23 November 1998

PHYSICS LETTERS A

ELSEVIER

Physics Letters A 249 (1998) 25-29

Nonclassical properties of the real and imaginary nonlinear Schrödinger cat states

B. Roy [1]

Physics & Applied Mathematics Unit, Indian Statistical Institute, Calcutta 700 035, India

Received 17 June 1998; accepted for publication 5 August 1998
Communicated by V.M. Agranovich

Abstract

The real and imaginary nonlinear Schrödinger cat states are introduced. The oscillatory nature of the photon distribution function resulting from the quantum interferences between the two components is shown and quadrature squeezing and antibunching are observed. © 1998 Elsevier Science B.V.

1. Introduction

Recently there has been much interest in the study of nonlinear coherent states [1,2], which are right-hand eigenstates of the product of the boson annihilation operator and a nonlinear function f of the number operator. In Ref. [2], it has been shown that these states may appear as stationary states of the centre-of-mass motion of a trapped and bichromatically laser-driven ion far from the Lamb-Dicke regime. The speciality of these nonlinear coherent states are that besides coherence properties, they exhibit nonclassical features such as amplitude squeezing and self-splitting accompanied by pronounced quantum interference effects. The even and odd nonlinear coherent states (superposition of nonlinear coherent states) were constructed in Ref. [3] and it was found that these states have rather different statistical properties from those of the usual even and odd coherent states.

In the present article we shall study the nonclassical properties of another representative of superposition states: the "real" and "imaginary" nonlinear Schrödinger cat states

$$|\mu_\pm\rangle = N_\pm(\mu,\mu^*)\,(|\mu\rangle \pm |\mu^*\rangle), \tag{1}$$

where $|\mu\rangle$ is the nonlinear coherent state given by Eq. (7), and $N_\pm(\mu,\mu^*)$ are the normalisation constants of real and imaginary nonlinear Schrödinger cat states given by (12) and (13). Similar superpositions were considered by Dodonov et al. [4] for the usual harmonic oscillator coherent states.

2. Real and imaginary nonlinear Schrödinger cat states

The annihilation operator A and the creation operator $A^\dagger$ of f-oscillators are distortions of the annihilation and creation operators a and $a^\dagger$ of the usual harmonic oscillator and are given by [1,2]

$$A = af(\hat{n}) = f(\hat{n}+1)\,a,$$
$$A^\dagger = f^\dagger(\hat{n})\,a^\dagger = a^\dagger f^\dagger(\hat{n}+1)\,, \tag{2}$$

[1] E-mail: barnana@isical.ac.in.

0375-9601/98/$ – see front matter
PII S0375-9601(98)00642-2

where

$$\hat{n} = a^\dagger a\,, \quad [A,\hat{n}] = A\,, \quad [A^\dagger,\hat{n}] = -A^\dagger\,, \tag{3}$$

f being an operator valued function of the Hermitian number operator $\hat{n}$.

The commutator between A and $A^\dagger$ can be easily computed using the relations

$$A = \sum_{n=0}^{\infty} \sqrt{n}\, f(n)\, |n-1\rangle\langle n|\,,$$

$$A^\dagger = \sum_{n=0}^{\infty} \sqrt{n}\, f^*(n)\, |n\rangle\langle n-1| \tag{4}$$

and it reads

$$[A, A^\dagger] = (n+1) f^2(\hat{n}+1) - n f^2(\hat{n})\,, \tag{5}$$

where f is chosen to be real, nonnegative and $f^\dagger(\hat{n}) = f(\hat{n})$.

