting itemwise satisfying levels is not possible in this case. $\bar{I}_2, \bar{I}_3, \ldots, \bar{I}_{k-1}$ can be similarly be interpreted as the *satisfying* levels of rank 2 to rank $k-1$ items respectively.

The expenditure function $e(u, p)$ consistent with the above utility function can be shown to be

$$
\begin{aligned}
e(u,p) = &\quad e_1(u,p_1) && if\ u < u_1^* = \bar{u}_1 \\
= &\quad e_2(u,p_1,p_2) && if\ \bar{u}_1 \le u < u_1^* + u_2^* = \bar{u}_2 \\
= &\quad \ldots && \\
= &\quad e_k(u,p_1,\ldots,p_k) && if\ u \ge u_1^* + \ldots + u_{k-1}^* = \bar{u}_{k-1}
\end{aligned} \tag{6.7}
$$

where, u_i^* is the utility corresponding to the satisfying level of composite characteristic i, $\bar{u}_i$ is the cumulative utility from satisfying levels of composite characteristics $1, 2, \ldots, i$; p_i is the composite price index for items in group i and e_i's are functions satisfying all the properties of an expenditure function. That is, each $e_i(.)$ is homogeneous of degree one in prices, increasing in u, non-decreasing in prices (and increasing in at least one price) and concave in prices.

To illustrate the general formula in (6.6) let us consider the following recursive utility function :

$$
\begin{aligned}
G(I_1,\ldots,I_k) = &\quad I_1 && if\ I_1 < \bar{I}_1 \\
= &\quad \bar{I}_1 + (I_1 - \bar{I}_1).I_2 && if\ I_1 \ge \bar{I}_1\ and\ I_2 < \bar{I}_2 \\
= &\quad \bar{I}_1 + (I_1 - \bar{I}_1).\bar{I}_2 + (I_1 - \bar{I}_1).(I_2 - \bar{I}_2).I_3 && if\ I_1 \ge \bar{I}_1, \\
&&& I_2 \ge \bar{I}_2\ and\ I_3 < \bar{I}_3 \\
= &\quad \ldots &&
\end{aligned} \tag{6.8}
$$

The corresponding expenditure function can be shown to be

$$
\begin{aligned}
e(u,p) = &\quad p_1.u && if\ u < u_1^* = \bar{u}_1 \\
= &\quad p_1.\bar{u}_1 + (u - \bar{u}_1).p_1^{a_1}.p_2^{a_2} && if\ \bar{u}_1 \le u < u_1^* + u_2^* = \bar{u}_2 \\
&&& 0 < a_i < 1,\ a_1 + a_2 = 1 \\
= &\quad p_1.\bar{u}_1 + (\bar{u}_2 - u_1).p_1^{a_1}.p_2^{a_2} && +(u - \bar{u}_2).p_1^{b_1}.p_2^{b_2}.p_3^{b_3} \\
&&& if\ \bar{u}_2 \le u < u_1^* + u_2^* + u_3^* = u_3 \\
&&& 0 < b_i < 1,\ b_1 + b_2 + b_3 = 1 \\
= &\quad \ldots &&
\end{aligned} \tag{6.9}
$$

Note that once the satisfying level of composite characteristic 1 is reached, in the next stage the cost of this enters as a fixed cost in the expenditure function along with an incremental cost component. We may also note that each component of the above expenditure function is structurally similar to the Linear Expenditure System (LES) of Stone (1954).

