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Steady thermocapillary and buoyancy driven flow in two-dimensional slots

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ABSTRACT. – The effect of buoyancy on the flow in a two-dimensional slot, induced by a temperature gradient along the free surface between two immiscible fluids, is investigated. For small aspect ratio slots, the complete flow structure is calculated using asymptotic matching of the outer core flow and the inner boundary layer flow near the side walls. The shape of the deformed free surface is also obtained. Buoyancy causes a decrease in the magnitude of the deflection, but does not affect the shape. The theory is validated against some earlier experiments using acetone. © Elsevier, Paris.

1. Introduction

Variation of surface tension due to temperature gradient can initiate the onset of convection even in the absence of buoyancy, as first shown by Pearson (1958). This convection, due to the shearing forces produced in the surface layer by gradients of surface tension, is known as Marangoni convection. Several authors (e.g. Nield (1964), McConaghy and Finlayson (1969), Dandapat (1990)) have studied the effects of different kinds of body forces and thermal conditions on Marangoni convection. In most of these studies, the free surface is considered to be undeformed, which is unrealistic. In recent years, attempts have been made to include the deformation of the free surface and study its effect on the onset of convection. Among these, Benjuria and Depassier (1987), Gouesbet *et al.* (1990), Dandapat and Kumar (1992a, b) deserve some attention. But all the above mentioned works are mainly confined to considering a vertical temperature gradient in the presence/absence of gravity.

Steady thermocapillary flow can also be generated by applying a temperature gradient along the interface between two immiscible fluids. This general class of flows has been well-documented in the review articles by Kenning (1968), Levich and Krylov (1969) and Ostrach (1977, 1982). Marangoni convection plays a crucial role in crystal growth under microgravity conditions. It is known that the nature of the crystal formed depends largely on what occurs in the vicinity of the fluid-solid interface. In that thin zone, the physical and/or chemical transformation that occurs during the crystal growth mostly depends on the change in composition across the interface. It is also known that the convection increases the overall transport and hence the growth rate, but on the other hand, it seems to affect the morphology of the solid adversely. The result, in any case, depends on density gradients in the fluid that vary in direction and generate buoyancy-driven convection that can alter the transport of heat and other constituent chemicals which may change its shape under some circumstances, periodically in others and chaotically in still others. Thus, flow conditions in the vicinity of the fluid-solid interface deserve some careful attention for the understanding of the crystal growth. This motivated Pimputkar and Ostrach (1980), Sen and Davis (1982), Strani, Piva and Graziani (1983), Chan, Mazumdar and Chin (1984), Srinivasan and Basu (1986), Zebib, Homsy and Meiburg (1985), Sen (1986), Rybicki and Floryan (1987a, b), Kuhlmann (1989), Hadid and Roux (1991) to study the flow under different geometric conditions. Several

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authors, including Kirdyashkin (1984), and Villers and Platten (1987, 1992) have undertaken experimental investigations on Marangoni convection.

The aim of the present paper is to calculate the shape of the interface analytically for small aspect ratio and provide details of the flow structure inside the slot. To obtain the complete flow structure we need to calculate the boundary layer flow near the hot and cold boundaries by a finite difference method and match them with the core flow by an asymptotic matching principle.

2. Formulation of the problem

Consider a rectangular cavity of height d and length ℓ filled with an incompressible liquid of density ρ, thermal diffusivity κ and viscosity μ (*Fig.* 1). The end walls are maintained at constant temperature T_H and T_C with $T_H > T_C$ and the lower horizontal boundary is thermally insulated. The upper boundary is a free surface defined by $y = h(x)$ and bounded by a passive gas of negligible density and viscosity. When $\Delta T = T_H - T_C$ differs from zero, a thermocapillary flow is induced due to the variation of the surface tension σ. It is convenient to introduce dimensionless lubrication-type (primed) variables: $x = lx'$, $y = dy'$, $h = dh'$, $u = u_* u'$, $v = \varepsilon u_* v'$, $p = (\mu u_* l/d^2)p'$, $T - T_{avg} = (\Delta T)T'$ where $\varepsilon = d/l$ is the aspect ratio and T_{avg} is the average of the hot and cold wall temperatures. The characteristic velocity u_* is obtained by considering the balance between the shear stress and the thermal stress acting along the interface and is given by $u_* = \gamma\varepsilon\Delta T/\mu$, where $\gamma = \frac{d\sigma}{dT}$. The dimensionless governing equations (with the primes dropped) can be written as

