Journal of Development Economics
Vol. 58 (1999) 359–385

JOURNAL OF Development ECONOMICS

Growth versus welfare in a model of nonrival infrastructure

Dipankar Dasgupta

Planning Unit, Indian Statistical Institute, 7 Shaheed Jeet Singh Marg, New Delhi 110 016, India

Revised 1 October 1998

Abstract

The paper constructs a model of endogenous growth where infrastructure acts as an accumulable stock generating a nonrival input service. Steady state growth paths are studied for Market and Command Economies. In the former, a final good is produced privately and, as in many developing economies, infrastructure accumulated on noncompetitive basis by the State. The Command Economy allocates resources by solving a grand optimization exercise. The transitionally stable steady growth rate for the Market Economy dominates the Command Economy growth rate due to the joint presence of noncompetitive behaviour by the State and noninternalizable externalities. © 1999 Elsevier Science B.V. All rights reserved.

JEL classification: O41; H54; H41; H21; D61

Keywords: Endogenous growth; Inefficiency; Infrastructure; Nonrival input; Optimal tax; Stability

1. Introduction

Modern Growth Theory is concerned, amongst other things, with a modelling of endogenously generated efforts to remove constraints on the growth of an economy. Neoclassical economics locates in the force of diminishing returns one such obstacle to growth. Following Solow (1956), diminishing returns are normally traced back to the accumulation of capital in the face of nonaccumulable labour and a way to overcome declining productivity of capital is found in the artifact of labour augmenting technical progress.

PII: S0304-3878(98)00127-8

Uzawa (1965), Lucas (1988) and Rebelo (1991) pushed the Solow theory to its logical limits by modelling the possibility of a conscious effort by economic agents to invest in human capital. [1] This gave rise to a human capital led theory of economic growth and development.

Inadequate accumulation of human capital is only one of the reasons constraining the rate of economic development. An equally important bottleneck comes in the shape of infrastructure. The importance of infrastructure to economic development was clearly brought out by the World Bank Development Report (World Bank, 1994) with reference to China's inter-city transport system. Due to a low investment (around 1.3% of its GNP during the 1980s) in transport networks, the high growth rate (around 9%) ushered in by China's open door policies led to severe rationing of transport infrastructure. This gave rise to a shortage of coal to the thermal power plants which generate 76% of China's electricity. It is estimated that the annual economic costs of inadequate transport infrastructure in China is at least 1% of its GNP. The India Infrastructure Report (1996) expresses a similar sentiment. To quote from Volume II of this report:

> Examination of the records of the fast-growing East and South East Asian countries shows a similar pattern. As their gross domestic investment rates increased to over 30% of GDP, rates of infrastructure investment rose correspondingly to levels of 7 to 8% of GDP. . . . (W)e have projected gross domestic investment in infrastructure in India to grow from the current level of 5.5% of GDP to about 7% in 2000–01 and 8% in 2005–06. . . .

While the significance of infrastructure for growth is thus well recognized, there have been only a few theoretical attempts to analyze the different aspects of the problem. Barro (1990) represents one of the first attempts to capture the role of infrastructure in the form of publicly provided services which enter as inputs into the production process. [2] Such services are physically indistinguishable from the final good produced by the economy. A part of the latter is taxed away from households and routed back into the productive sector as input. Thus, Barro's public service is a pure flow and its rate of growth a flow rate. [3]

As opposed to this, the World Bank Development Report (World Bank, 1994) found that a 1% rise in the *stock* of infrastructure leads to a 1% improvement in the GDP across countries. In other words, at any moment of time, an economy is endowed with stocks of infrastructure, out of which the services flow into the different productive activities. It would appear (as in the Chinese example quoted above), that the growth of the economy is limited by the growth of these *stocks*.

[1] See also Rafzin (1972).

[2] See Aghion and Howitt (1998), Chap. 1, p. 46 for an interpretation of Barro's public services as infrastructure.

[3] Alesina and Rodrik (1994) adopt the Barro view also.

To capture this aspect of the problem, one must specify explicitly the processes governing the accumulation of infrastructural stocks.

The present paper is concerned with such an exercise. Infrastructure is treated as an input *flow* into the production process, emanating from an aggregate *stock*, owned and controlled by a single macro agent. For reasons outlined below, this agent is best interpreted as the State. [4]

Infrastructural services are often in the nature of public goods. Hence, they are *nonrival* in consumption and generate externalities. This is adequately recognized by The India Infrastructure Report (1996):

> Most infrastructure services have some elements of public good in them in the sense that they are generally publicly available and also exhibit significant positive externalities. To take the simplest example ... consumption of public lighting by one citizen has no effect on the consumption by another.

Truong (1993a,b) studies the case of nonrival infrastructure. On the other hand, Barro views public services (including defense) as rival, though he points out the possibility of nonrival services, such as the space programme. In our view, a significant part of infrastructure, roadways, transport services, telecommunications, etc., seems to satisfy the property of nonrivalry. One might argue that if environment is nonrival then so is a highway, since several agents enjoy its service simultaneously. The highway of course is more prone to congestion than environment is to pollution. But that is a question of degree rather than essence. [5]

In what follows we shall adopt a two sector breakdown of the macroeconomy into a private sector producing final goods and a public sector accumulating infrastructure and selling its services to the private sector as input. Being nonrival in nature, the services are available free of charge to the public sector itself, for further accumulation of infrastructure with the help of privately supplied inputs. The latter are purchased, amongst other things, out of the revenue flows from the sale of infrastructural services. [6]

In principle, infrastructure might be privately accumulated also, though this is rarely the case for developing economies. In India, for example, there have been attempts in the recent past to open up segments of infrastructural services to the private sector. Nevertheless, the bulk of infrastructure continues to be the realm of the State. Thus, railways, airlines, highways, a large part of telecommunications

[4] Thus, although the physical form assumed by infrastructure differs in the present exercise from Barro's public services, its provider continues to be the State.

