

Classification of Rotated Textures using Overcomplete Wavelet Frames

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In this paper we present an approach to characterize textures at multiple scales using wavelet transforms and propose a texture classification algorithm that is invariant to rotation and translation. The nonseparable discrete wavelet frame is used as the wavelet transform that decompose the texture images into a set of frequency channels. In each channel we take the variance as feature. Classification experiments using twenty Brodatz textures indicate that texture signatures based on wavelet frame analysis are beneficial for accomplishing subtle discrimination of textures and robust classification against rotation and translation.

Indexing Term: Feature extraction, Texture classification, Wavelet frames, Non-separable filters.

TEXTURE analysis plays an important role in pattern recognition and computer vision and is widely applied to areas like image analysis, remote sensing, robot vision, query by content in large image data bases and many others.

A number of texture descriptors have been developed for texture analysis and discrimination, descriptors based on structural model [1] and statistical model [2]. In the last decade there has been an extensive study on model based approaches like Markov random fields [3].

Psychovisual studies reveal that the human visual system processes images by decomposing them into filtered images of various frequencies and orientation [4], that is capable of preserving both local and global information. It has been observed that the response corresponds to Gabor-like function. This multiscale processing, which humans apply to texture perception, is a strong motivation for texture analysis methods based on these concepts. There has been an extensive study providing a clear demonstration of the superiority of the these multiscale processing over the more traditional ones mentioned above. Wavelet theory provides a more formal, precise and unified approach to multiresolution representations [5,6].

Carter [7] first reported texture classification results using Mexican hat wavelets. He achieved 98 per cent classification accuracy on 6 types of natural textures. The standard wavelet transform gives a frequency splitting in octave bands and is too coarse for textures consisting of high frequencies. The work by Chang and Kuo [8], however, indicate that texture features are most prevalent

in intermediate frequency bands. This proposal has been studied with a particular attention to the use of wavelet packets [9]. Unser [10] has also introduced wavelet frames for texture feature extraction.

Many texture identification methods presented in the literature approach the problem by assuming that samples of a texture all possess the same orientation. When this assumption is not valid, most of these methods performs poorly. In this paper we address the problem of rotation invariant texture identification i.e. the classification system should be able to identify textures with any arbitrary orientation. Previous works on rotation invariant classification were done by Kashyap *et al* [11] and Cohen *et al* [12]. More recent works on rotational invariance of textures involves incorporation of rotated examples in the training data [13], or spiral resampling of the data, to obtain a 1-dimensional signal, where rotation invariance is simulated as translation invariance [14]. They achieved 93.33% and 95.14% correct classification respectively. Most recently Fountain *et al* [15] have worked on rotation invariant texture classification by taking the Fourier transform (FT) of the gradient direction histograms of the textures. The direction histograms being a period function of 2π , a rotation is reflected as a translation in the fourier domain. Therefore, fourier coefficients give the rotational invariant features.

In this paper we propose extracting features from the texture itself and incorporating rotation-invariance in the features. In the present work, texture properties are characterized by wavelet frame analysis. While discrete wavelet transform gives a non redundant representation of the textures, the discrete wavelet frame (DWF) gives an overcomplete representation. This technique is employed to study the performance of a texture classifier with respect

to rotational and translational invariance.

A discretization of the transform parameters of the Continuous Wavelet Transform (CWT) gives the Discrete Wavelet Transform (DWT). The one dimensional implementation of the DWT using a filterbank is given in [6,16]. The orthogonal 2-D DWT is expressed in separable forms so as to exploit the advantage of separability property. The two-dimensional extension is obtained in two steps by successive application of the 1-D filtering along rows and columns of an image. Due to the separable nature of implementation of the two dimensional discrete wavelet transform, it is strongly oriented in the horizontal and vertical directions. Such a decomposition cannot efficiently characterize directions other than 0° and 90° . This is particularly inadequate while dealing with oriented textures. So what we need is a non-separable nature of implementation of the wavelet transform in 2-D.

Another important drawback of the DWT is that, a simple integer shift of the input signal will yield a completely different wavelet transform. A feature extraction scheme has to be independent of any translational shift, for texture has translation invariance (or stationary) property. An obvious way to overcome this limitation is to compute the discrete wavelet transform for all possible integer shifts of the input signal [10]. This approach leads to the redundant DWT in which the output of the filterbank is not subsampled. This should yield a better estimation of texture statistics and a more detailed texture characterization at region boundaries.