The coherent state for the above f-deformed algebras satisfies the equation

$$A|\mu\rangle = \mu|\mu\rangle \tag{6}$$

and is given by

$$|\mu\rangle = N_\mu \sum_{n=0}^{\infty} \frac{\mu^n}{\sqrt{n!} f(n)!} |n\rangle\,, \tag{7}$$

where

$$N_\mu = \left(\sum_{n=0}^{\infty} \frac{|\mu|^{2n}}{n! [f(n)!]^2} \right)^{-1/2}, \tag{8}$$

where

$$[f(n)]! = f(0)\, f(1) \cdots f(n) \tag{9}$$

and $\mu = |\mu| e^{i\phi} = r e^{i\phi}$, $0 \leqslant \phi < 2\pi$. Using (7) it is easy to derive the corresponding expansions for real and imaginary nonlinear Schrödinger cat states,

$$\begin{aligned} |\mu_+\rangle &= |\mu\rangle + |\mu^*\rangle \\ &= N_+ \sum_{n=0}^{\infty} d_n r^n \cos(n\phi)\, |n\rangle\,, \\ |\mu_-\rangle &= |\mu\rangle - |\mu^*\rangle \\ &= N_- \sum_{n=0}^{\infty} d_{n+1} r^{n+1} \sin[(n+1)\phi]\, |n+1\rangle\,. \end{aligned} \tag{10}$$

The normalisation constants $N_\pm$ are calculated from the normalisation condition

$$\langle \pm\mu | \mu_\pm \rangle = 1 \tag{11}$$

and are given by

$$N_+ = \left(\sum_{n=0}^{\infty} d_n^2 r^{2n} \cos^2(n\phi) \right)^{-1/2}, \tag{12}$$

$$N_- = \left(\sum_{n=0}^{\infty} d_{n+1}^2 f^{2n+2} \sin^2[(n+1)\phi] \right)^{-1/2}, \tag{13}$$

d_n being given by

$$d_n = \frac{1}{\sqrt{n!}\, [f(n)]!}\,. \tag{14}$$

3. Statistical properties of real and imaginary nonlinear Schrödinger cat states

3.1. Photon distribution

Using Eqs. (10) and (12), (13), the photon distribution function for real and imaginary nonlinear Schrödinger cat states are given as

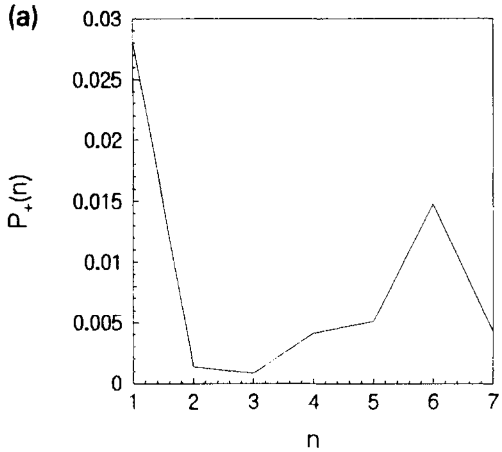
$$P_\pm(n) = |\langle n|\mu_\pm\rangle|^2 = \frac{d_n^2 r^{2n} [1 \pm \cos(2n\phi)]}{B_\pm}\,, \tag{15}$$

where

$$B_+ = \sum_{n=0}^{\infty} d_n^2 r^{2n} [1 + \cos(2n\phi)]\,, \tag{16}$$

$$B_- = \sum_{n=0}^{\infty} d_{n+1}^2 r^{2n+2} \{1 - \cos[(2n+2)\phi]\}\,. \tag{17}$$

Oscillations of photon distribution functions are demonstrated in Figs. 1a,b both for real and imaginary nonlinear Schrödinger cat states taking $f(n) = L_n^1(\eta^2)\,[(n+1) L_n^0(\eta^2)]^{-1}$, where L_m^n is an associated Laguerre polynomial. As shown in Ref. [2] such a nonlinearity can be generated in a trapped ion. Oscillations of photon distribution functions indicate