The corresponding budget share functions for the composite characteristic classes are

Expenditure Levels	Budget Shares of Composite Characteristic Group			
	1	2	3	...
$c < c_{01}$	1	0	0	0
$c_{01} \le c < c_{02}$	$a_1 + (1 - a_1)c_{01}/c$	$a_2(1 - c_{01}/c)$	0	0
$c_{02} \le c < c_{03}$	$b_1 + (1 - a_1)c_{01}/c$ $+(a_1 - b_1)c_{02}/c$	$b_2 + (a_2 - b_2)c_{02}/c$ $-a_2c_{01}/c$	$b_3(1 - c_{02}/c)$	0
...	...	...	...	...

$$\begin{aligned} \textit{where, } c_{01} &= p_1\bar{u}_1 \\ c_{02} &= p_1\bar{u}_1 + (\bar{u}_2 - \bar{u}_1)p_1^{a_1}p_2^{a_2} \\ c_{03} &= p_1\bar{u}_1 + (\bar{u}_2 - \bar{u}_1)p_1^{a_1}p_2^{a_2} + (\bar{u}_3 - u_2)p_1^{b_1}p_2^{b_2}p_3^{b_3} \\ \cdots & \qquad \cdots \end{aligned}$$

and c_{0j} is the threshold total expenditure level for the choice to be passed on to the level of composite characteristic $j+1$. a_i's, b_i's and c_{0j}'s are to be determined from the data.

We now specify the sub-expenditure function defined on the set of items representing a composite characteristic i. We have already noted that the functions $I_i(.)$, $i = 1, 2, \ldots, k$ are standard utility functions defined on the space of simple characteristics. Hence the form of the corresponding sub-expenditure functions $e_i(u_i, P_{i1}, \ldots, P_{ik_i})$, $i = 1, 2 \ldots, k$, where P_{ij} is the price index of the group of items providing j^{th} simple characteristic, will also be a standard one. For example, we can take the following functional form

$$e_i(u_i, P_{i1}, \ldots, P_{ik_i}) = \sum_{j=1}^{k_i} P_{ij}.\alpha_j + u_i.\prod_{j=1}^{k_i} P_{ij}^{\beta_j} \tag{6.10}$$

with $\sum_{j=1}^{k_i} \beta_j = 1$, $\beta_j > 0$ for all j. Note that this is the expenditure function of the LES defined on a restricted commodity space.

Finally, the expenditure functions $e_{ij}(.)$, $i = 1, 2, \ldots, k$; $j = 1, 2, \ldots, k_i$ at the item level, corresponding to the utility functions $g_{ij}(.)$'s, can be formulated in a manner described in (6.9). The arguments of $e_{ij}(.)$ are the level of utility derived from consumption of items providing the simple characteristic j (belonging to composite characteristic i) and the prices of the individual items in that group.

6.4 Empirical Evidence

The data used here have been obtained from the Indian National Sample Survey (NSS) 38th round consumer expenditure enquiry covering the period January - December, 1983. To ensure that the consumers covered in the empirical study have similar preference structure we have confined our study to the analysis of the household level consumer expenditure data of one particular region, viz., the state of Uttar Pradesh (UP)[4], for one particular season, viz., subround 1 (January - March, i.e., winter). The data for urban UP have been used to minimize the effect of consumption out of home-grown products. In the sample considered there are 1077 households.

To examine the extent of variation of consumption baskets with rising level of living (strictly, per capita consumption expenditure (PCE)), we have looked at frequency[5] of reporting of items across decile groups of PCE[6]. Table 6.1 shows the distribution of items by their first appearance over the various decile groups of PCE. This clearly supports the hypothesis of hierarchical income thresholds for consumption of various items. Out of 480 items considered in the survey, only 234 were reported by the first decile group. The remaining items gradually come into the picture as we move up the decile classes.

Next, to implement our model, we need to identify the composite characteristics and the items associated with them as well. Now grouping of items can be done by looking at the shape of budget share curve of each item in relation to the PCE.

[4] The staple diet of majority of the people of UP is hand-made wheat bread and bread made from a mix of other cereals. The climate of this state is characterised by harsh summers and winters.

[5] All the computations done here are estimates for the population obtained from the sample households, using design-based probability weights (given in the form of multipliers in NSS data).

[6] The decile groups are obtained by ordering the households by PCE. Each group represents (more or less) equal number of households in the population.