[5] The possibility of congestion in the use of nonrival infrastructure is considered by Truong (1993a).

[6] An alternative treatment of infrastructure as a stock may be found in Futagami et al. (1993). This work, which is interesting in its own right, differs from the present one in several respects. Noteworthy amongst these are that it does not (a) deal with nonrival infrastructure, (b) concern itself with the allocation of private inputs between final goods production and infrastructure accumulation and (c) compare the relative performances of the Market and the Command Economies in steady state.

and so on fall under the public sector. In fact, attempts to privatize air transport in India have largely been unsuccessful. Thus, the assumption that infrastructure is publicly provided is to be viewed as a structural feature of developing economies. As noted in Section 6, however, the results of this paper have implications for efforts to privatize the infrastructure sector in developing economies.

The rule for financing private inputs used by the infrastructural sector gives rise to a pseudo-free enterprise economy, where all sectors do not maximize profits. At the cost of terminological abuse, we refer to it as a Market Economy. [7] The paper starts off with such an economy and proceeds to compare it with an economy where resources are allocated by solving a grand optimization exercise. The latter is called a Command Economy, which may be thought of as a fully planned system.

Although the paper looks into the question of the transitional dynamics of the Market Economy, the analysis is restricted to steady states alone when it comes to a relative evaluation of the growth performances of the two systems. This allows for a clear comparison of the results of the paper with the ones proved for some of the well-known models of Endogenous Growth, such as Romer (1986, 1990), Lucas (1988), Barro (1990) and others. These latter have shown that the Command Economy, by virtue of its ability to internalize different kinds of externalities, grows faster than the Market Economy *in steady state*. By contrast, the present exercise demonstrates the Command Economy to be endowed with a *smaller* steady state growth rate. The lone instance of this phenomenon in the literature is provided by Aghion and Howitt (1992), who demonstrated that a Command Economy grows more slowly because it internalizes the *losses* caused by technical progress in the shape of obsolescence. The slower growth rate derived by the present paper is explained of course by different factors (discussed in Section 4 below). [8]

In steady state, all sectors of the economy, including infrastructure, grow at the same positive rate. Traditional wisdom might suggest that it is in the achievement of a minimal *level* of infrastructure, rather than in its steady accumulation, that a solution to the problem of sustainable development lies. How meaningful is it then to study steady state growth paths involving unlimited growth of infrastructure? The question may be answered in at least two ways. First, the present study is partly motivated by Barro's seminal work, which *does* consider steady growth of the public service input which, as already noted, is a proxy for infrastructure in his model. [9] Secondly, to the extent that one subscribes to the basic tenets of neoclassical economics (as is the case with the present author), the law of

[7] *Mixed Economy* may be a better expression.

[8] Since the paper introduces the Command Economy merely to provide a contrast for the steady state behaviour of the Market Economy, it abstracts from the transitional dynamics of the former.

[9] Futagami et al. (1993) are also concerned with a steady growth of infrastructural stocks.

diminishing returns suggests the impossibility of maintaining steady state growth in the face of a fixed factor (such as infrastructure).[10] Alternatively put, in the presence of steady growth of the economy, a fixed or slowly growing infrastructure is likely to give rise to congestion problems, as in the Chinese example quoted above, or, to indulge in a bit of casual empiricism, in the case of airline reservations during rush seasons even in the most developed of economies.

The paper is organized as follows. Section 2 sets out the preliminary features of the model. The one following discusses the static equilibrium of the Market Economy as well as its steady state equilibrium when infrastructure accumulation is financed out of the sale of infrastructural services and a tax on profits. Section 4 considers the workings of the Command Economy and proves that it grows slower than the Market Economy even though the welfare achieved by it dominates the one for the latter. Section 5 looks into the possibility of attaining the Command Economy solution through the market channel. The paper concludes in Section 6. The transitional dynamics of the Market Economy is worked out in Appendix A.

2. Model preliminaries

There are two productive sectors, referred to as the Y and the G-sector, respectively. Commodity Y is used for consumption as well as capital accumulation. As in Solow (1956), K denotes both the stock of capital and the flow of its services. It may be accumulated physically or as human capital (similar to Barro, 1990), the latter through expenditure on education. Both forms involve a one-to-one transformation of savings out of Y-sector income. The paper abstracts completely from unskilled labour and concentrates on the limits on growth imposed by the truly scarce resources in a developing economy, viz., infrastructure on the one hand and physical and human capital on the other.

The infrastructural good is G. The stock of G generates a flow that enters all production (including its own) as an essential input. As with K, the stock-flow ratio is a constant so that G represents a flow also. The flow of G is a nonrival commodity by assumption, though excludable. The stock of G is accumulated by means of a technology that is subject to diminishing returns.

The technology for Y is given by

$$Y = AK_y^{\alpha} G^{1-\alpha}, 0 < \alpha < 1, \tag{1}$$

where K_y = flow of capital services used to produce Y and A is parametrically specified. Throughout the paper, our attention will be restricted to growth paths

[10] Of course, Rebelo (1991) has argued to the contrary, provided there exists an indecomposable subsector of the economy which is free from the operation of diminishing returns. While this has helped to locate the *genesis* of endogeneity in the so called *AK*-type models of growth, the latter cannot be taken as a prototype of *all* observed economies.

consistent with a full utilization of the available nonrival infrastructure in *both* sectors, a natural assumption for a model concerned with infrastructural constraints on growth. [11] Thus, the technology for $\dot{G}$, the change in the *stock* of G, is

$$\dot{G} = BK_g^{\beta} G^{1-\beta}, 0 < \beta < 1, \tag{2}$$

where K_g = flow of capital services used to produce $\dot{G}$ and B is parametrically specified.

