Acta Mechanica 130, 143–146 (1998)

ACTA MECHANICA

Note

On the stability of MHD flow of a viscoelastic fluid past a stretching sheet

B. S. Dandapat, Calcutta, India, and **L. E. Holmedal** and **H. I. Andersson,** Trondheim, Norway

(Received February 24, 1997)

Summary. The effects of a magnetic field on the stability characteristics of a viscoelastic fluid flow due to the stretching sheet are investigated. A three-dimensional linear stability analysis is performed by means of the Method of Weighted Residuals for disturbances of the Taylor-Görtler type. It is found that the magnetic field exerts a stabilizing influence on the flow.

1 Introduction

The flow past a stretching sheet has several important engineering applications, viz. in the polymer processing unit of a chemical engineering plant, and for the metal working process in metallurgy. Crane [1] first initiated the study of a steady two-dimensional boundary-layer flow caused by the stretching of the sheet which moves in its own plane with a velocity that varies linearly with the distance from a fixed point on the sheet. This flow problem was extended to non-Newtonian fluids without or with heat and mass transfer by several authors [2]–[7]. Extensions to magnetohydrodynamic (MHD) flow for both Newtonian and non-Newtonian fluids without or with heat transfer were also done by [8]–[13]. In other words, this particular problem of a stretching sheet has been extensively studied by several authors considering different aspects of the problem. It is therefore interesting to note that the corresponding stability analysis does not seem to have received any adequate attention so far.

To the best of our knowledge only Bhattacharyya and Gupta [14] and Takhar, Ali and Gupta [15] have investigated the viscous flow of a Newtonian fluid over a stretching sheet (without and with magnetic field) with respect to three-dimensional disturbances of the Taylor-Görtler type. Recently, Dandapat, Holmedal and Andersson [16] studied the stability of a viscoelastic non-Newtonian fluid flow caused by the stretching of a sheet. The aim of this paper is to study the effect of a magnetic field on the stability characteristics of the viscoelastic flow arising due to the stretching sheet. In particular we focus on the stability of the MHD flow considered recently by Andersson [11].

2 Mathematical formulation

We consider the flow of an incompressible electrically conducting viscoelastic fluid which arises due to the stretching of an impermeable flat sheet. Further we assume that the sheet is in the xz-plane and is stretched along the x-axis. A uniform transverse magnetic field B_0 acts parallel to the y-axis, and the conducting viscoelstic fluid occupies the half space $y > 0$.

The basic velocity field $[u_0(x, y), v_0(x, y), 0]$ developed due to the stretching of the sheet will satisfy the boundary-layer equations

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0, \tag{2.1}$$

$$u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} = \nu \frac{\partial^2 u_0}{\partial y^2} - \varkappa_0^* \left[\frac{\partial}{\partial x}\left(u_0 \frac{\partial^2 u_0}{\partial y^2}\right) + \frac{\partial u_0}{\partial y}\frac{\partial^2 v_0}{\partial y^2} + v_0 \frac{\partial^3 u_0}{\partial y^3}\right] - \frac{\sigma B_0{}^2}{\varrho} u_0 \tag{2.2}$$

and the boundary conditions

$$u_0 = cx, \quad v_0 = 0 \quad \text{at} \quad y = 0 \tag{2.3}$$

$$u_0 \to 0 \quad \text{as} \quad y \to \infty \tag{2.4}$$

where c is a constant and ν, $\varkappa_0^*$, σ, and ϱ denote kinematic viscosity, viscoelastic parameter, electrical conductivity, and density of the fluid, respectively.

Following Andersson [11] an exact similarity solution of the above system can be obtained as

$$u_0 = cxe^{-Q\eta} = cxf'(\eta), \tag{2.5}$$

$$v_0 = -(c\nu)^{1/2}(1 - e^{-Q\eta})/Q = -(c\nu)^{1/2} f(\eta), \tag{2.6}$$

where $Q = \left(\frac{1+N}{1-\gamma}\right)^{1/2}$, $N = \sigma B_0{}^2/\varrho c$, $\gamma = \varkappa_0^* c/\nu$, and the similarity variable is $\eta = (c/\nu)^{1/2} y$.

We are now considering the stability of the above solutions (2.5), (2.6). Following the procedure of Dandapat et al. [16] we obtain the following perturbed equations after eliminating all variables except u and v:

$$\begin{aligned} &u_1'' + fu_1' - (\bar{\alpha}^2 + 2f' + \bar{\beta} + N)\, u_1 + f''v_1 + \gamma[u_1'''f - (\bar{\beta} + 2f')\, u_1'' - (\bar{\alpha}^2 f - 3f'')\, u_1' \\ &+ \bar{\beta}\bar{\alpha}^2 u_1 - f''v_1'' - 2f'''v_1' + (\bar{\alpha}^2 f'' + f^{iv})\, v_1] = 0 \end{aligned} \tag{2.7}$$