Table 6.1 : The Minimum Expenditure for Positive Consumption of Items

Decile groups of PCE	Range of PCE (Rs.)	Percentage of items with first positive reporting in the decile group
(1)	(2)	(3)
1	0.00 – 73.03	48.75
2	73.03 – 87.82	11.25
3	87.82 – 103.11	5.83
4	103.11 – 116.03	5.63
5	116.03 – 136.37	6.04
6	136.37 – 156.87	4.17
7	156.87 – 187.54	5.63
8	187.54 – 229.14	3.33
9	229.14 – 304.07	2.50
10	304.07 –	6.88
Total		100.00
Total no. of items = 480		

Such shapes have various possibilities : a falling curve indicating *necessaries* and a rising one signalling *luxuries*. Therefore, we can take *subsistence* as the composite characteristic representing necessary items and *societal status* as the composite characteristic representing luxuries. To give credence to our model, we need at least one more composite characteristic which will be of *intermediate* nature between these two extremes, *subsistence* and *societal status*. Let us call this attribute *comfort*. The items corresponding to this may be called *conventional necessities*. Thus conventional necessities are *relative luxuries* which are luxuries for a poor person and necessities for a rich person. Therefore, the expected budget share curves of these items are *inverted U-shaped*. By looking at the budget shares of all the items in our data, most of them could be classified into these three categories barring some exceptions[7].

Mention of *relative luxuries* dates back to Törnqvist (1941). (See also Wold and Jureen (1953) and Rudra (1969).) In fact, in demand analysis using inter-country data with countries as units, the inverted U-shape has been observed and regarded as the 'most representative formula for a demand function if one takes an income range covering the most poor to the most rich countries' (Rudra (1969)). These items cannot be classified by expenditure elasticities as in the case of necessaries or luxuries.

In table 6.2, we present the budget shares of some items which appear to be conventional necessities[8]. To look into the aspect of sampling fluctuation, we have obtained budget shares of these items for the two independent and interpenetrating sub-samples available in the data. The divergence between sub-sample estimates indicates the margin of uncertainty associated with the combined sample estimates. Due to high level of disaggregation and sampling fluctuations, for some items the inverted U-shape may not be very clear. Figure 6.2 shows the budget share curves for two coventional necessaries where the inverted U-shape is quite pronounced. Thus we may conclude that the class of conventional necessities is not merely a theoretical possibility.

In table 6.2, we also present percentages of households reporting each item by decile groups of PCE. These tend to corroborate the relative luxury nature of the items covered in the table.

Finally, we need to identify the simple characteristic groups within each com-

[7]For the item *cooked meals* the budget share curve turned out to be U-shaped. This implies that this item behaves as *necessary* at the lower income level and then becomes a *luxury* with increase in income. This is probably due to grouping of *cheap* road-side meals and *expensive* restaurant food together under *cooked meals* or some other recording inconsistency. In fact, Minhas (1991) has clearly mentioned some problems in recording the consumption of cooked meals in the NSS enquiry. All cooked meals, including those served to other people not belonging to the host household, are recorded in the consumption of the host household. These households will generally belong to the richer sections of the society. (see, for details, Minhas (1991)). Such items can be classified into *necessary* (*luxury*) according as the PCE is *below* (*above*) the expenditure at which the budget share is minimum.

[8]Roy and Dhar (1960) have fitted Törnqvist forms of engel curves in a study of consumer expenditure pattern using Indian NSS 7^{th} round data (October 1953 - March 1954). In their study they used 9 broad commodity groups separately for rural and urban India. In both the rural and urban regions, the Törnqvist form for relative luxuries gave the best fit for the commodity groups *sugar* and *clothing*.