Society's welfare is identical with that of the representative Household and given by

$$W = \int_0^{\infty} \frac{C(\theta)^{1-\sigma}}{1-\sigma} e^{-\rho\theta} d\theta, \tag{3}$$

where $C(\theta)$ represents consumption at point of time θ, the constant ρ a positive discount parameter and $1 \neq \sigma > 0$ the elasticity of instantaneous marginal utility. A large value of σ indicates a sharp fall in this marginal utility in response to a rise in consumption. [12]

3. Market Economy

In the Market Economy, the Household owns all capital. The final product Y is produced by a representative Firm that maximizes profit assuming all prices to be parametrically specified. It is charged a price τ (fixed for all time in steady state) per unit flow of G consumed and r (also fixed over time) by the Household per unit flow of capital services consumed. Further, there is a proportional tax t on profit income the proceeds from which are passed on entirely to the G-sector.

Profit maximization will give rise to demands for K and G services by the Y-sector. The former is added to the demand for K services by the G-sector to

[11] This is further explained in Section 3.

[12] It is more common in Endogenous Growth Theory to deal with the welfare function

$$\int_0^{\infty} \frac{C(\theta)^{1-\sigma} - 1}{1-\sigma} e^{-\rho\theta} d\theta.$$

When $\sigma = 1$, it reduces to $\int_0^{\infty} \ln C(\theta) e^{-\rho\theta} d\theta$. Since the introduction of the latter form does not change the results of the paper in any way, we concentrate on the case $\sigma \neq 1$. On the other hand, we shall see that the phase diagram analysis in Appendix A divides up into two cases depending on whether $\sigma > 1$ or < 1.

yield the aggregate demand for K. As in Solow (1956), the aggregate of capital services are thrown inelastically on the factor market at each point of time, its rental r being determined by equating the inelastic supply to the aggregate demand for capital services.

To accumulate infrastructure, the G-sector pays K the competitive rental, but unlike the competitive Firm, has free use of G services. Further, it does not equate the marginal product of K to its rental. Instead, K services are purchased from the proceeds of the sale of G services to the Y-sector and the profit tax. This determines the G-sector's demand for K.

As in the case of K, the aggregate of G-services available is also supplied inelastically at each point of time. Its price τ is determined (once again, as in Solow) by equating this supply to the demand for it by the Y-sector, the only sector that pays for G-services. Thus, total consumption of G-services in the economy must be bounded above by the consumption of the Y-sector. Given its nonrival nature then, the aggregate demand for G will be the same as the demand by the Y-sector.[13] Further, infrastructure being a productive service that causes bottlenecks for any developing economy, the G-sector is best understood to be nonsatiated in its use. In other words, the G-sector will use up the entire supply of nonrival G at each point of time.

The mechanics of the model falls into two parts: Static Equilibrium at each θ and the Dynamic Steady State Equilibrium. These will be described in turn.

3.1. Static equilibrium

For the Y-sector to behave competitively, the ratio of marginal products of the factors should equal their price ratio. This means

$$\frac{\tau}{r}=\frac{A(1-\alpha)\left(K_y/G\right)^{\alpha}}{A\alpha\left(K_y/G\right)^{\alpha-1}}=\frac{1-\alpha}{\alpha}\frac{K_y}{G}$$

or, alternatively,

$$K_y=\frac{\alpha}{1-\alpha}G\frac{\tau}{r}. \tag{4}$$

Given the inelastically supplied value of G, Eq. (4) defines the competitive sector's demand for K as a function of τ/r when all G is used up.

[13] This analytical feature derives from Samuelson's (Samuelson, 1954) original treatment of a public good.

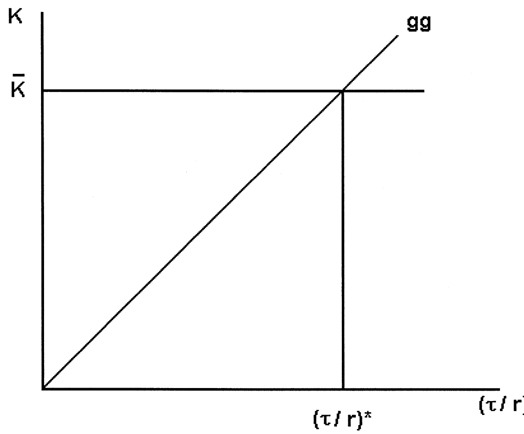

Fig. 1. Static equilibrium price ratio.

As noted above, the G-sector employment of K is governed by a budget constraint, viz.,

$$rK_g = \tau G + trK. \tag{5}$$

Hence, the demand for K by the G-sector is

$$K_g = \frac{\tau}{r} G + tK. \tag{6}$$

Using $K = K_y + K_g$, aggregate demand for K is then

$$K_y + K_g = \frac{1}{1-t}\left[\frac{\alpha}{1-\alpha} + 1\right] G \frac{\tau}{r}. \tag{7}$$

Eq. (7) gives the locus of $(K_y + K_g, \tau/r)$ combinations consistent with the full employment of G by the Y-sector.[14] The curve representing this relationship in the $(K, \tau/r)$ plane will be called the gg curve. The static equilibrium value of the relative price τ/r is determined by the intersection of the gg curve and the inelastic supply curve of K. This is shown in Fig. 1 under the assumption that the supply of $K = \bar{K}$. The equilibrium relative price is shown by $(\tau/r)*$. The demand for K falls with a fall in t, thereby raising the value of $(\tau/r)*$.

Proposition 1. *At any given point of time, each possible specification of the tax rate and the stocks of capital and infrastructure gives rise to a unique positive value of the relative returns to their services such that the market for capital services is in equilibrium and the services of infrastructure fully are utilized.*

Given the equilibrium price ratio established by Proposition 1, the ratio K_y/G employed in the Y-sector is known from Eq. (4). Since the technology displays

[14] At the cost of repetition, let us recall that G is used to the same extent by both sectors.

constant returns to scale, the marginal productivity of each factor is known for the Y-sector. For competitive behaviour, it is necessary that τ and r be chosen equal to the respective marginal productivities. Thus, infrastructure price and the real rate of interest are given by

$$\tau = A(1-\alpha)\left(\frac{K_y}{G}\right)^{\alpha} \tag{8}$$

and

$$r = A\alpha\left(\frac{K_y}{G}\right)^{\alpha-1} \tag{9}$$

Eq. (7) shows that a rise in K/G increases the equilibrium τ/r. In fact, $\tau/r \to \infty$ as $K/G \to \infty$ and $\tau/r \to 0$ as $K/G \to 0$. Hence, Eqs. (4) and (8) imply that the static equilibrium choice of τ satisfies a similar property. Thus, $\tau \to \infty$ as $K/G \to \infty$ and $\tau \to 0$ as $K/G \to 0$. We shall denote this monotonic relationship between the market clearing τ and K/G by $\tau = f(K/G)$ (see Fig. 2, left hand panel).