WAVELET TRANSFORMS AND WAVELET FRAMES

In this section the wavelet transform and wavelet frame transform are described briefly. An exhaustive mathematical treatment is given in [9]. The Continuous Wavelet Transform (CWT) of a 1-D signal $f(x)$ is defined as,

$$Wf_a(b) = \int f(x) \psi^*_{a,b}(x) dx \quad (1)$$

where ψ is the mother wavelet and a and b are dilation and translation parameters.

Discrete Wavelet Transform

DWT is obtained by discretizing the parameters a and b , a popular choice being $a = 2^m$ and $b = n2^m$ with $m, n \in \mathbb{Z}$.

The wavelet decomposition can be interpreted as signal decomposition in a set of independent, spatially oriented frequency channels. Under these constraints an efficient real space implementation of the transform using quadrature mirror filter exists [6]. The full discrete wavelet expansion of a signal $x \in l_2$ (l_2 is the space of square

summable functions) is given as,

$$x(k) = \sum_{l \in \mathbb{Z}} s_{(l)}(l) \phi_{l,l} + \sum_{i=1}^I \sum_{l \in \mathbb{Z}} d_{(i)}(l) \psi_{i,l} \quad (2)$$

where ϕ and ψ are the scaling and wavelet functions respectively and are associated with the analyzing / synthesizing filters h_i and g_i . $d_{(i)}$'s are the wavelet coefficients and $s_{(l)}$'s are the expansion coefficients of the coarser signal approximation $x_{(l)}$. It also follows from this construction that the family of sequences $\{\phi_{l,l}, \psi_{1,l}, \psi_{2,l}, \dots, \psi_{l,l}\} l \in \mathbb{Z}$ constitute an orthonormal basis. The discrete normalized basis functions are defined as,

$$\phi_{i,l}(k) = 2^{i/2} h_i(2^i k - l) \quad (3)$$

$$\psi_{i,l}(k) = 2^{i/2} g_i(2^i k - l) \quad (4)$$

where i and l are the scale and translation indices respectively, the factor $2^{i/2}$ is an inner product normalization. The s_l 's and d_i 's can be interpreted in terms of simple filtering and downsampling operations.

$$\begin{cases} s_{(l)}(l) &= 2^{l/2} [h_l^T * x] \downarrow 2^l(l) \\ d_{(i)}(l) &= 2^{i/2} [g_i^T * x] \downarrow 2^i(l) \end{cases} \quad (5)$$

where the symbol T denotes the transpose operation (i.e. $h^T(k) = h(-k)$) and where $[\cdot] \downarrow m$ is the downsampling by factor m . Let h_i and g_i be a perfect lowpass and perfect bandpass filter respectively. The extension of the Discrete Wavelet Transform (DWT) to the 2-D case is usually performed by using a product of 1-D filters. In practice the 2-D DWT is computed by applying a separable filter bank to the image.

$$S_i(x, y) = [h_x * [h_y * S_{i-1}] \downarrow_{2,1}] \downarrow_{1,2}(x, y) \quad (6)$$

$$D_i^1(x, y) = [h_x * [g_y * S_{i-1}] \downarrow_{2,1}] \downarrow_{1,2}(x, y) \quad (7)$$

$$D_i^2(x, y) = [g_x * [h_y * S_{i-1}] \downarrow_{2,1}] \downarrow_{1,2}(x, y) \quad (8)$$

$$D_i^3(x, y) = [g_x * [g_y * S_{i-1}] \downarrow_{2,1}] \downarrow_{1,2}(x, y) \quad (9)$$

* denotes the convolution operator, $\downarrow_{2,1}$ ($\downarrow_{1,2}$) denote subsampling along the rows (columns) and $S_0 = I(x, y)$ the original 2-D signal. $S_i(x, y)$ corresponds to the lowest frequencies, the D_i^n are obtained by bandpass filtering in a specific direction and thus contain the detail information at scale i . $D_i^1(x, y)$ corresponds to the vertical high frequencies (horizontal edges), $D_i^2(x, y)$ the horizontal high frequencies (vertical edges) and $D_i^3(x, y)$ the high frequencies in both direction (the corners). $I(x, y)$ is represented at several scales by, $\{S_d, D_i^n | n = 1, 2, 3, i = 1, \dots, d\}$.

Wavelet Frames

As already pointed out in the introduction that the standard subsampled wavelet transform is inadequate for

translational and rotational invariance. A natural way to overcome this limitation is to perform an analysis of the input signal in terms of the overcomplete family of frames. Wavelet frame leads to an overcomplete decomposition of the signal

$$\begin{cases} d_i^{DWF}(k) = \langle g_i(k-l) \dots x(k) \rangle_{l_2} \\ s_l^{DWF}(k) = \langle h_l(k-l) \dots x(k) \rangle_{l_2} \end{cases} \quad (10)$$

which is a non-sampled version of (5). Because of the special structure of the analysis filter bank, this decomposition has a number of remarkable properties that are associated with the mathematical concept of frame [5]. The frame is a spanning set, that requires finite limits on an inequality bound of inner products. If we want the coefficient in an expansion of a signal to represent the signal well, these coefficients should have certain properties, that are stated best in terms of energy and energy bounds.