and

$$\begin{aligned} &v_1^{iv} + fv_1''' + (f' - 2\bar{\alpha}^2 - \bar{\beta} - N)\, v_1'' + (f'' - \bar{\alpha}^2 f)\, v_1' + \{f''' + \bar{\alpha}^2(\bar{\beta} + \bar{\alpha}^2 - f')\}\, v_1 \\ &- 2f'u_1' - 2f''u_1 + \gamma[4f''u_1'' + 4f'''u_1' + fv_1^{v} - (\bar{\beta} + f')\, v_1^{iv} - (4f'' + 2\bar{\alpha}^2 f)\, v_1''' \\ &+ (2\bar{\beta}\bar{\alpha}^2 + 2\bar{\alpha}^2 f' - 4f''')\, v_1'' + \{\bar{\alpha}^2 f'' - f^{iv} + \bar{\alpha}^2(\bar{\alpha}^2 f + 3f'')\}\, v_1' \\ &+ \{f^{v} + \bar{\alpha}^2 f''' - \bar{\alpha}^2(\bar{\beta}\bar{\alpha}^2 + f''' + \bar{\alpha}^2 f')\}\, v_1] = 0, \end{aligned} \tag{2.8}$$

where $\bar{\alpha}^2 = \alpha^2 \nu/c$ and $\bar{\beta} = \beta/c$. It is to be noted that in deriving Eqs. (2.7) and (2.8) we have assumed that all the perturbed quantities have periodicity in the direction normal to the basic flow with usual exponential time-dependence. Here α and β denote the wave number and the phase speed, respectively, and u_1, v_1 are the corresponding normal mode components for u and v (cf. Dandapat et al. [16] for details). The corresponding boundary conditions are

$$u_1 = v_1 = v_1' = 0 \quad \text{at} \quad \eta = 0 \quad \text{and} \quad \eta \to \infty. \tag{2.9}$$

Following Dandapat et al. [16] we have transformed the η variable into T according to

$$T = e^{-Q\eta}, \quad L = -T\frac{d}{dt} \tag{2.10}$$

and then used the Method of Weighted Residuals in the transformed equations by expanding u_1 and v_1 as

$$u_1 = \sum_{i=1}^{\infty} A_i u_i(T), \quad v_1 = \sum_{i=1}^{\infty} B_i v_i(T). \tag{2.11}$$

$u_i(T)$ and $v_i(T)$ are chosen by satisfying the transformed boundary conditions

$$u_1 = v_1 = Lv_1 = 0 \quad \text{at} \quad T = 0 \quad \text{and} \quad T = 1,$$

as

$$u_i = T^i(1 - T) \quad \text{and} \quad v_i = T^i(1 - T)^2. \tag{2.12}$$

The growth rate $\bar{\beta}$ is calculated as the eigenvalue of the matrix which is formed by using (2.11) in the transformed equations and restricting to the orthogonality condition as demanded by the Method of Weighted Residuals (Finlayson [17]). In this calculation we have restricted ourselves to 10-term trial functions, although one may obtain good results with even a 2-term approximation.

3 Results and discussions

The mathematical formulation in the preceding Section enables the exploration of the combined effects of fluid viscoelasticity and an external magnetic field on the stability of the boundary layer flow along a stretching sheet. It should be recalled, however, that for an inelastic fluid ($\gamma = 0$) the problem reduces to the Newtonian flow problem considered by Takhar et al. [15], whereas the non-magnetic case investigated by Dandapat et al. [16] is recovered for $N = 0$.

Figure 1 shows the variation of the decay rate of the disturbances $-\bar{\beta}$ with the wave number $\bar{\alpha}$ for different values of the viscoelastic parameter γ and the magnetic parameter N. It is clear from the figure that $-\bar{\beta}$ increases with the increase of $\bar{\alpha}$. Here $-\bar{\beta}$ is positive since the growth parameter $\bar{\beta} < 0$. This shows that the flow is stable for disturbances of the Taylor-Görtler type. It is evident from the graph that the magnetic field stabilizes the flow for both Newtonian ($\gamma = 0$) and non-Newtonian ($\gamma \neq 0$) fluids. Further one can observe that high wave-number disturbances are more effectively damped than low wave-number modes. It is also clear that the disturbances of low wave-number modes are practically unaffected by the viscoelasticity, whereas the