Notice that under the Cobb–Douglas framework assumed, there is another restriction on the model. This arises from the fact that the share of each factor in the Y-sector output must be a constant under competitive conditions. Thus,

$$rK_y = \alpha Y \tag{10}$$

and

$$\tau G = (1-\alpha)Y.$$

According to the G-sector's budget constraint, however, $rK_g = \tau G + trK$, so that

$$rK_g = (1-\alpha)Y + tr(K_y + K_g) = (1-\alpha)Y + t\alpha Y + trK_g. \tag{11}$$

Dividing out Eq. (11) by Eq. (10), we see that

$$\frac{K_g}{K_y} = \left(\frac{1}{1-t}\right)\left(\frac{1-\alpha}{\alpha}\right) + \left(\frac{t}{1-t}\right) \tag{12}$$

It should be noted that the allocative rule represented by Eq. (12) is inefficient. This follows from two facts. First, as is evident from Eq. (5), the G-sector does not equate the marginal product of K to its market rate of return r. Secondly, the Y-sector equates the price τ of G to its *private* rather than *social* marginal product. The latter is higher than the former given the nonrivalry of G. Thus, on the one hand, the rival commodity fails to satisfy a marginal condition and on the

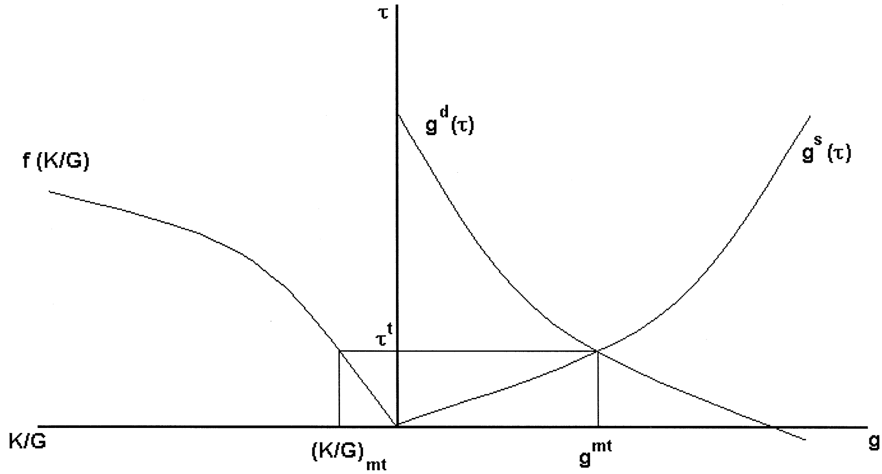

Fig. 2. The dual role of τ.

other, the nonrival commodity satisfies the wrong one. These inefficiencies will have a bearing on the major result of the paper.

3.2. Dynamic steady state equilibrium

As noted in Section 1, the economy is studied for a state of steady growth. Such an equilibrium involves:

$$\begin{aligned} \frac{K_y}{K} &= \text{constant}, \\ \frac{K_y}{G} &= \text{constant}, \\ \frac{\dot{G}}{G} &= \frac{\dot{Y}}{Y} \\ &= \frac{\dot{K}}{K} \\ &= \frac{\dot{C}}{C} \\ &= \text{constant}. \end{aligned}$$

This means, in particular, that K/G is a constant for all θ. We now proceed to show that $\exists$ a 1–1 correspondence between t and K/G.

In this connection, $\dot{G}/G$ will be called the 'supply rate of growth' g^{s} of the system, since it imposes an upper bound on the rate of indefinite growth achievable by the economy. Similarly, $\dot{C}/C$ is the 'demand rate of growth' g^{d}, for, as shown below, it is chosen optimally by the Representative Household. A dynamic steady state equilibrium occurs when $g^{s}=g^{d}$.

The variable τ has two different roles to play in the model. First, as shown in Section 3.1, each specification of K/G leads to a market clearing value of τ. This may be referred to as the *static* role of τ. Second, as will soon be established, for any given t, $\exists$ a value of τ equating g^{s} to g^{d}. This constitutes the *dynamic* role of τ. As far as the static role is concerned, it follows from the invertibility of f (left hand panel of Fig. 2) that *each* value of τ can be viewed as the static equilibrium price of infrastructure for *some* unique value of K/G.

We proceed now to the dynamic role of τ. For each t, both g^{d} and g^{s} are functions of τ. To see this, solve first for K^{y}/G as a function of τ from Eq. (8). Substitution in Eq. (9) gives r as a function of τ. Using this relationship along with Eqs. (8) and (6), it follows that

$$\frac{K_{g}}{G}=\frac{1}{1-t}\left[\frac{\tau^{1/\alpha}}{A\alpha(A(1-\alpha))^{(1-\alpha)/\alpha}}+t\left(\frac{\tau}{A(1-\alpha)}\right)^{1/\alpha}\right]. \tag{13}$$

From Eqs. (2) and (13), the supply rate of growth turns out to be

$$\begin{aligned} g^{s}(\tau) &= \frac{\dot{G}}{G} \\ &= B\left(\frac{K_{g}}{G}\right)^{\beta} \qquad (14) \\ &= B\left[\frac{1}{1-t}\left[\frac{\tau^{1/\alpha}}{A\alpha(A(1-\alpha))^{(1-\alpha)/\alpha}}+t\left(\frac{\tau}{A(1-\alpha)}\right)^{1/\alpha}\right]\right]^{\beta}. \qquad (15) \end{aligned}$$