The family of sequences S constitutes a frame of l_2 , of the Hilbert space l_2 if there exists two constants A and B such that

$$\begin{aligned} A \cdot \|x\|_{l_2}^2 &\leq \sum_{l \in Z} \langle x(k), h_l(k-l) \rangle^2 + \\ &\sum_{i=1}^I \sum_{l \in Z} \langle x(k), g_i(k-l) \rangle^2 \leq B \cdot \|x\|_{l_2}^2 \end{aligned} \quad (11)$$

We start by using Parseval's formula to compute the energies in the different channels

$$\begin{aligned} \|d_i\|_{l_2}^2 &= \int_0^1 |G_i(e^{j2\pi f})|^2 |X(f)|^2 df \\ \|s_l\|_{l_2}^2 &= \int_0^1 |H_l(e^{j2\pi f})|^2 |X(f)|^2 df \end{aligned}$$

where $X(f)$ denotes the Fourier transform of the input signal. We sum the individual terms and it is easy to show that the energy conservation property is preserved

$$\|x\|_{l_2}^2 = \int_0^1 |X(f)|^2 df = \|s_l\|_{l_2}^2 + \sum_{i=1}^I \|d_i\|_{l_2}^2 \quad (12)$$

By definition, $s_i(l) = \langle x(k), h_i(k-l) \rangle$ and $d_i(l) = \langle x(k), g_i(k-l) \rangle$ where $\langle \cdot, \cdot \rangle$ is the corresponding inner product, the fundamental difference with an orthogonal system is that the representation may be redundant. In the present context where $H = l_2$, this property together with the definition of wavelet coefficients in (10), leads to the simple reconstruction formula

$$x(k) = \sum_{l \in Z} s_l(l) h_l(k-l) + \sum_{i=1}^I \sum_{l \in Z} d_i(l) g_i(k-l) \quad (13)$$

A fast iterative decomposition algorithm is ,

$$\begin{cases} s_{i+1}(k) = [h_i] \uparrow_{2^i} * s_i(k) \\ d_{i+1}(k) = [g_i] \uparrow_{2^i} * s_i(k), (i = 0, \dots, I) \end{cases} \quad (14)$$

with the initial condition $s_0 = x$. Each step involves a convolution with the basic filters h and g , which are expanded by inserting an appropriate number of zeros between taps.

PROPOSED METHOD FOR FEATURE EXTRACTION

We discuss in this section the choice of the filter bank, and the computation of the wavelet parameters used as texture features.

What we need is a nonorthogonal basis set. This facilitates the decomposition of a signal in 2-dimensional space by applying one dimensional non separable filters along rows and columns independently.

The wavelet function in 2 dimensions is defined by $\psi_a(\vec{b}) = \frac{1}{\sqrt{a}} \psi(\frac{\vec{x}-\vec{b}}{a})$. The directional information, can be

incorporated in the wavelet function, by including a rotational parameter in it [5].

$$\psi_{a,\theta}(\vec{b}) = \frac{1}{a} \psi \quad R^\theta \quad \frac{\vec{x}-\vec{b}}{a} \quad (15)$$

where R^θ is the rotation operator denoted by the matrix.

$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

Now if the wavelet is circularly symmetric i.e. R^θ has no influence in (15), such an wavelet would generate rotation invariant features. Therefore the wavelet function has to satisfy two conditions one is $\hat{\psi}(0) = 0$, which implies that the wavelet function has to be a zero mean function and $\hat{\psi}(\vec{r}, \theta) = \hat{\psi}(\vec{r}, 0)$, this ensures that the rotation operator has no effect in (15).

We have chosen the wavelet used by Mallat in [17], which is the second derivative of a smoothing function. This choice have been made due to the following reasons. Firstly, this closely approximates the second derivative of Gaussian, which has circular symmetry. Secondly, the basis functions are symmetrical, which means that there is no phase distortion, and that the spatial localization of the wavelet coefficients is well preserved, moreover these filters are also useful in alleviating boundary effects by

simple methods of mirror extension. The filter corresponding to the second derivative of a smoothing function can be written as $F(w) = w^2 L(w)$, where $L(w)$ is the frequency response of the smoothing low pass filter and should have a non zero value at $w = 0$. For the filter used $F(w)$ has an order 2 zero at $w = 0$, and exhibits sharper outband attenuation and thus better frequency separation.