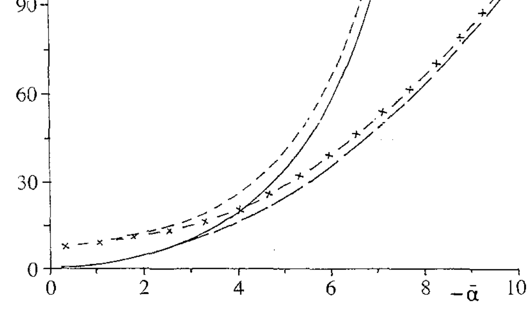

Fig. 1. Predicted variation of decay rate $-\bar{\beta}$ versus wave number $\bar{\alpha}$ for different values of the viscoelastic parameter γ and the magnetic parameter N. Inelastic fluid $\gamma = 0$: — — $N = 0$; × — × — $N = 5$. Viscoelastic fluid $\gamma = 0.01$: ——— $N = 0$; — — — — — $N = 5$

influence of the magnetic field is prominent. As was pointed out by Dandapat et al. [16] the viscoelastic fluid will produce stabilization for those disturbance wavelengths which are shorter than the viscoelastic length scale. The stabilizing effect of the magnetic field can be explained as the disturbance kinetic energy gets exhausted in order to overcome the resistive force due to the tension along the magnetic lines of force.

References

[1] Crane, L. J.: Flow past a stretching plate. J. Appl. Math. Phys. (ZAMP) **21,** 645–647 (1970).
[2] Chiam, T. C.: Micropolar fluid flow over a stretching sheet. ZAMM **62,** 565–568 (1982).
[3] Rajagopal, K. R., Na, T. Y., Gupta, A. S.: Flow of a viscoelastic fluid over a stretching sheet. Rheol. Acta **23,** 213–215 (1984).
[4] Andersson, H. I., Dandapat, B. S.: Flow of a power-law fluid over a stretching sheet. Stability Appl. Anal. Continuous Media **1,** 339–347 (1991).
[5] Dandapat, B. S., Gupta, A. S.: Flow and heat transfer in a viscoelastic fluid over a stretching sheet. Int. J. Non-Linear Mech. **24,** 215–219 (1989).
[6] Andersson, H. I., Hansen, O. R.: Chemical reactions in the viscoelastic boundary layer along a stretching surface. In: Some applied problems in fluid mechanics (Mazumdar, H. P., ed.), pp. 37–44. Calcutta: ISI 1993.
[7] Andersson, H. I., Hansen, O. R., Holmedal, B.: Diffusion of a chemically reactive species from a stretching sheet. Int. J. Heat Mass Trans. **37,** 659–664 (1994).
[8] Pavlov, K. B.: Magnetohydrodynamic flow of an incompressible viscous fluid caused by deformation of a plane surface. Magnitnaya Gidrodinamika **4,** 146–147 (1974).
[9] Chakrabarti, A., Gupta, A.S.: Hydromagnetic flow and heat transfer over a stretching sheet. Q. Appl. Math. **37,** 73–78 (1979).
[10] Mandal, G., Gupta, A. S., Pop, I.: Magnetohydrodynamic flow of an incompressible viscous fluid caused by the axisymmetric stretching of a plane sheet. Magnitnaya Gidrodinamika **1,** 10–14 (1987).
[11] Andersson, H. I.: MHD flow of a viscoelastic fluid past a stretching surface. Acta Mech. **95,** 227–230 (1992).
[12] Andersson, H. I.: An exact solution of the Navier-Stokes equations for magnetohydrodynamic flow. Acta Mech. **113,** 241–244 (1995).
[13] Andersson, H. I., Bech, K. H., Dandapat, B. S.: Magnetohydrodynamic flow of a power-law fluid over a stretching sheet. Int. J. Non-Linear Mech. **27,** 929–936 (1992).
[14] Bhattacharyya, S. N., Gupta, A. S.: On the linear stability of viscous flow over a stretching sheet. Q. Appl. Math. **43,** 359–367 (1985).
[15] Takhar, H. S., Ali, M. A., Gupta, A. S.: Stability of magnetohydrodynamic flow over a stretching sheet. In: Liquid metal hydrodynamics (Lielpeteris J., Moreau, R., eds.), pp. 465–471. Dordrecht: Kluwer 1989.
[16] Dandapat, B. S., Holmedal, L. E., Andersson, H. I.: Stability of flow of a viscoelastic fluid over a stretching sheet. Arch. Mech. **46,** 829–838 (1994).
[17] Finlayson, B. A.: The method of weighted residuals and variational principles. New York: Academic Press 1972.

Authors' address: B. S. Dandapat, Physics and Applied Mathematics Unit, Indian Statistical Institute, 203, B. T. Road, Calcutta 700 035, India; L. E. Holmedal and H. I. Andersson, Division of Applied Mechanics, Norwegian University of Science and Technology, N-7034 Trondheim, Norway

Verleger: Springer-Verlag KG, Sachsenplatz 4–6, A-1201 Wien. – Herausgeber: Prof. Dipl.-Ing. Dr. Hans Troger und Prof. Dipl.-Ing. Dr. Franz Ziegler, beide Technische Universität, Karlsplatz 13, A-1040 Wien. – Redaktion: Karlsplatz 13, A-1040 Wien. – Hersteller: Druckerei zu Altenburg GmbH, D-04600 Altenburg – Verlagsort: Wien. – Herstellungsort: Altenburg. – Printed in Germany.