(1) $$u_x + v_y = 0$$

(2) $$R_e\varepsilon\,[uu_x + vu_y] = -p_x + \varepsilon^2 u_{xx} + u_{yy}$$

(3) $$R_e\varepsilon^3\,[uv_x + vv_y] = -p_y + \varepsilon^2\,[\varepsilon^2 v_{xx} + v_{yy}] + \varepsilon G_r T/R_e.$$

(4) $$M_a\varepsilon\,[uT_x + vT_y] = \varepsilon^2 T_{xx} + T_{yy}.$$

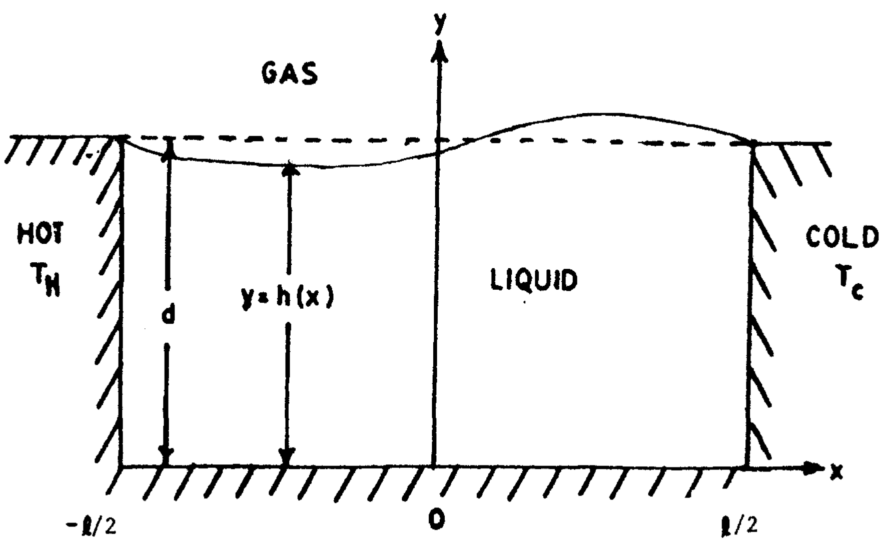

Fig. 1. – Rectangular cavity.

The Reynolds number R_e, Marangoni number M_a and Grashof number G_r are defined by $R_e = \frac{u_* d}{\nu}$, $M_a = \frac{u_* d}{\kappa}$, and $G_r = \frac{\alpha g (\Delta T) d^3}{\nu^2}$, where α is the volumetric expansion coefficient, $\nu = \mu/\rho$ is kinematic viscosity and g is acceleration due to gravity. The boundary conditions for the dimensionless variables on the solid walls are

$$u = v = 0, \quad T = \mp\frac{1}{2} \text{ on } x = \pm\frac{1}{2}$$
$$u = v = 0, \quad T_y = 0 \text{ on } y = 0. \tag{5}$$

On the free boundary, $y = h(x)$, the following must hold:

$$p_a - p + 2\varepsilon^2 (1 + \varepsilon^2 h'^2)^{-1} [v_y - h' u_y + \varepsilon^2 h' (h' u_x - v_x)]$$
$$= \varepsilon^3 C^{-1} h'' (1 + \varepsilon^2 h'^2)^{\frac{-3}{2}} (1 - \varepsilon^{-1} CT),$$
$$(u_y + \varepsilon^2 v_x)(1 - \varepsilon^2 h'^2) + 2\varepsilon^2 h' (v_y - u_x) = -(1 + \varepsilon^2 h'^2)^{\frac{1}{2}} (T_x + h' T_y),$$
$$(1 + \varepsilon^2 h'^2)^{-\frac{1}{2}} (T_y - \varepsilon^2 h' T_x) + L(T + x) = 0,$$
$$v = uh', \tag{6}$$

where $C = \mu u_* / \sigma (T_0)$ is the capillary number, $L = k_g d/k$ is the Biot number, p_a is atmospheric pressure, k_g is the heat coefficient in the gas and T_g is the gas temperature. Furthermore, conservation of total liquid volume and the requirement of zero net mass flow into and out of the cavity leads to