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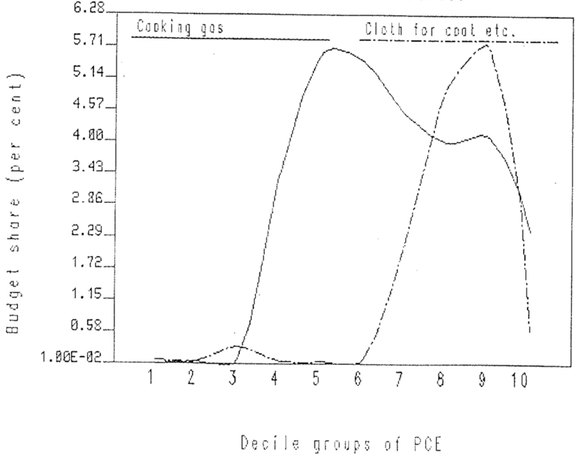
Table 6.2 : Budget Shares (per cent) and Percentage of Households Reporting for Some Selected *Conventional Necessaries* by Decile Groups of PCE

ITEM		Type of sample	Decile groups of PCE 1	2	3	4	5	6	7	8	9	10
(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Railway Fare	Budget shares	combined	2.01	3.26	3.72	4.46	2.32	3.93	6.65	2.33	4.66	3.29
		subsample 1	1.78	2.86	1.42	8.86	2.96	10.52	6.91	1.66	4.74	3.19
		subsample 2	2.80	3.56	5.04	3.56	2.35	1.81	6.19	3.17	4.19	3.41
	Per cent reporting	combined	6.40	5.30	5.00	14.20	10.30	10.20	11.30	11.40	14.30	11.70
Tyres Tubes	Budget shares	combined	0.00	0.59	0.47	0.38	0.53	0.37	0.49	0.97	1.38	0.38
		subsample 1	0.00	0.69	0.29	0.30	0.4 9	0.35	0.74	0.97	0.00	0.20
		subsample 2	0.00	0.19	0.62	0.58	0.54	0.39	0.41	0.96	1.38	0.45
	Per cent reporting	combined	0.00	7.70	13.00	8.70	15.90	9.40	14.20	13.80	9.90	7.30
Books Journals	Budget shares	combined	0.88	0.78	0.86	1.60	0.91	1.48	1.04	1.56	1. 54	0.97
		subsample 1	1.12	0.88	1.01	1.96	1.56	1.72	1.43	1.70	2.13	1.25
		subsample 2	0.67	0.73	0.71	1.26	0.90	0.55	0.78	1.23	1.01	0.65
	Per cent reporting	combined	7.60	8.10	10.00	9.50	10.70	11.60	13.20	11.20	7.80	10.20
Ghee	Budget shares	combined	2.85	3.93	5.62	4.35	5.44	4.15	4.07	4.37	4. 83	3.81
		subsample 1	2.97	2.50	4.86	4.45	6.47	4.08	4.61	4.15	4.93	4.12
		subsample 2	1.99	4.36	6.42	4.42	4.73	3.58	3.67	4.66	4.60	3.46
	Per cent reporting	combined	2.00	1.40	4.20	4.00	9.40	7.70	17.80	15.20	14.60	23.50
Khandsari Sugar	Budget shares	combined	0.00	2.14	2.64	0.00	6.10	0.00	0.96	0.65	0. 45	0.12
		subsample 1	0.00	0.00	0.00	0.00	6 .10	0.00	0.08	0.00	0.00	0.00
		subsample 2	0.00	2.25	2.64	0.00	0.00	0 .00	1.77	0.65	0.45	0.12
	Per cent reporting	combined	0.00	36.50	6.00	0.00	15.70	0.00	11.00	6.40	19.60	4.90

Table 6.2 (contd.)