Wavelet Frame Representation of an Image

In the present work we have employed the transform based on Mallat's non-orthogonal (redundant) discrete wavelet frames [18]. Let $\theta(x, y)$ be a 2-D smoothing function. We call the smoothing function the impulse response of a lowpass filter. The convolution of a function $I(x, y)$ with a smoothing function attenuates part of its high frequency without modifying the lowest frequency and hence smooths $I(x, y)$. Supposing $\theta(x, y)$ is differentiable, we define two wavelet functions, $\psi^1(x, y)$ and $\psi^2(x, y)$ such that,

$$\psi^1(x, y) = \frac{\delta^2 \theta(x, y)}{\delta^2 x} \text{ and } \psi^2 = \frac{\delta^2 \theta(x, y)}{\delta^2 y} \quad (16)$$

Let,

$$\psi_s^1(x, y) = \frac{1}{s^3} \psi^1\left(\frac{x}{s}, \frac{y}{s}\right) \text{ and } \psi_s^2(x, y) = \frac{1}{s^3} \psi^2\left(\frac{x}{s}, \frac{y}{s}\right) \quad (17)$$

be the dilations of the functions ψ^i by a factor s and $\theta_s(x, y) = \frac{1}{s} \theta\left(\frac{x}{s}\right)$ the dilation of $\theta(x, y)$ by s .

Let $I(x, y)$ be an image in 2-D and $I(x, y) \in L_2(R^2)$. The wavelet transform of I at scale s has two components defined by,

$$W_s^1(x, y) = I * \psi_s^1(x, y) \text{ and } W_s^2(x, y) = I * \psi_s^2(x, y) \quad (18)$$

s is the scale parameter which commonly is set equal to 2^i with $i = 1, \dots, n$. This yields the so called dyadic wavelet transform of depth n . W_s^1 and W_s^2 are referred to as the detail images, since they contain the horizontal and vertical details of I at scale s .

This transform is computed by iterative filtering by a set of low and high pass filters h_i and g_i , associated with the smoothing function θ and the wavelet functions ψ^1 and ψ^2 respectively. These filters have finite impulse responses, which makes the transform fast and easy to implement (Fig 1).

$$S_{2^{i+1}}^d(x, y) = [h_{i,x} * [h_{i,y} * S_{2^i}^d]](x, y) \quad (19)$$

$$W_{2^{i+1}}^{1d}(x, y) = [D_{i,x} * [g_{i,y} * S_{2^i}^d]](x, y) \quad (20)$$

$$W_{2^{i+1}}^{2d}(x, y) = [g_{i,x} * [D_{i,y} * S_{2^i}^d]](x, y) \quad (21)$$

$S_1^d I$ is the original image and D is the Dirac filter whose impulse response equals 1 at 0 and 0 otherwise. $*$ denotes the convolution operator. $A * (B * C)$ denotes separate convolution of the rows and columns respectively of the image A with 1-D filters B and C .

Then the wavelet representation of depth n of the digitized image I consists of the low resolution images $\{S_{2^i}^d\}$ and detail images $\{W_{2^i}^{jd}\}$ for $\{j = 1, 2\}$ and $\{i = 1, \dots, n\}$.

Texture Features Computation

We now discuss the computation of the rotation and translation invariant parameters from wavelet transformed image.

Substitution of (16) and (18) in (17) yields the following interesting property.

