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Optimal subsidy for the poor

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Abstract

This paper considers the problem of allocating an antipoverty budget (subsidy) among the poor in a society in an optimal and inequality reducing manner. Optimality requires minimization of some exogenously given poverty index. On the other hand, for inequality reduction, a poorer poor should receive a higher amount of subsidy than a richer poor (minimal progressivity) and the ranks of the persons in the pre- and post-subsidized distributions should be the same. An illustration of the general optimal subsidy function using some specific poverty index and an income distribution is provided.

Keywords: Inequality; Minimal progressivity; Poverty alleviation; Rank preservation; Subsidy

JEL classification: D31; D63; H20; I38

1. Introduction

Since poverty exists in a large number of countries, a targeted poverty alleviation programme is an important issue in such countries. A literature has emerged in which the targeting problem is usually formalized as one of minimizing some particular index of poverty subject to a budgetary constraint (see Kanbur, 1987; Besley and Kanbur, 1988, 1993; Besley, 1990; Keen, 1992; Kanbur et al., 1994; for an empirical illustration, see Ravallion and Chao, 1989). However, since a test of eligibility of claimants may be required, the administrative costs of ensuring that the targeted group actually benefits may be high (Besley, 1990; Besley and Kanbur, 1993).

An important aspect of targeting that has received particular attention is the optimal allocation of resources across heterogeneous groups (Keen, 1992). In a recent paper, Bourguignon and Fields (1997) showed that, among all poverty measures, only those which are discontinuous at the poverty line optimally allocate an antipoverty budget either to the poorest of the poor or to the richest poor or to both. It is evident that such formulations may ignore incentive effects. Kanbur et al. (1994) took into consideration incentive effects and investigated the central features of the nonlinear income tax schedules that minimize measures of poverty.

In this paper we address the problem of allocating an antipoverty subsidy in a minimally progressive, rank preserving and optimal manner. In order to do this we begin by specifying a general

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subsidy function for the poor. By minimal progressivity of this function we mean that a richer poor person receives a lower amount of subsidy than a poorer poor. Rank preservation demands that ranks of the poor persons in the pre- and post-subsidized income distributions remain the same. Clearly, this latter requirement is an incentive preservation condition. Minimal progressivity and rank preservation ensure that the post-subsidized income distribution of the poor becomes more equal than the pre-subsidized distribution by all strictly S-convex inequality indices. According to strict S-convexity a rank preserving transfer of income from a person to anyone who has lower income decreases inequality. Finally, optimality requires minimization of a poverty index subject to a given budgetary constraint. Thus, we distribute the subsidy in an inequality reducing and poverty minimizing manner. The size of the antipoverty budget does not allow anybody to cross the poverty line so that the total number of poor in the pre- and post-subsidized income profiles remains the same. This constraint on the budget size, which ensures that nonpoor persons do not receive any subsidy, may arise for several reasons. For instance, if the typical unimodal income distribution has a thick lower tail and if the poverty line does not exceed the modal income, then a substantial portion of the budget has to be allocated to the extremely poor persons. In such a case, relatively richer poor persons will receive only an insignificant fraction of the total subsidy. This, along with rank preservation, establishes the possibility that the post-subsidized income of a poor person may not exceed the poverty line. The smallness of the antipoverty budget may also be a consequence of greatly constrained expenditure budgets of an economy. In either case the smallness can be ensured if we impose a boundary constraint which states that a person with an income equal to the poverty line does not receive any subsidy at all.

The paper is organized as follows. The next section presents the notation and definitions. In Section 3 we identify the optimal subsidy function and Section 4 concludes.