(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Tea (cups)	Budget shares	combined	2.99	2.83	2.52	2.51	2.97	3.90	3.18	3.16	5 .37	3.12
		subsample 1	2.98	2.73	2.60	2.59	3.17	3.92	3.02	3.20	5.78	2.69
		subsample 2	3.02	2.92	2.72	2.33	2.61	3.98	3.15	3.37	5.00	3.53
	Per cent reporting	combined	4.90	7.20	7.50	9.10	8.80	9.90	13.50	9.60	15.10	14.30
Biscuit	Budget shares	combined	0.74	1.07	0.83	0.79	0.84	1.29	1.46	1.00	1 .30	0.79
		subsample 1	0.92	1.13	0.98	1.03	0.9 2	1.41	1.12	1.06	1.23	0.76
		subsample 2	0.46	1.02	0.74	0.70	0.81	1.03	1.81	0.96	1.68	0.78
	Per cent reporting	combined	4.90	5.60	5.00	4.50	8.50	11.60	14.50	16.50	10.90	18.10
Cooking Gas	Budget shares	combined	0.00	0.00	0.00	3.49	5.61	5.45	4.50	4.00	4. 11	2.42
		subsample 1	0.00	0.00	0.00	0.00	7.2 6	5.12	4.18	4.39	4.60	2.23
		subsample 2	0.00	0.00	0.00	5.32	5.18	3. 78	5.71	3.86	2.65	2.62
	Per cent reporting	combined	0.00	0.00	0.00	0.30	1.40	7.30	14.10	20.60	16.60	39.70
Woolen Cloth for Coat etc.	Budget shares	combined	0.08	0.00	0.33	0.00	0.00	0.00	2.17	4.98	5. 71	0.62
		subsample 1	0.00	0.00	0.00	0.00	0.00	0.00	3.88	5.06	6.55	1.42
		subsample 2	0.08	0.00	0.33	0.00	0.00	0.00	1.06	4.33	0.00	0.00
	Per cent reporting	combined	5.20	0.00	5.90	0.00	0.00	0.00	22.40	30.80	26.90	8.90
Woolen Shawl	Budget shares	combined	0.00	0.00	0.00	0.00	0.58	0.71	1.50	0.73	0. 51	0.38
		subsample 1	0.00	0.00	0.00	0.00	0.73	0.71	1.67	0.45	0.51	0.38
		subsample 2	0.00	0.00	0.00	0.00	0.34	0 .00	0.88	0.76	0.00	0.00
	Per cent reporting	combined	0.00	0.00	0.00	0.00	11.90	7.30	21.30	21.20	23.80	14.60
Pillow ,Quilt	Budget shares	combined	1.32	2.22	1.12	2.61	1.61	4.26	0.50	1.36	1. 37	0.94
		subsample 1	1.40	1.28	1.12	1.39	1.45	2.71	0.29	1.30	1.40	1.02
		subsample 2	1.28	2.77	0.00	3.75	1.63	7.23	0.73	2.57	1.16	0.41
	Per cent reporting	combined	7.10	10.90	6.00	4.40	15.30	9.20	10.90	11.40	13.70	11.20

Figure showing budget shares of two conventional necessaries



posite characteristic group. We observe that the groupings *food*, *clothing* and a *residual* miscellaneous category shows positive values of budget shares for each of the composite characteristics. These groups, can, therefore, be identified as the simple characteristic groups within each composite charactetistic group.

It is thus evident from the data that there does exist a hierarchical ordering in the commodity space and with increase in PCE the dimension of consumers' basket increases systematically as depicted in the model.

6.5 Conclusions

In a society, individuals may have a clear preference of some items over others in terms of needs and the items can be grouped according to certain basic composite characteristics inherent in them. Each composite characteristic has an associated threshold level of income such that only the consumers with income greater than this threshold level consume the items providing this characteristic. We have formalised this behaviour in terms of a recursive utility function. Empirically, this implies that the number of composite characteristics relevant for a society would, in principle, determine the number of Per Capita Expenditure (PCE) strata into which the consumers can be classified. The threshold level of PCE corresponding to each successive composite characteristic would represent a well defined hierarchy of absolute levels of living. One can also get an idea of relative deprivation of consumers in different PCE strata in terms of items consumed exclusively by consumers of higher PCE strata. From a planner's point of view, the results thrown up by this type of studies may be useful for designing commodity tax reform and public policies involving considerations of basic needs.

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