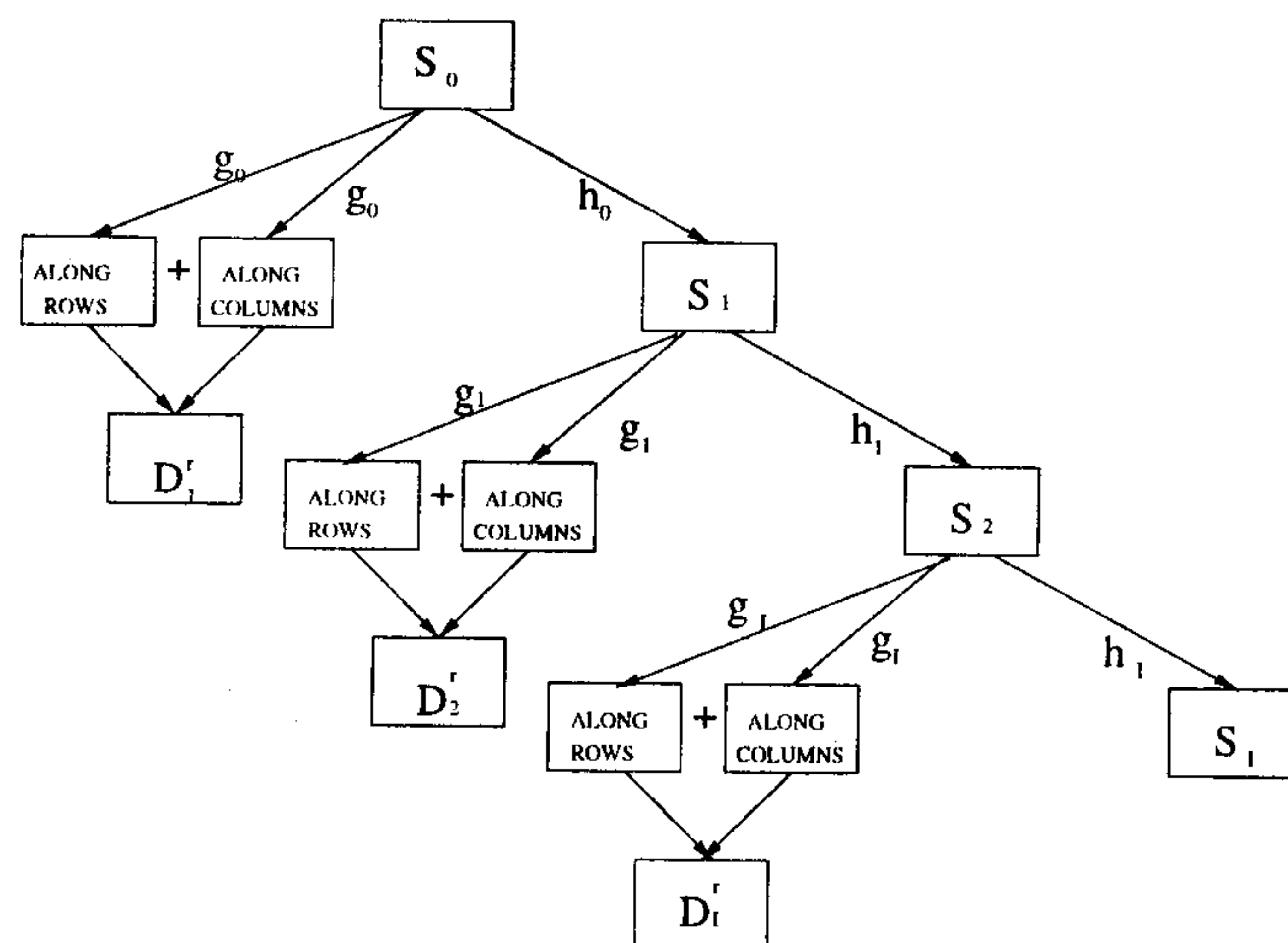


Fig 1 Fast iterative implementation of the algorithm used for extracting texture features

$$\begin{pmatrix} W_s^{1d}(x, y) \\ W_s^{2d}(x, y) \end{pmatrix} = s^2 \begin{pmatrix} \frac{\delta^2}{\delta_x^2} (I * \theta_s)(x, y) \\ \frac{\delta^2}{\delta_y^2} (I * \theta_s)(x, y) \end{pmatrix} = s^2 \nabla^2 (I * \theta_s)(x, y) \quad (22)$$

where ∇^2 denotes the Laplacian and gives zero value at the location of the signal sharper variation points. It defines edge magnitudes of the image and since it has the same property in all directions is invariant to rotations in the image. That is the wavelet transform of an image consists of the components which give a measure of the edge magnitudes of the image, smoothed by the dilated smoothing function θ_s . The edge magnitude of the image is given as,

$$w_s^r(x, y) = \sqrt{(w_s^1(x, y))^2 + (w_s^2(x, y))^2} \quad (23)$$

The $w_s^r(x, y)$ contain a measure of the edge magnitude which is proportional to the magnitude of the local gray level variation of the image and clearly yields a rotation invariant multiscale representation $\{(w_s^r)_{(s=1, \dots, 2)}, c_n\}$. The nonsampled DWF representation gives translation invariance.