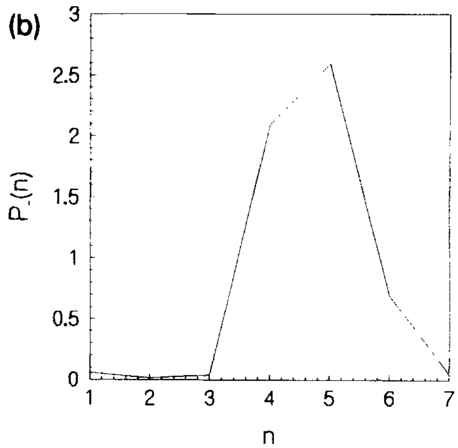


Fig. 1. (a) Photon distribution function $P_+(n)$ of the real cat state for $r = 0.5$, $\phi = \pi/9$ and $\eta^2 = 0.88$. (b) Photon distribution function $P_-(n)$ of the imaginary cat state for $r = 0.5$, $\phi = \pi/9$ and $\eta^2 = 0.88$.

a highly nonclassical state and the profile of the distribution will be determined by the function f. The second order correlation function

$$g_{\pm}^{(2)}(0) = \frac{\langle \pm\mu | a^{\dagger 2} a^2 | \mu_\pm \rangle^2}{\langle \pm\mu | a^\dagger a | \mu_\pm \rangle} \tag{18}$$

follows immediately from the relation

$$\langle \pm\mu | a^{\dagger 2} a^2 | \mu_\pm \rangle = \tfrac{1}{2} N_\pm^2 \sum_{n=0}^{\infty} (n+1)(n+2) d_{n+2}^2 r^{2n+4} \times [1 \pm \cos(2n+4)\phi], \tag{19}$$

$$\langle \pm\mu | a^\dagger a | \mu_\pm \rangle = \tfrac{1}{2} N_\pm^2 \sum_{n=0}^{\infty} (n+1) d_{n+1}^2 r^{2n+2} \times [1 \pm \cos(2n+2)\phi]. \tag{20}$$

The right-hand side of Eq. (18) can be less or greater than 1, depending on the particular form of f, producing either antibunching (sub-Poissonian statistics) or bunching (super-Poissonian statistics). Figs. 2a,b (with $f(n) = L_n^1(\eta^2)\,[(n+1)L_n^0(\eta^2)]^{-1}$) show that it is possible to get antibunching both in the case of real and imaginary nonlinear Schrödinger cat states unlike usual harmonic oscillator cat states where the antibunching effects are shown only by odd coherent states [5].

3.2. Squeezing and correlation

The dispersion and correlation of the quadratures

$$x = \frac{a + a^\dagger}{\sqrt{2}}, \quad p = \frac{a - a^\dagger}{\sqrt{2}} \tag{21}$$

can be obtained by noting that

$$\langle \pm\mu | x | \mu_\pm \rangle = \sqrt{2}\, \langle \pm\mu | a | \mu_\pm \rangle = \frac{N_\pm^2}{\sqrt{2}} \sum_{n=0}^{\infty} \sqrt{n+1}\, d_n d_{n+1} r^{2n+1} \times [\cos\phi \pm \cos(2n+1)\phi], \tag{22}$$

$$\langle \pm\mu | p | \mu_\pm \rangle = 0, \tag{23}$$

$$\langle \pm\mu | a^2 | \mu_\pm \rangle = \langle \pm\mu | a^{\dagger 2} | \mu_\pm \rangle = \tfrac{1}{2} N_\pm^2 \sum_{n=0}^{\infty} \sqrt{(n+1)(n+2)}\, d_n d_{n+2} r^{2n+2} \times [\cos\phi \pm \cos(2n+2)\phi]. \tag{24}$$

Therefore by using Eqs. (20)-(24), the x and p quadrature dispersions can be written as

$$\sigma_{x,\pm} = \tfrac{1}{2} + \tfrac{1}{2} N_\pm^2 \sum_{n=0}^{\infty} (n+1) d_{n+1}^2 r^{2n+2} [1 \pm \cos(2n+2)\phi] + \tfrac{1}{2} N_\pm^2 \sum_{n=0}^{\infty} \sqrt{(n+1)(n+2)}\, d_n d_{n+2} r^{2n+2} \times [\cos 2\phi \pm \cos(2n+2)\phi]$$