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} h(x)\, dx = \overline{V}, \quad \int_0^{h(x)} u dy = 0, \tag{7}$$

where $\overline{V}$ is the total volume (per unit depth) occupied by the liquid. Eliminating the pressure from (2) and (3) and introducing the stream function $u = \psi_y$ and $v = -\psi_x$ gives

$$R_e \varepsilon [\psi_y \psi_{xyy} - \psi_x \psi_{yyy} + \varepsilon^2 (\psi_y \psi_{xxx} - \psi_x \psi_{xxy})] = \psi_{yyyy} + 2\varepsilon^2 \psi_{xxyy} + \varepsilon^4 \psi_{xxxx} - \varepsilon (G_r/R_e) T_x. \tag{8}$$

For long shallow cavities, *i.e.* for small aspect ratio ε, the entire flow region will be divided into two distinct parts, viz. an outer core flow which will be away from the side wall, and an inner flow (boundary layer) near each end wall. This inner flow will turn around to conserve mass and recirculation. The core flow, the shape of the interface and the turning flows are all coupled together. Therefore, to complete the flow structure the inner and outer flows can be calculated separately and then matched through the asymptotic matching procedure. Following Sen and Davis (1982), assume

$$R_e = \varepsilon \overline{R}_e, \quad M_a = \varepsilon \overline{M}_a, \quad C = \varepsilon^4 \overline{c}, \quad G_r = \varepsilon \overline{G}_r, \quad L = 0(1). \tag{9}$$

The basis of these assumptions lies in the fact that we consider only small departure from the state of pure conduction. For small capillary number, the deflection of the interface due to normal stress generated by the flow is in general small and thus the free-boundary location may be obtained as a perturbation about the rest state of capillary statics.

3. Core flow

To obtain the solution of equation (8), and (2)-(4) for temperature and pressure, subject to the conditions (6) and (7), the variables are expanded as

$$f = f_0(x, y) + \varepsilon f_1(x, y) + \varepsilon^2 f_2(x, y) + 0(\varepsilon^3), \tag{10}$$

where f is any one of ψ, p, T or h and $h_0 = 1$.

The zeroth order solution is given by

$$\psi_0 = \frac{1}{4} y^2 (y-1), \quad T_0 = -x, \quad p_0 = \frac{3}{2} x + \text{constant} \tag{11}$$

$$h_1 = \overline{c}\left[-\frac{1}{4}x^3 + H_1 x^2 + H_2 x + H_3\right] \tag{12}$$

where H_1, H_2 and H_3 are constants. The zeroth order solution does not satisfy all the boundary conditions at the end walls and hence is only valid in the core region of the cavity. The constants in (12) will be determined by matching inner and outer solutions. The next order approximation of (8), *i.e.* $0(\varepsilon)$, is

$$\psi_{1yyyy} = ST_{0x} \tag{13}$$

where $S = \overline{G}_r/\overline{R}_e = R_a/M_a$ is the dynamic Bond number $\left(R_a = \frac{\alpha g (\Delta T) d^3}{\nu\kappa}\text{ is the Rayleigh number}\right)$, which leads to the solution

$$\psi_1(x, y) = -\frac{S}{24} y^4 + \left(\frac{5S}{48} - \frac{1}{4} h_1\right) y^3 - \frac{S}{16} y^2 \tag{14}$$

satisfying the zero mass flux condition and boundary conditions at $y = 0$ and $y = 1$.
Integration of (2)-(4) gives, to $0(\varepsilon)$,

$$T_1(y) = 0, \quad p_{1_x} = -Sy + \frac{5S}{8} - \frac{3}{2} h_1(x). \tag{15}$$