Note that Eq. (12) is satisfied along the $g^{s}(\tau)$ curve. This follows since Eq. (15) is derived using Eq. (6) and the marginal productivity conditions for the *Y*-sector. Obviously, g^{s} is a monotone increasing function of τ. Moreover, $g^{s}(\tau)\rightarrow 0$ as $\tau\rightarrow 0$ and $g^{s}(\tau)\rightarrow\infty$ as $\tau\rightarrow\infty$ (see right hand panel of Fig. 2). Finally, Eq. (15) shows that $g^{s}(\tau)$ is monotone increasing in t. The intuition behind these results lies in the fact that with a rise in revenue, more infrastructural accumulation is feasible.

The Representative Household is assumed to maximize Eq. (3) subject to the instantaneous budget constraint

$$C+\dot{K}=(1-t)rK. \tag{16}$$

The household optimization exercise yields the demand rate of growth [15]

$$g^{\mathrm{d}}(\tau) = \frac{\dot{C}}{C}$$

$$= \frac{(1-t)r - \rho}{\sigma} \tag{17}$$

$$= \frac{(1-t)\,A\alpha\left(K_{\mathrm{y}}/G\right)^{\alpha-1} - \rho}{\sigma} \tag{18}$$

$$= \frac{(1-t)\,A\alpha\left(\tau/A(1-\alpha)\right)^{(\alpha-1)/\alpha} - \rho}{\sigma}. \tag{19}$$

Unlike $g^{\mathrm{s}}(\tau)$, the demand rate of growth does not necessarily satisfy Eq. (6). Consequently, Eq. (12) need not hold at all points lying on $g^{\mathrm{d}}(\tau)$. The demand rate of growth g^{d} is monotone decreasing in τ, with $g^{\mathrm{d}}(\tau) \to \infty$ as $\tau \to 0$ and $g^{\mathrm{d}}(\tau) < 0$ for τ large enough. This monotone relationship follows from the fact that with a rise in τ, the K_{y}/G ratio employed by the profit maximizing *Y*-sector rises, thereby reducing the marginal productivity of capital and along with it, the demand rate of growth (see right hand panel of Fig. 2). Further, $g^{\mathrm{d}}(\tau)$ decreases with t. The intuition for this result is straightforward also. The effective return from a postponement of consumption decreases with a rise in the tax rate. This has a dampening effect on the household's desire to grow.

Given these properties of g^{d}, g^{s}, there is, corresponding to each value of $t \in [0,1)$, a unique value of τ such that $g^{\mathrm{d}} = g^{\mathrm{s}}$. At this value of τ, both functions satisfy Eq. (12). This establishes the dynamic role of τ discussed above. Denote the equilibrium (g,τ) pair by $(g^{\mathrm{m}t},\tau^{t})$. Since the demand rate falls and the supply rate rises with a rise in the tax rate, it is intuitively obvious that τ^{t} is monotone decreasing in t (shifting curves in the right hand side of Fig. 2 reveals this clearly).

These observations are summarized in

Proposition 2. *For the Market Economy, each possible value of the proportional tax rate is associated with unique positive steady state values of the rate of growth, infrastructural price and aggregate capital–infrastructure ratio. The equilibrium value of the infrastructural price falls as the tax rate rises.*

The model then works as follows. Any value of t gives rise to an equilibrium value τ^{t} of τ. The function f^{-1} finds the unique ratio K/G, say $(K/G)_{\mathrm{m}t}$, that

[15] The result is standard for the assumed form of the utility function. See, for example, Romer (1990).

simultaneously makes τ^t a static equilibrium price also. In other words, *had the economy started out with the ratio* $(K/G)_{mt}$, i.e., if $K(0)/G(0) = (K/G)_{mt}$, then it would remain there indefinitely with K and G growing at the rate g^{mt}, *provided* that τ is also held fixed at τ^t forever. [16]

The triplet $(g^{mt}, \tau^t, (K/G)_{mt})$ established by Proposition 2 does not correspond to a true optimum for the household unless the associated growth path for C leads to a finite value of Eq. (3). Given any steady growth rate g of C (as well as of K, G and Y) it is straightforward to conclude that for the assumed form of W, a necessary condition for the convergence of the integral in Eq. (3) is

$$\rho > (1-\sigma) g. \tag{20}$$

The condition imposes no restriction when $\sigma > 1$. When $\sigma < 1$, there will be a nontrivial upper bound on admissible values of the rate of growth. Proposition 2 will not automatically guarantee that g^{mt} will satisfy this restriction. The condition will be satisfied if A and B are not too large.

Proposition 2 established that τ^t is a monotone declining function of t and this followed from the fact that g^d fell and g^s rose with a rise in t. However, it was not clear from this what the effect would be on g^{mt}, the equilibrium rate of growth corresponding to t. Thus, g^{mt} might rise or fall, depending on the relative magnitudes of the shifts in the demand and supply functions. We proceed now to investigate this question by representing the equilibrium of Proposition 2 in an alternative manner. Inverting Eq. (18), we have

$$\frac{K_y}{G} = \left(\frac{g^d\sigma + \rho}{(1-t) A\alpha} \right)^{1/(\alpha-1)}. \tag{21}$$

On the other hand, Eq. (14) yields

$$\frac{K_g}{G} = \left(\frac{g^s}{B} \right)^{1/\beta}. \tag{22}$$

Writing the common value of g^d and g^s as g, division of Eq. (22) by Eq. (21) gives rise to

$$\frac{K_g}{K_y} = \left(\frac{g}{B} \right)^{(1/\beta)} \left(\frac{g\sigma + \rho}{(1-t) A\alpha} \right)^{1/(1-\alpha)}. \tag{23}$$