2. Notation, definitions and preliminaries

The set of income distributions in an n-person society is $D^n = \{\underline{y} \in R^n | m \leq y_1 \leq y_2 \leq \cdots \leq y_n\}$, where R^n is the Euclidean n-space and $m \geq 0$ is the minimum income existing in the society. Thus, we have assumed at the outset that all income distributions are illfare ranked. Let $z > 0$ be the exogenously given poverty line, the income level necessary to maintain subsistence standard of living. Suppose that there are q poor persons and $(n-q)$ rich persons, that is $y_i \leq z$ for $i = 1,2,\ldots,q$ and $y_i > z$ for $i = q+1,\ldots,n$. Thus, for any $\underline{y} \in D^n$, we can partition $\underline{y}$ as $(\underline{y}^{\mathrm{p}},\underline{y}^{\mathrm{r}})$, where $\underline{y}^{\mathrm{p}}$ $(\underline{y}^{\mathrm{r}})$ is the income distribution of the poor (rich).

Now, let $s:[m,z] \to R^1$ be a subsidy function. For any $t \in [m,z]$, the quantity $s(t)$ represents the amount of subsidy received by the person with income t. For subsidy to make sense we must have $s(t) \geq 0$, otherwise we are speaking of a tax. Let us suppose that s is continuously differentiable. Minimal progressivity of s means that

$$s'(t) < 0 \tag{1}$$

for all $t \in (m,z)$. On the other hand, for preservation of original ranks of the poor individuals in the final or post-subsidized distribution, we need

$$s'(t) > -1 \tag{2}$$

for all $t \in (m,z)$. For any original distribution $\underline{y}$ we will denote the final distribution of the poor $(y_1 + s(y_1), \ldots, y_q + s(y_q))$ by $(\underline{y}^{\mathrm{p}} + s(\underline{y}^{\mathrm{p}}))$.

In order to study the impact of subsidy on the income inequality of the poor, we first note that inequality indices can be of relative or absolute type. While a relative index remains invariant under equiproportional variations in all incomes, an absolute index does not alter if all incomes are changed by the same absolute amount. Examples of absolute indices are the variance $V(\underline{y}^{\mathrm{p}}) = \Sigma_{i=1}^{q}(y_i - m(\underline{y}^{\mathrm{p}}))^2/q$ and the absolute Gini index $A(\underline{y}^{\mathrm{p}}) = m(\underline{y}^{\mathrm{p}}) - \Sigma_{i=1}^{q}(2(q-i)+1)y_i/q^2$, where $m(\cdot)$ denotes the mean. Standard relative indices are the squared coefficient of variation $V(\underline{y}^{\mathrm{p}})/m^2(\underline{y}^{\mathrm{p}})$ and the Gini coefficient $A(\underline{y}^{\mathrm{p}})/m(\underline{y}^{\mathrm{p}})$.

Given conditions (1) and (2), taking a cue from Moyes (1988), we can state that the final income distribution of the poor $(\underline{y}^{\mathrm{p}} + s(\underline{y}^{\mathrm{p}}))$ is more equitable than the original one $\underline{y}^{\mathrm{p}}$ by all strictly S-convex absolute inequality indices. Further, note that condition (1) along with nonnegativity of s ensures that $s(t)/t$ is decreasing. In view of the results established by Eichhorn et al. (1984) it then follows that $(\underline{y}^{\mathrm{p}} + s(\underline{y}^{\mathrm{p}}))$ is regarded as more equal than $\underline{y}^{\mathrm{p}}$ by all strictly S-convex relative inequality indices. We summarize these observations in the following proposition.

Proposition 1. *Suppose that the subsidy function $s : [m,z] \to R^1$ is minimally progressive and rank preserving. Then for any arbitrary income distribution of the poor $\underline{y}^{\mathrm{p}}$, $I(\underline{y}^{\mathrm{p}}) > I(\underline{y}^{\mathrm{p}} + s(\underline{y}^{\mathrm{p}}))$, where I is any strictly S-convex absolute (or relative) inequality index.*