The energies of the detailed images decomposed into different frequency channels at different resolutions are given by

$$Ed_i = \frac{1}{MN} \sum_{x,y} (w_{2i}^r(x, y))^2, \quad i = 1, \dots, n \quad (24)$$

Energies of the low resolution images are

$$Es_i = \frac{1}{MN} \sum_{x,y} (c_{2i}(x, y))^2, \quad i = 1, \dots, n \quad (25)$$

M and N are the number of rows and columns of the digitized image $I(x, y)$. In this approach we arrange the output of the filterbank into n component subbands

$$\begin{aligned} f(x, y) &= (f_{2i}(x, y))_{i=1, \dots, n} \\ &= [c_1(x, y), \dots, c_n(x, y) d_1(x, y), \dots, \\ &\quad d_n(x, y)]^T \end{aligned} \quad (26)$$

We can get a more compact representation in terms of the channel variances $\text{Var}\{y_i\}$. In practice the channel variances are estimated from the average sum of squares over a region Reg of the given texture. Each channel extracts a particular aspect of the textures at different frequencies and resolutions.

$$v_i = \frac{1}{Reg} \sum_{(k,l) \in Reg} (y_i(k, l) - \bar{y}_i(k, l))^2 \quad (27)$$

where Reg denotes the number of pixels in its region and $\bar{y}_i(k, l)$ is the mean value of energy of each channel.

TEXTURE CLASSIFICATION

In texture classification task, the generated features span a high dimensional feature space, which is subdivided into a set of classes. A feature vector is assigned a class label according to its position in feature space.

Discrimination using k_nn Classifier

Here each image is represented by a single feature vector. A training set contains several labeled images. By applying classification on this set, texture classes are defined according to the labels of the textures in the training set. We use a non-parametric classifier (k_nn) in our experiment. Here an unknown datapoint is given the same class label as the majority of its k nearest neighbors from the training set [19]. Classification of feature vector \bar{x} is performed by searching its k nearest training vectors according to some metric $d(\bar{x}, \bar{y})$. The vector \bar{x} is assigned to that class to which the majority of these k nearest neighbors belong. The metric $d(\bar{x}, \bar{y})$ express the Euclidean distance between two points x and y in feature space.

$$d(\bar{x}, \bar{y}) = \sum_{i=1}^d |(x_i - y_i)|^2,$$

Training such a classifier requires a set of reasonable size which reflects all properties of the complete set of possible images. This is achieved by the *leave k out method*. That is out of N data sets each time k distinct samples were taken as test data without repetition and the rest were used for training. By this a large design set and also independent test set is available.

Experimental Data Set

We performed classification experiments using 20 Brodatz textures [24]. The original photographs were digitized and converted into 128×128 image arrays.

So far in texture classification problems researchers have usually divided each images into overlapping (or non overlapping) subregions and evaluated the feature vectors for each subregion in each class. This was adopted to increase the number of data sets. But here in this paper we have adopted a different method for extracting experimental data sets. Each class were subjected to five different translations in all directions (left, right, up and down), and oriented by five different rotations. There were 25 different samples of size 128×128 for each class which were all different from each other either in translation or rotation. The wavelet frame decomposition of (23) was performed by processing individual images without

subsampling. Each image was subjected to three levels (scale) of decomposition, and each level was represented by 6 features as shown in Fig 1. The channel variances for all the 6 subbands were computed. We chose the log of variance as the information measure.

Figures 2 and 3 shows some of the translated and rotated textures used in our work.

RESULTS AND CRITICAL COMMENTS

In the first series of experiments the data sets were evaluated using the five set of parameters shown in

Table 1. We get an excellent classification result while considering the redundant DWF decomposition which is consistent with our expectation. We simulated rotation of the image about an axis through the centre merely by transporting the gray level of the pixels from their original position to their approximate modified positions after transformation of the axes due to a rotation θ . So no appreciable distortion is introduced in these rotated images due to this approximation (truncation). The texture features that we have extracted are proportional to the gray level values of the images, which in turn are not changed due to rotation. The observation from Table 1 is that, a 45°