Eq. (23) gives the relative allocation of capital between the two sectors necessary

[16] There is no guarantee of course that the initial condition corresponding to a chosen t will be exactly satisfied. This problem will be taken up in Appendix A on transitional dynamics.

to maintain equality between the demand and the supply rates at the level g. Let us call this the *necessary* ratio. From Proposition 2, we know that each t will correspond to a particular g only. Hence, Eq. (23) will not be meaningful for arbitrary choices of (g,t) pairs. To find the right pairs, we recall that Eq. (12) gives the constraint exogenously imposed on (K_g/K_y) by the G-sector's budget constraint. The equilibrium g corresponding to t must be the one that equates the *necessary* ratio to the exogenous ratio. Taking this into account, the equilibrium condition turns out to be

$$\left(\frac{g}{B}\right)^{1/\beta}\left(\frac{\sigma g+\rho}{\alpha A}\right)^{1/(1-\alpha)}=(1-t)^{\alpha/(1-\alpha)}\left[\frac{1-\alpha}{\alpha}+t\right]. \tag{24}$$

Fig. 3 shows the alternative representation of the market equilibrium. The LHS of Eq. (24) can be shown to be a strictly convex and increasing function of g. Also, the function vanishes at $g=0$. Given that the RHS is a positive constant for each t, the conclusion of Proposition 2 on the existence of a unique value of g^{mt} is confirmed. Further, it is quite obvious from Fig. 3 that g^{mt} is not too large when A and B are small. The RHS of Eq. (24) falls with a rise in t, a fact easily checked by differentiation. Hence, given the monotone rising property of the LHS with respect to g, it follows that g^{mt} falls with a rise in t. Going back to Fig. 2, this answers the question raised above about the relative magnitudes of the shifts

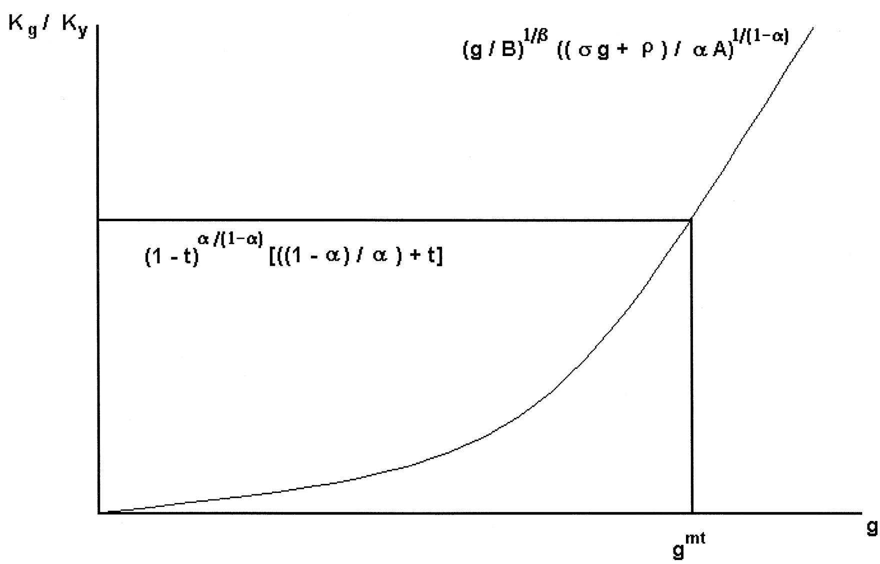

Fig. 3. Alternative representation of dynamic equilibrium.

in g^d and g^s in response to a rise in t. Clearly, g^d falls relatively more than the rise in g^s.[17] Accordingly, we have

Proposition 3. *The unique equilibrium value for the steady state growth rate falls with a rise in the tax rate.*

Following Barro, a further task before the Government is to find a t that leads to the best of the steady state paths, viz., the counterpart of the Golden Rule of Phelps (1961). We proceed now to a search for this optimal value of t. Using Eqs. (16) and (17),

$$C(\theta)^{mt} = \{\rho + (\sigma - 1) g^{mt}\} K(0) e^{g^{mt}\theta}, \tag{25}$$

where the $C(\cdot)^{mt}$ stands for the Market Economy's consumption at tax rate t. Substituting this in Eq. (3), the welfare associated with the choice of t is

$$W_{mt} = \frac{K(0)^{1-\sigma}}{1-\sigma} \frac{1}{\left(\rho + (\sigma - 1) g^{mt}\right)^{\sigma}} \tag{26}$$

where W_{mt} represents the welfare in question. Differentiating with respect to g^{mt}, the function W_{mt} is seen to be rising with g^{mt}. In other words, the welfare of the Market Economy rises with the rate of growth and the Household would wish to grow as fast as feasible.[18] Hence, Proposition 3 implies that the optimal value of the tax rate in the Market Economy is zero. Therefore, we have

Proposition 4. *Maximum welfare for the Market Economy corresponds to the maximum rate of growth. The optimal profit tax rate is zero.*

We shall denote the maximal, hence optimal, growth rate for the Market Economy by g^m. The first part of the Proposition is similar to the result of Barro (1990) for the Cobb–Douglas case. Thus, even with stock infrastructure, the Market Economy wishes to grow as fast as possible. The result, as with Barro, follows from the fact that for the chosen form of the utility function, society's welfare is an increasing function of the equilibrium rate of growth. We have noted earlier that the growth rate g^m will be attained by the Market Economy by means of an inefficient allocation of resources. Hence, it will not represent the best possible growth rate for the economy. On the other hand, the Command Economy of Section 4 is concerned with the achievement of the full optimum.

[17] As paraphrased by one of the referees, the Ramsey consumption effect dominates the *G*-sector budget constraint effect.

[18] Barro (1990) notes the same behaviour for the Cobb–Douglas production structure, though not necessarily so for more general technologies.