The first condition in (6) implies that $h_2(x)$ must satisfy a third order differential equation, which can be integrated to give

$$h_2(x) = \overline{c}\left[\int\int\int\left(\frac{3}{8}S + \frac{3}{2}h_1\right)dx + k_1 x^2 + k_2 x + k_3\right], \tag{16}$$

where k_1, k_2 and k_3 are constants which are to be determined by matching the three-term outer solution of h, namely $h \sim 1 + \varepsilon h_1 + \varepsilon^2 h_2$, with a three-term inner solution near the boundary wall and using the condition $(i = 1, 2)$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} h_{ic}(x)\, dx = 0, \tag{17}$$

where subscript c refers to the uniformly valid composite expansion of the free-surface height h. It is clear from (11) and (15) that the solution for temperature field up to $O(\varepsilon^2)$ does not show any boundary layer behaviour.

Moreover, to complete the solution up to this order we need to calculate $T_2(x, y)$ since this will give the necessary $0(\varepsilon)$ correction of the stream function near the end walls. The solution for $T_2(x, y)$ is

$$T_2(x, y) = -\frac{\overline{M}_a}{48}(3y^4 - 4y^3 + 1) \tag{18}$$

It is interesting to note that $T_2(x, y)$ does not satisfy the thermal condition at $x = \pm\frac{1}{2}$, so it is expected that the thermal boundary layer is present at this order.

4. Inner flow

To examine the inner flow near the hot side wall at $x = -\frac{1}{2}$, the x-coordinate is stretched as

$$\xi = \varepsilon^{-1}\left(x + \frac{1}{2}\right), \tag{19}$$

and an overbar is used to indicate the inner dependent variables. Under this transformation, the governing equations for $\overline{\psi}$ and $\overline{T}$ become (with $\eta = y$),

$$\begin{gathered} \nabla^4\overline{\psi} - S\overline{T}_\xi = \overline{R}_e\varepsilon\,[\overline{\psi}_\eta\,(\overline{\psi}_{\xi\eta\eta} + \overline{\psi}_{\xi\xi\xi}) - \overline{\psi}_\xi\,(\overline{\psi}_{\eta\eta\eta} + \overline{\psi}_{\xi\xi\eta})] \\ \nabla^2\overline{T} = \overline{M}_a\varepsilon\,[\overline{\psi}_\eta\overline{T}_\xi - \overline{\psi}_\xi\overline{T}_\eta], \end{gathered} \tag{20}$$

where ∇^4 and ∇^2 represented the biharmonic and Laplacian operators respectively. To solve (20) with appropriate conditions obtained by transforming (6) and (7), all variables are expanded as functions of ξ, η according to (10).

The zeroth order functions $\overline{\psi}_0$ and $\overline{T}_0$ satisfy a biharmonic and Laplace equation, respectively. Applying boundary conditions and matching the outer (core) solution T_0 $(cf.eqn.\,(11))$ with the inner solution $\overline{T}_0$ leads to

$$\overline{T}_0(\xi, \eta) = \frac{1}{2}. \tag{21}$$

The biharmonic for $\overline{\psi}_0$ has been solved numerically using the standard technique of splitting it into a pair of coupled second order equations $\nabla^2\overline{\psi}_0 = g$, $\nabla^2 g = 0$. These equations are discretized using three-point central differencing. Boundary conditions for the auxiliary function g are obtained in a manner similar to that used for vorticity in the numerical solution of the 2D Navier-Stokes equations in stream function-vorticity formulation. The resulting system of algebraic equations is solved by successive-line-over-relaxation (SLOR), with convergence tolerance of 10^{-5}. Systematic grid refinement was performed to ensure convergence to the exact solution as $\Delta\xi$, $\Delta\eta \to 0$.

Similarly, the solution for $\overline{T}_1$ is

$$\overline{T}_1(\xi, \eta) = -\xi \tag{22}$$

and the nonhomogeneous biharmonic equation for $\overline{\psi}_1$ is solved numerically following the same procedure as for $\overline{\psi}_0$.

To obtain the first-order correction in the boundary layer flow we need $\overline{T}_2$ which is governed by

$$\nabla^2 \overline{T}_2 = -M_a \overline{\psi}_{0\eta} \tag{23}$$

The boundary conditions for (23) involve the shape of the interface. Once $\overline{h}_1$ is determined, (23) is solved using SLOR.