In order to identify the optimal subsidy function we need to specify a poverty index. For this, let us assume that income distributions are defined on the continuum and represented by the distribution function $G : [m,\infty] \to [0,1]$. $G(t)$ gives the proportion of the population with income less than or equal to t, $G(m) = 0$, $G(\infty) = 1$ and G is nondecreasing. Suppose that G is absolutely continuous and write g for the density function. Then the entire class of additively decomposable poverty indices is given by

$$P(G;z) = \int_m^z f(t)g(t)\mathrm{d}t, \tag{3}$$

where $f : [m,\infty] \to R^1$ is decreasing, strictly convex and $f(t) = 0$ for all $t > z$ (Foster and Shorrocks, 1991). Decreasing f means that an increase in a poor person's income decreases P (monotonicity axiom; Sen, 1976). On the other hand, by strict convexity of f, P increases under a transfer of income from a poor person to anyone who is richer (transfer axiom; Sen, 1976). The assumption $f(t) = 0$ for all $t > z$ shows that P is independent of nonpoor incomes. Assume that f is twice differentiable and that $f'(z) = 0$. The last assumption is required because, as $f(t) = 0$ for all $t > z$, $f'(z)$ has to be 0 if it is defined and continuous. It may be important to note that this assumption, which is made for simple technical convenience only, is satisfied by many existing continuous poverty measures. [See, for example, additively decomposable poverty indices suggested by Chakravarty (1983) and Foster et al. (1984).] Evidently, different specifications of f will generate alternative forms of P (see, for example, Chakravarty, 1983, 1990; Foster et al., 1984).

The average amount of subsidy which is to be distributed among the poor in an optimal manner is given by $B > 0$. That is

$$\int_m^z s(t)g(t)\mathrm{d}t = B. \tag{4}$$

Given that B is not very large and that the amount is to be distributed in a decreasing way, we may impose the boundary constraint

$$s(z) = 0. \tag{5}$$

3. Optimal subsidy function

The purpose of this section is to derive the poverty minimizing subsidy function. Given that s is order preserving, the poverty index for the post-subsidized distribution of income becomes

$$P(G;z,s) = \int_m^z f(t + s(t))g(t)\mathrm{d}t. \tag{6}$$

Definition. A subsidy function is called optimal if it minimizes the poverty index (6) subject to the budgetary constraint (4) and the boundary constraint (5).

Proposition 2. *The optimal subsidy function is given by*

$$s(t) = (f')^{-1}\left(\frac{k}{g(t)} - \frac{k}{g(z)}\right) - t,\ \ m \leq t \leq z, \tag{7}$$

where k is a constant.

Proof. We will invoke the Euler–Lagrange technique to prove the proposition. Let

$$L(s) = \int_m^z f(t + s(t))g(t)\mathrm{d}t - \lambda\left(\int_m^z s(t)g(t)\mathrm{d}t - B\right), \tag{8}$$

where λ is the Lagrange multiplier. Consider any function $h : [m,z] \rightarrow R^1$ such that $\int_m^z h(t)\mathrm{d}t = 0$. For any θ denote $L(s + \theta h)$ by $H(\theta)$. If P attains a minimum for some s, then $H(\theta)$ attains a minimum for $\theta = 0$.

Now

$$H(\theta) = \int_m^z f(t + s(t) + \theta h(t))g(t)\mathrm{d}t - \lambda\left(\int_m^z (s(t) + \theta h(t))g(t)\mathrm{d}t - B\right). \tag{9}$$

Therefore,

$$H'(\theta) = \int_m^z f'(t + s(t) + \theta h(t))h(t)g(t)\mathrm{d}t - \lambda\int_m^z h(t)g(t)\mathrm{d}t. \tag{10}$$

Hence, $H'(0)=0$ gives

$$\int_m^z f'(t+s(t))h(t)g(t)\mathrm{d}t - \lambda\int_m^z h(t)g(t)\mathrm{d}t = 0$$

or

$$\int_m^z (f'(t+s(t))-\lambda)h(t)g(t)\mathrm{d}t = 0. \tag{11}$$

Since (11) holds for all h such that $\int_m^z h(t)\mathrm{d}t = 0$, it follows that

$$(f'(t+s(t))-\lambda)g(t) = k, \tag{12}$$

where k is a constant. From (12) we get

$$s(t) = (f')^{-1}\left(\frac{k}{g(t)}+\lambda\right) - t. \tag{13}$$

Using $s(z)=0=f'(z)$ in (13) we have

$$\lambda = -\frac{k}{g(z)}. \tag{14}$$

Substituting the value of λ from (14) into (13) we obtain the desired result. Note that

$$H''(\theta) = \int_m^z f''(t+s(t)+\theta h(t))h^2(t)g(t)\mathrm{d}t.$$

Therefore,

$$H''(0) = \int_m^z f''(t+s(t))h^2(t)g(t)\mathrm{d}t, \tag{15}$$

which is positive due to the positivity of f'', h^2 and g. Thus, the second order condition for minimization is fulfilled. ■