The second part of Proposition 4 is reminiscent of Lucas (1990) who found that the socially optimum profit tax rate for a model of human capital accumulation is zero in steady state. The difference between our result and his is that the zero tax rate of our model, though welfare maximizing for the Market Economy, is nevertheless socially suboptimal. The reasons underlying the divergence are the same as those outlined in the last paragraph.

4. Command Economy

As opposed to the Market Economy, resource allocation in the Command Economy is carried out by solving a grand optimization exercise. It may help to think in terms of an altruistic social planner organizing production in both sectors and allocating resources between them. The welfare function of the planner is identically the same as that of the representative Household. The result of the planner's optimum is a first best situation, since it is unconstrained by institutional requirements (such as the G-sector's budget constraint (Eq. (5)) in the Market Economy and hence, Eq. (12)). Unlike the representative Household of the Market Economy, the planner maximizes Eq. (3) subject to

$$\dot{K} = A(\phi K)^{\alpha} G^{1-\alpha} - C \tag{27}$$

and

$$\dot{G} = B((1-\phi)K)^{\beta} G^{1-\beta}, \tag{28}$$

where ϕ and $1-\phi$ are, respectively, the shares of K in the Y and G sectors. The problem is solved by maximizing the current value Hamiltonian

$$H = \frac{C^{1-\sigma}}{1-\sigma} + \eta\left[A(\phi K)^{\alpha} G^{1-\alpha} - C\right] + \xi\left[B((1-\phi)K)^{\beta} G^{1-\beta}\right], \tag{29}$$

where η and ξ are the costate variables associated with the stocks of K and G. The steady state solution for this problem may be called the Command Equilibrium and the associated rate of growth denoted g^{c}. The first order optimality conditions for the maximization of H are

$$\frac{\partial H}{\partial C} = 0 \tag{30}$$

$$\frac{\partial H}{\partial \phi} = 0 \tag{31}$$

$$\dot{\eta} = -\frac{\partial H}{\partial K} + \eta\rho \tag{32}$$

$$\dot{\xi} = -\frac{\partial H}{\partial G} + \xi\rho. \tag{33}$$

Since both the instantaneous utility function and the two production functions are strictly concave, it follows from Cass (1965) that Eq. (30) through Eq. (33), along with the transversality conditions

$$\eta(\theta)e^{-\rho\theta} \to 0 \text{ as } \theta \to \infty$$

and

$$\xi(\theta)e^{-\rho\theta} \to 0 \text{ as } \theta \to \infty,$$

are a sufficient characterization of the *unique* optimum path solving the planner's problem.

Using Eqs. (30)–(32)

$$\frac{\dot{C}}{C} = \frac{A\alpha(\phi K)^{\alpha-1}G^{1-\alpha} - \rho}{\sigma}. \tag{34}$$

Analogous to a Market Equilibrium,

$$\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{G}}{G} = \frac{\dot{Y}}{Y} \tag{35}$$

in steady state. Denote the common growth rate by g. Consistency between the demand and the supply rate requires (using Eq. (2)) that

$$g = B\left(\frac{(1-\phi)K}{G}\right)^{\beta}. \tag{36}$$

Differentiating Eq. (31) and using Eq. (30)

$$\frac{\dot{\eta}}{\eta} = \frac{\dot{\xi}}{\xi} = -\sigma g. \tag{37}$$

Eq. (37) implies that the transversality conditions are satisfied. Eq. (33) may be manipulated to yield

$$\frac{\dot{\xi}}{\xi} = -g\left[\frac{(1-\alpha)\beta}{\alpha}\frac{\phi}{1-\phi} + (1-\beta)\right] + \rho. \tag{38}$$

Combining Eqs. (37) and (38),

$$\frac{1-\phi}{\phi} = \frac{1-\alpha}{\alpha}\frac{\beta g}{\rho + (\sigma - (1-\beta))g}. \tag{39}$$

Eq. (39) represents the relationship that must hold between indefinitely maintained values of $(1-\phi)/\phi$ (i.e., K_g/K_y) and g if resources are allocated efficiently. This, in other words, is the efficient counterpart of Eq. (12) in the Market Economy. Again, Eqs. (34) and (36) together imply that

$$\frac{1-\phi}{\phi} = \left(\frac{g}{B}\right)^{1/\beta}\left(\frac{\sigma g + \rho}{\alpha A}\right)^{1/(1-\alpha)}. \tag{40}$$

Eq. (40) yields the value of $(1-\phi)/\phi$ to be maintained in steady state in order for Eqs. (34) and (36) to lead to the same value of g. Needless to say, varying specifications of g give rise to varying values of $(1-\phi)/\phi$. We may look upon Eqs. (39) and (40) as the reduced form optimality conditions for the planner's problem. The optimal rate of growth is found by solving these two equations simultaneously for $(1-\phi)/\phi$ and g. Eliminating $(1-\phi)/\phi$ between Eqs. (39) and (40),

$$\left(\frac{g}{B}\right)^{1/\beta}\left(\frac{\sigma g+\rho}{\alpha A}\right)^{1/(1-\alpha)}=\frac{1-\alpha}{\alpha}\frac{\beta g}{\rho+(\sigma-(1-\beta))g}. \quad (41)$$

In what follows, the LHS will be represented by the function $\Gamma(g)$ and the RHS by $\Psi(g)$. Eq. (41) has a solution $g=0$. This solution is ignored, however, since according to Eq. (38), $1-\phi\neq 0$, 1. A condition under which a positive solution