The analysis near the cold boundary (at $x = 1/2$) can be performed in a similar manner by stretching the x-coordinate as $\zeta = \varepsilon^{-1}\left(\frac{1}{2} - x\right)$.

5. Shape of the interface

It can be shown that the functions $\overline{h}_1$, $\overline{h}_2$ must be linear functions of ξ, satisfying homogeneous boundary conditions $\overline{h}_i(0) = 0$. Hence, the inner expansion for the shape of the interface is

$$\overline{h} = 1 + \varepsilon l_1 \xi + \varepsilon^2 m_1 \xi + \ldots$$

Matching three terms in the outer expansion of h with three terms in the inner expansion gives

$$l_1 = 0, \quad m_1 = h'\left(-\frac{1}{2}\right) \tag{24}$$

and

$$\frac{H_1}{4} - \frac{H_2}{2} + H_3 = -\frac{1}{32} \tag{25}$$

Similarly, matching at the cold wall leads to

$$\frac{H_1}{4} + \frac{H_2}{2} + H_3 = \frac{1}{32} \tag{26}$$

From (25), (26) and (17), we get

$$H_1 = 0, \quad H_2 = \frac{1}{16}, \quad H_3 = 0$$

and hence the first-order deflection of the free-surface from its flat profile is given by

$$h_1(x) = -\frac{1}{4}\overline{c}x\left(x^2 - \frac{1}{4}\right) \tag{27}$$

Now, using (27) in (24) and conditions at the cold wall, one can obtain values for k_1, k_2, k_3, and (16) gives

$$h_2(x) = \overline{c}\left(x^2 - \frac{1}{4}\right)\left[\frac{Sx}{16} - \frac{\overline{c}}{320}\left(x^4 - x^2 + \frac{5}{112}\right)\right]. \tag{28}$$

6. Results and discussion

Experiments have confirmed that monocellular convection occurs for low R_a and M_a (*cf.* e.g. Villers and Platten (1992)). These conditions are met when the layer is thin and ΔT is small. Furthermore, under certain conditions, a multicelllar structure will appear for small ε, especially when $l \gg d$. The present theory does not apply as $d \to 0$ since then $u_* \to 0$ and the scaling breaks down.

The theory presented in this work can be validated against some experiments conducted by Villers and Platten (1992) using acetone. For example, consider the three cases: $(a) d = 1.75mm$, $l = 30mm$, $\Delta T = 1.2K$ ($R_a = 2212$, $\overline{M}_a = 7980$, $S = 4.755$); $(b) d = 2.5mm$, $l = 30mm$, $\Delta T = 0.8K$ ($R_a = 4300$, $\overline{M}_a = 7600$, $S = 6.792$); $(c) d = 2.5mm$, $l = 30mm$, $\Delta T = 1.2K$ ($R_a = 6450$, $\overline{M}_a = 11400$, $S = 6.792$). To first order, in the core, the dimensional surface velocity along the vertical centreline of the cavity is given by

$$u = \left(\frac{1}{4} + \frac{1}{48}\varepsilon S\right) u_*.$$

For cases (a) and (c), the ratio of surface velocities is 1.46, which is close to the ratio of thicknesses, 1.43, in agreement with the analysis of Villers and Platten (1987). For cases (b) and (c), in which only ΔT is changed, the ratio of the surface velocities is exactly the ratio of temperature differences, *i.e.* 1.5.

To first order, the maximum deflection of the interface from the mean, $\Delta h_{\max}$, occurs at $x = \pm l/2\sqrt{3}$. For $d = 6mm$, $l = 30mm$ and capillary number $\overline{c} = 0.5$ we find that $\Delta h_{\max} = \pm 0.0072mm$ at $x = \pm 8.66mm$. Correcting to second order gives non-symmetric deflection, with $\Delta h_{\max} = -0.0066mm$ at $x \approx -9.0mm$. For the range of parameters ε, S and $\overline{c}$ for which the present theory applies, one can show from (27) and (28) or from numerical tests that, for the right half of the slot, the deflection is an increasing function of capillary number $\overline{c}$ and a decreasing function of Bond number S. The opposite trend exists for the left half of the slot. It is interesting to note that, for small ε and $\overline{c}$ (neglecting $0(\varepsilon^2 \overline{c}^2)$), buoyancy does not change the shape of the deflection, but decreases its amplitude by an amount $\varepsilon^2 S\overline{c}/16$.