In order to discuss and illustrate the optimal subsidy function, we need to determine the signs of λ and k. Note that λ measures responsiveness of the budget to the optimum. That is, if we increase B by 1 unit, then λ determines the amount by which the optimum should change. From (14), note that λ and k must be of opposite sign as $g(t)>0$ for all $m<t<z$. Differentiating (7) and simplifying, we get

$$kg'(t) + f''(t+s(t))g^2(t)[1+s'(t)] = 0. \tag{16}$$

As s must satisfy (2), from (16) and the fact that f'' and g are positive, we conclude that $kg'(t)<0$ for all $m<t<z$. Now, if the income distribution is unimodal and the poverty line is below the mode, then $g'(t)>0$ in (m,z). In that case we have $k<0$ and, consequently, $\lambda>0$. This seems intuitively reasonable since the Lagrangian objective function (8) should represent a trade-off between the extent of poverty reduction and the budgetary increase. Next, using (16) and noting that s must satisfy (1), we find the following inequality:

$$k > -\frac{f''(t+s(t))g^2(t)}{g'(t)},\ m < t < z. \tag{17}$$

Though the condition (17) looks complicated, for specific choices of income density and poverty index it can be simplified substantially.

In order to illustrate the optimal subsidy function, suppose that the income distribution defined on $[m,\infty]$ is represented by the following density function:

$$g(t) = \frac{t}{\alpha M^2}\mathrm{e}^{-t/M},\ m \le t \le \infty, \tag{18}$$

where the mode of the income distribution is at $M > z$ and $\alpha = \int_{m/M}^{\infty} u\mathrm{e}^{-u}\,\mathrm{d}u$. For the poverty measure defined in (3), take

$$f(t) = (z-t)^2,\ 0 \le t \le z. \tag{19}$$

That is, P is a member of the Foster et al. (1984) family. This measure satisfies all the assumptions laid down in Section 2. We can now calculate the optimal subsidy function which turns out to be

$$s(t) = (z-t) + \frac{\alpha k M^2}{2}\left[\frac{\mathrm{e}^{t/M}}{t} - \frac{\mathrm{e}^{z/M}}{z}\right],\ m \le t \le z. \tag{20}$$

Here, k is any negative real number satisfying

$$k > -\frac{2t^2}{\alpha M[M-t]}\mathrm{e}^{-t/M},\ m \le t \le z. \tag{21}$$

A sufficient condition for this is given by

$$k > -\frac{2m^2}{\alpha \mathrm{e} M^2}. \tag{22}$$

4. Conclusion

The subject of this paper is the distribution of a given amount of subsidy among the poor persons in a society in an optimal and inequality reducing manner. Optimality means that, given budget size, poverty should be minimized to the extent possible. Inequality reduction can be assured if the subsidy is distributed in minimally progressive and rank preserving manner, where minimal progressivity means that a richer poor gets a lower amount of subsidy than a poorer poor and rank preservation requires that the ranks of the individuals in the original and the final or post-subsidized distributions are the same. A problem associated with the allocation of an antipoverty budget is that of leakage of some part of the budget to the nonpoor. We have therefore imposed a boundary condition to avoid the leakage problem.

It should be noted that since the optimal subsidy function is derived by minimizing some particular index of poverty, the implied policy prescription is contingent on the implicit valuation of the poverty index. However, it may be useful to explore the implications of doing so for two reasons. First, policy is often evaluated by the use of such indices and therefore it may be helpful to know what kind of policy would be implied if a particular index is adopted. Second, the nonwelfarist approach to policy analysis is often adopted in the literature (see, for example, Sen, 1985. For an earlier discussion, see Seade, 1980).

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