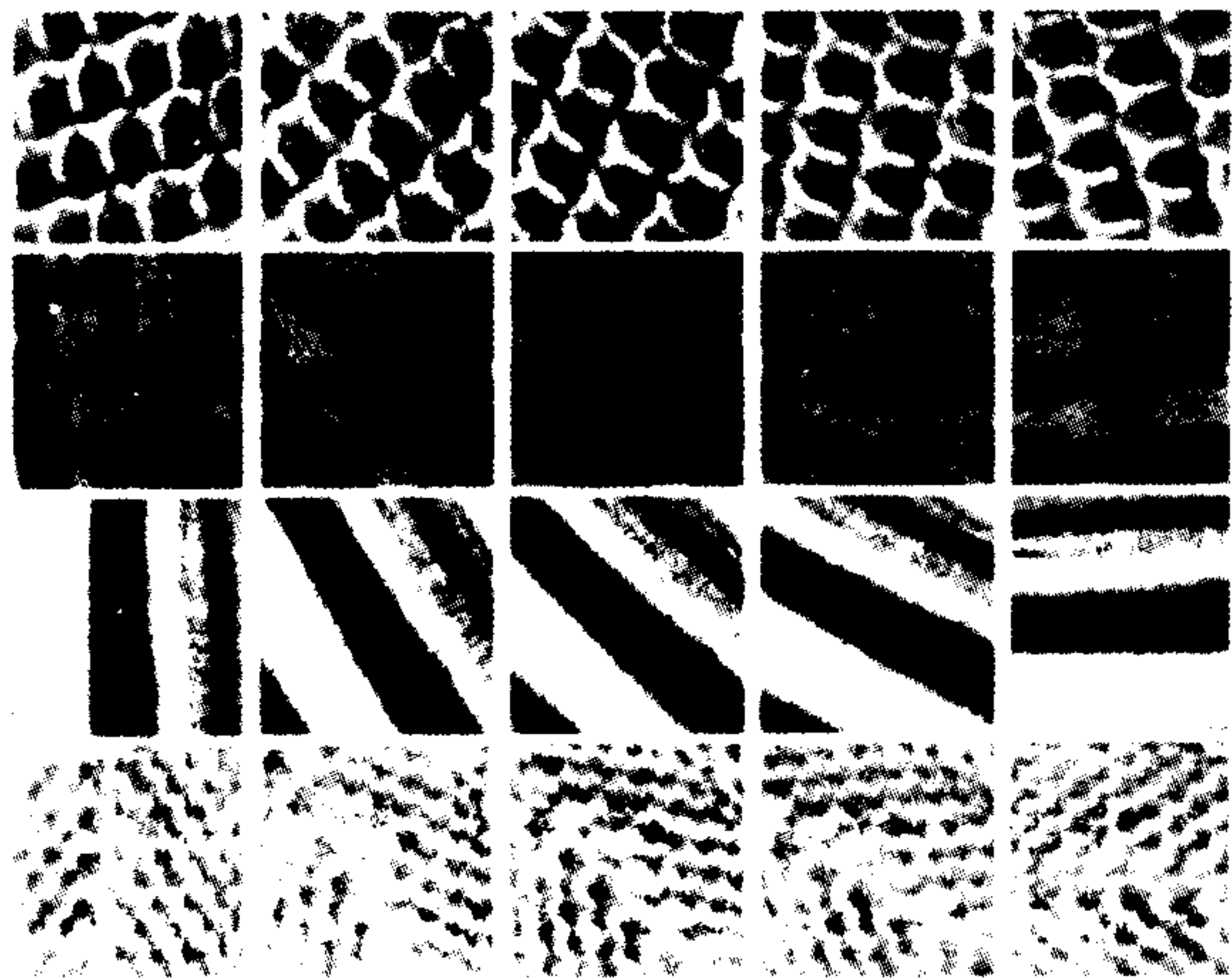


Fig 2 Rotated samples (0°, 30°, 45°, 60°, and 90°) of some of the textured images used in our experiment