Typical streamlines and isotherms are illustrated in Figure 2, for $\varepsilon = 0.2$, Biot number $L = 1.0$ and $S = 1.5$ (left half of cavity only). The theory predicts nearly straight isotherms in the core region, and the parameters L and S have very little effect on their shape. As L increases, the isotherms shift towards the hot wall. However, L has minimal effect on the streamlines. On the other hand, as S increases, the streamlines are pulled towards the hot wall, indicating a faster flow in that region.

The results of Sen and Davis (1982) are recovered from the present theory when $S = 0$. We see that buoyancy effects are felt in the core only at first order, in the horizontal pressure gradient and the streamfunction. The horizontal pressure gradient depends linearly on y for nonzero S. Throughout both the core and the boundary

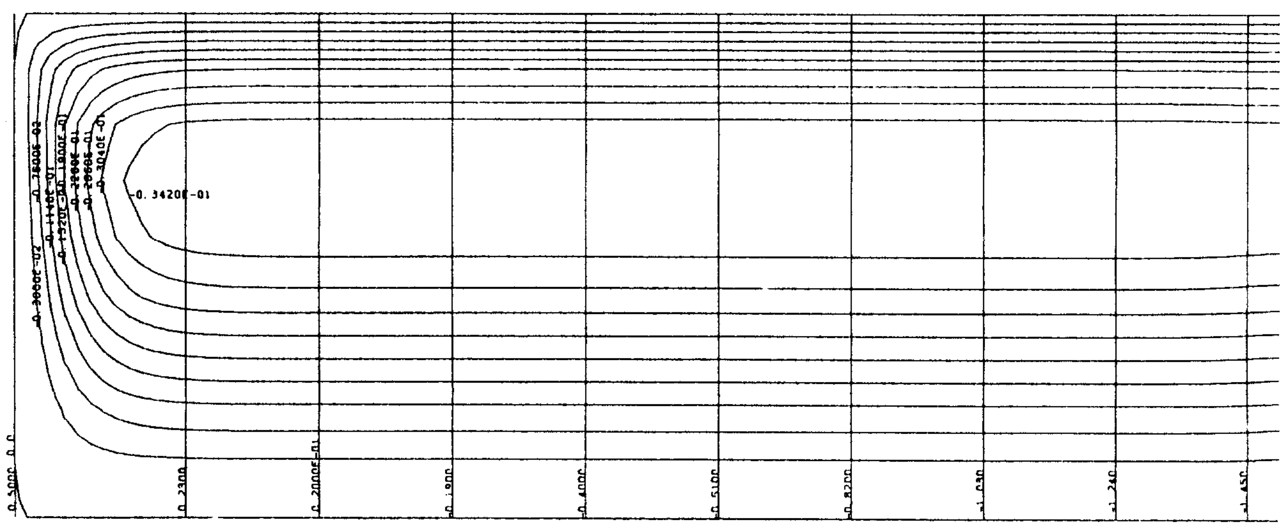

Fig. 2. – Streamlines and isotherms $L = 1$, $S = 3/2$.

layer region, the local temperature does not depend on S. However, the buoyancy does reduce the amplitude of the free surface deflection.

REFERENCES

BENJURIA R. D., DEPASSIER M. C., 1987, Oscillatory instabilities in the Rayleigh-Bénard problem with a free surface, *Phys. Fluids*, **30**, 1678-1682.

CHAN C., MAZUMDAR J., CHEN M. M., 1984, A two-dimensional transient model for convection in laser melted pool, *Metall. Trans.*, **A15**, 2175-2184.

DANDAPAT B. S., 1990, Onset of buoyancy and surface-tension driven instabilities in presence of random vibrations, *Z. Naturforsch*, **45a**, 1235-1240.

DANDAPAT B. S., KUMAR B. P., 1992, Onset of thermosolutal convection in a liquid layer having deformable free surface – I. Stationary convection, *Z. Naturforsch*, **47a**, 554-560.