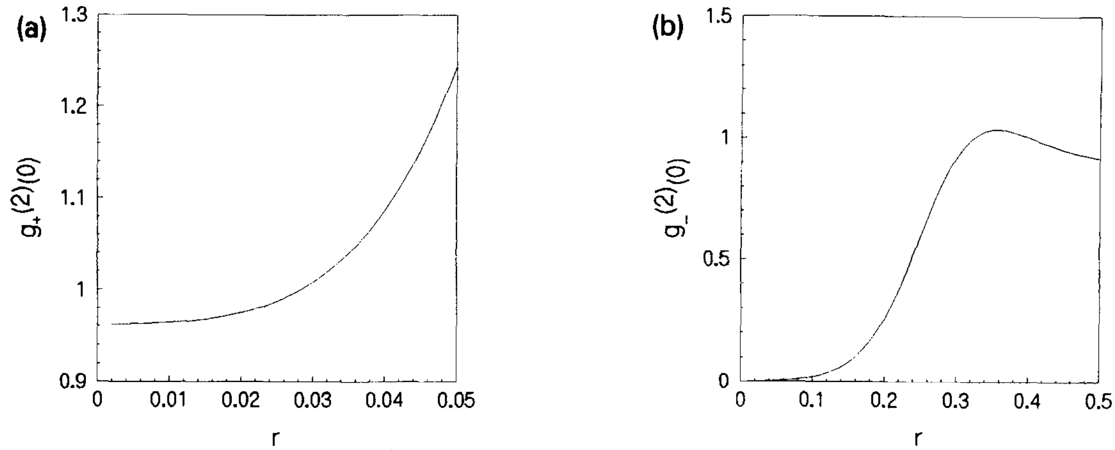


Fig. 2. (a) Second order correlation function $g_+^{(2)}(0)$ of the real cat state for $\phi = \pi/5$, $\eta^2 = 0.88$. (b) Second order correlation function $g_-^{(2)}(0)$ of the imaginary cat state for $\phi = \pi/5$, $\eta^2 = 0.88$.

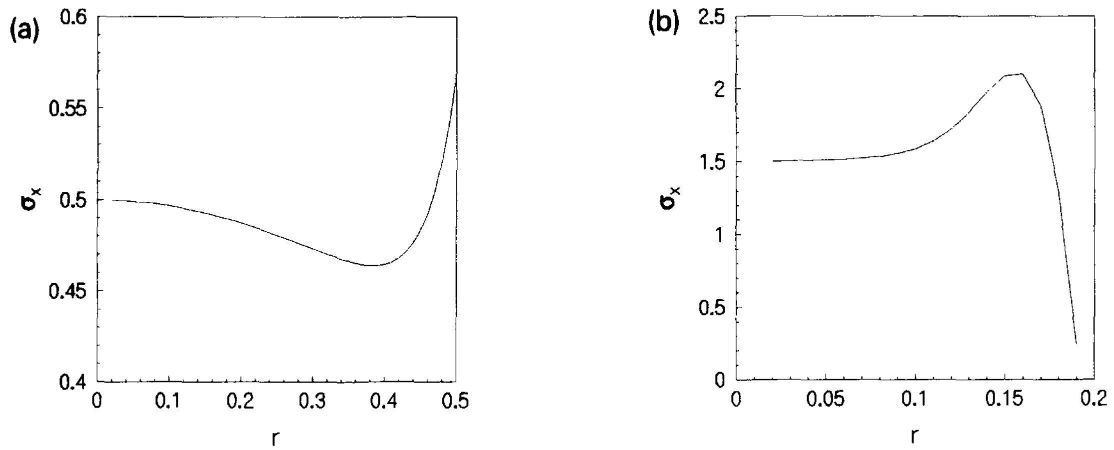


Fig. 3. (a) The x quadrature dispersion function σ_x versus the field amplitude r of the real cat state for $\phi = \pi/5$, $\eta^2 = 0.88$. (b) The x quadrature dispersion function σ_x versus the field amplitude r of the imaginary cat state for $\phi = \pi/7.88$, $\eta^2 = 0.88$.