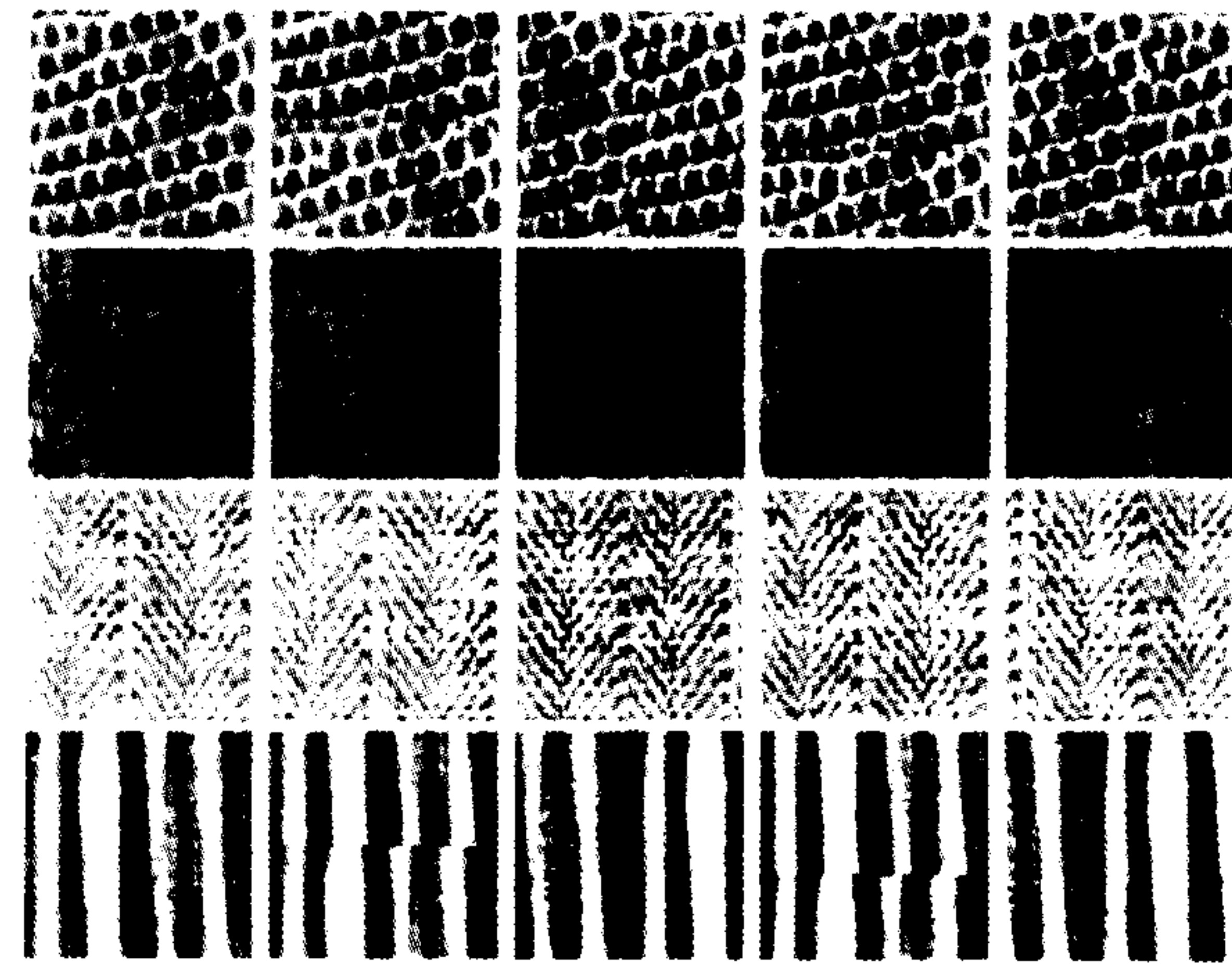


Fig 3 Translated samples of some of the textured images downshift - 64, 48 and rightshift -64, 48

TABLE 1 Classification performance considering the discrete wavelet frame transform

Type of Decomp	No. of features	Test data set		Training data set		%of classifn.	
		Transl.	Rotn	Transl.	Rotn		
DWF	6	8	5	16	20	99.99	
				8	30		
				16	60		
				8	45		
	16	20	16	20	8	5	99.92
					8	30	
					16	60	
					8	45	
	8	30	8	60	8	5	99.99
					16	20	
					16	60	
					8	45	
16	60	8	45	8	5	99.99	
				8	30		
				16	20		
				8	45		
8	45	8	20	8	5	95.92	
				8	30		
				16	60		
				16	20		

rotation affects classification results. This is because some of the textures that we have used are highly anisotropic and this anisotropy is more pronounced in the diagonal direction.

We performed another experiment in which we used features for 0° rotation to design a classifier and tested its performance in classifying features from rotated samples ranging from 20° to 90° . The results given in Table 2 clearly gives us an idea as how the classifier responds to the rotated samples of the textures. The 45° rotation classification is comparatively poor than the others.

TABLE 2 Percent of correct classification considering rotation only

Training data set rotation in deg	Test data set rotation in deg.	Information measure	Percent classifn.
0	20		98.82
0	30		98.82
0	45	Log variance	95.29
0	60		97.64
0	75		97.64
0	90		97.64

So we can summarize our results as, for classification of rotated textures, multiresolution representation of the images is clearly preferable. Although texture features are prevalent in the intermediate frequency bands, which suggests that wavelet packet signatures can classify textures very efficiently, they perform very poorly when rotation is taken into account. So an overcomplete wavelet decomposition, which leads to wavelet frames happens to be a preferred solution. We also find that number of features used is only six which is really very small compared to the number of features used for classification purpose found in the literature. We do not need the best features selection algorithm.

For this whole series of experiments classification error usually occurred between texture pairs that were almost difficult to discriminate visually and also from textures which were highly anisotropic.

We can conclude that texture signatures based on multiresolution wavelet frames analysis holds great potential for accomplishing subtle discrimination and robust classification against rotation, translation and is computationally simple and efficient.

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