DANDAPAT B. S., KUMAR B. P., 1992, Onset of thermosolutal convection in a liquid layer having deformable free surface – II. Overstability, *Z. Naturforsch*, **47a**, 561-564.

GOUESBET G., MAGNET J., ROZE C., DARRIGO R., 1990, Surface-tension and coupled buoyancy-driven instability in a horizontal liquid layer. Over-stability and exchange of stability, *Phys. Fluids*, **A2**, 903-911.

HADID H. B., ROUX B., 1991, Melt motion in differentially heated horizontal cavities; motion due to buoyancy and thermocapillarity, *J. Fluid Mech.*, **221**, 77-103.

KIRDYASHKIN A. G., 1984, Thermogravitational and thermocapillary flows in a horizontal liquid layer under the conditions of horizontal temperature gradient, *Int. J. Heat Mass Transfer*, **27**, 1205-1218.

KUHLMANN H., 1989, Small amplitude thermocapillary flow and surface deformations in a liquid bridge, *Phys. Fluids*, **A1**, 672-677.

KENNING D. B. R., 1968, Two-phase flow with nonuniform surface tension, *Appl. Mech. Rev.*, **21**, 1101-1111.

LEVICH V. G., KRYLOV V. S., 1969, Surface-tension-driven phenomena, *Ann. Rev. Fluid Mech.*, **1**, 293-316.

McCONAGHY G. A., FINLAYSON B. A., 1969, Surface tension driven oscillatory instability in a rotating fluid layer, *J. Fluid Mech.*, **39**, 49-55.

NIELD D. A., 1964, Surface tension and buoyancy effects in cellular convection, *J. Fluid Mech.*, **19**, 341-352.

OSTRACH S., 1977, Motion induced by capillarity, In Physicochemical Hydrodynamics, Ed. V. G. Levich, Festschrift, **2**, pp. 571-589, London, Advanced Publications.

OSTRACH S., 1982, Low-gravity fluid flows, *Ann. Rev. Fluid Mech.*, **14**, 313-345.

PEARSON J. R. A., 1958, On convection cells induced by surface tension, *J. Fluid Mech.*, **4**, 489-500.

PIMPUTKAR S. M., OSTRACH S., 1980, Transient thermocapillary flow in thin layers, *Phys. Fluids*, **23**, 1281-1285.

RYBICKI A., FLORYAN J. M., 1987, Thermocapillary effects in liquid bridge I. Thermocapillary convection, *Phys. Fluids*, **30**, 1956-1972.

RYBICKI A., FLORYAN J. M., 1987, Thermocapillary effects in liquid bridge II. Deformation of the interface and capillary instability, *Phys. Fluids*, **30**, 1973-1983.

SEN A., 1986, Thermocapillary convection in a rectangular cavity with deformable interface, *Phys. Fluids*, **29**, 3881-3883.

SEN A., DAVIS S. H., 1982, Steady thermocapillary flows in two-dimensional slots, *J. Fluid Mech.*, **121**, 163-186.

SRINIVASAN J., BASU B., 1986, A numerical study of thermocapillary flow in a rectangular cavity during laser melting, *Int. J. Heat Mass Transfer*, **29**, 563-572.

STRANI M., PIVA R., GRAZIANI G., 1983, Thermocapillary convection in a rectangular cavity: asymptotic theory and numerical simulation, *J. Fluid Mech.*, **130**, 347-376.

VAN DYKE M., Perturbation methods in fluid mechanics, Academic Press, New York, 1964.

VILLERS D., PLATTEN J. K., 1987, Separation of Marangoni convection from gravitational convection in earth experiments, *Physico-Chemical Hydrodyn.*, **8**, 173.

VILLERS D., PLATTEN J. K., 1992, Coupled buoyancy and Marangoni convection in acetone: experiments and comparison with numerical simulations, *J. Fluid Mech.*, **234**, 487-510.

ZEBIB A., HOMSY G. M., MEIBURG E., 1985, High Marangoni number convection in a square cavity, *Phys. Fluids*, **28**, 3467-3476.

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