$$- \tfrac{1}{2} N_\pm^4 \left(\sum_{n=0}^{\infty} \sqrt{(n+1)}\, d_n d_{n+1} r^{2n+1} \right. \\ \left. \times [\cos\phi \pm \cos(2n+1)\phi] \right)^2 , \tag{25}$$

$$\sigma_{p,\pm} = \tfrac{1}{2} \\ + \tfrac{1}{2} N_\pm^2 \sum_{n=0}^{\infty} (n+1) d_{n+1}^2 r^{2n+2} [1 \pm \cos(2n+2)\phi] \\ - \tfrac{1}{2} N_\pm^2 \sum_{n=0}^{\infty} \sqrt{(n+1)(n+2)}\, d_n d_{n+2} r^{2n+2} \\ \times [\cos 2\phi \pm \cos(2n+2)\phi] . \tag{26}$$

Depending on the function $f(n)$, the dispersions $\sigma_{x,\pm}(\sigma_{p,\pm})$ may become less than $\frac{1}{2}$, which means squeezing. The x-quadrature dispersion functions $\sigma_{x,\pm}$ are shown in Figs. 3a,b (taking $f(n) = L_n^1(\eta^2)\,[(n+1)L_n^0(\eta^2)]^{-1}$ for real and imaginary Schrödinger cat states respectively) from which it is evident that it is possible to get squeezing for both the states. This is again in contrast with the usual harmonic oscillator cat states where only the even coherent states exhibit squeezing.

One can calculate the correlation of the quadrature components in real and imaginary nonlinear Schrödinger cat states using (22)–(24) as

$$\sigma_{xp,\pm} = \frac{1}{2}\langle\pm\mu|\hat{x}\hat{p}+\hat{p}\hat{x}|\mu_\pm\rangle - \langle\pm\mu|\hat{p}|\mu_\pm\rangle\langle\pm\mu|\hat{x}|\mu_\pm\rangle = 0. \quad (27)$$

This equation shows that both real and imaginary nonlinear Schrödinger cat states have no correlation between the quadratures. Moreover $\sigma_{x,\pm}\sigma_{p,\pm} - \sigma^2_{xp,\pm} \geqslant \frac{1}{4}$, thus the real and imaginary Schrödinger cat states minimize the Schrödinger uncertainty relation for some values of r and ϕ.

4. Conclusions

We have introduced real and imaginary nonlinear Schrödinger cat states which are superpositions of nonlinear coherent states $|\mu\rangle$ and $|\mu^*\rangle$. The oscillatory character of the photon distribution functions of both the states emerging due to the quantum interference between the two components is shown. Both the states manifest quadrature squeezing and antibunching. These properties depend essentially on the introduced nonlinearity and not on the symmetry of the state. Interestingly enough, these two states have no correlation between the quadratures and they satisfy the Schrödinger uncertainty relation for some values of r and ϕ. Because of their singular properties, states of the type considered here might be of general interest in the optical and microwave fields, in molecular vibrations or nuclei vibrations for polyatamic molecules etc.

References

[1] V.I. Manko, G. Marmo, E.C.G. Sudarshan, F. Zaccaria, Phys. Scr. 55 (1996).
[2] R.L. de Matos Filho, W. Vogel, Phys. Rev. A 54 (1996) 4560.
[3] S. Mancini, Phys. Lett. A 233 (1997) 291.
[4] V.V. Dodonov, S.Yu. Kalmykov, V.I. Manko, Phys. Lett. A 199 (1995) 123.
[5] V.V. Dodonov, I.A. Malkin, V.I. Manko, Physica 72 (1994) 597.