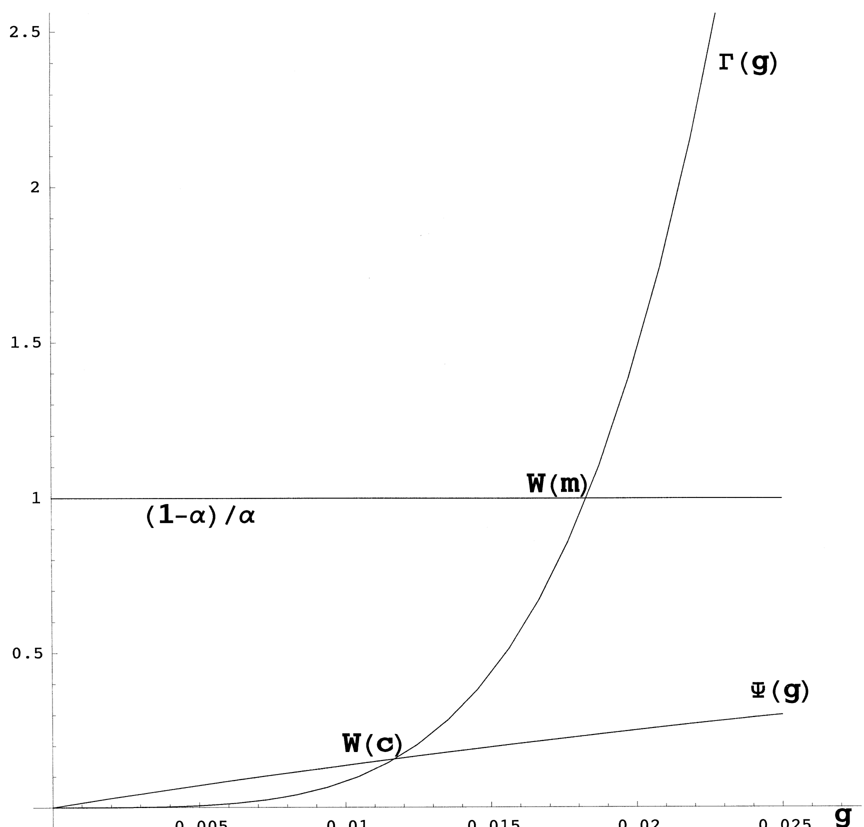

Fig. 4. Market growth rate dominates command growth rate $\alpha=0.5$, $\beta=0.3$, $\rho=0.02$, $\sigma=0.9$, $A=0.2$, $B=0.01$.

$g = g^c$ exists is discussed below in Proposition 5. A particular case is shown in Fig. 4.

The function $\Gamma(g)$ (viz., the *necessary* ratio) is shown in the diagram by the convex, monotone increasing function. On the other hand, $\Psi(g)$ is a concave function of g.[19] The positive solution g^c to the planner's exercise is represented by the abscissa of the point $W(c)$ in Fig. 4 where the two curves intersect. For later reference, denote the ordinate of $W(c)$ by $(1-\phi^c)/\phi^c$. Since the maximum is unique, a point such as $W(m)$, which does not fall on both curves, is expected to yield a lower level of welfare compared to $W(c)$. The point $W(m)$ is chosen to represent the solution to Eq. (24) when $t=0$. Its coordinates are given by the pair $(g^m, (1-\alpha)/\alpha)$. As drawn, $g^m > g^c$ and $(1-\alpha)/\alpha > (1-\phi^c)/\phi^c$. We proceed next to show that these are not peculiar features of the numerical example.

Proposition 5. *Suppose a solution exists to the Market Economy's problem characterized by a bounded value of the welfare function. Then a unique, strictly positive solution for the Command Economy's growth rate exists and the growth rate achieved by the Market Economy exceeds that of the Command Economy. Moreover, as compared to the Command Economy, the Market Economy allocates more capital for infrastructure accumulation relative to final good production.*

Proof: (See Fig. 4.) The function $\Psi(g)$ has the following easily verifiable properties:

$$\Psi'(g) = \frac{1-\alpha}{\alpha}\frac{\beta\rho}{(\rho+(\sigma-(1-\beta))g)^2} > 0$$

and

$$\Psi'(g) \to \frac{1-\alpha}{\alpha}\frac{\beta}{\rho} > 0 \text{ as } g \to 0.$$

On the other hand, the function $\Gamma(g)$ satisfies the property that

$$\Gamma'(g) \to 0 \text{ as } g \to 0.$$

Thus, for g arbitrarily close to zero, $\Psi(g) > \Gamma(g)$. Again, since g^m exists, Eq. (20) implies that for $g = g^m$,

$$\Psi(g) < \frac{1-\alpha}{\alpha}.$$

The continuity of $\Psi(g)$ and $\Gamma(g)$ implies now from the Intermediate Value Theorem that $\exists\ \bar{g}$ satisfying Eq. (41) such that $0 < \bar{g} < g^m$. The uniqueness of the solution to the planner's problem implies $\bar{g} = g^c$.

[19] It is actually strictly concave, though for the chosen values of the parameters, its curvature is mild, making it *visually* indistinguishable from a linear function.

The relative magnitudes of K_g/K_y under the two systems follow trivially. QED

The fact that the Market Economy grows faster than the Command Economy stands in contrast to Barro's result that the opposite is the case. In Barro's analysis, the Command Economy grows faster since it is able to internalize the social productivity of capital in calculating the demand rate of growth. Given that the planner is aware of the fixed proportion between capital and public services, the marginal productivity of capital in the Command Economy turns out to be the same as its *average* product. In the Market Economy on the other hand, it equals the *private* marginal product of capital. Under concavity, the average is larger than the marginal, so that the Market Economy grows slower.

The reason underlying the reversal of the Barro result in the present paper is discussed in Section 5.

5. Command solution through markets?

Given that the full command solution is not achievable in the Market Economy, is it possible at least to grow at the command rate? The answer is 'yes', since according to Eq. (24), $g^{mt} \to 0$ as $t \to 1$ and $g^{mt} = g^m > g^c$ when $t = 0$. Hence, by continuity, $\exists$ a $t = t^c \in (0,1)$ such that $g^{mt^c} = g^c$. Thus, the Market Economy can *in fact* attain the growth rate of the Command Economy.

Proposition 6. *There exists a tax rate for which the market equilibrium rate of growth is the same as that of the Command Economy.*

From our discussion following Eq. (12), we know that the Market Economy attains g^{mt^c} inefficiently. Consequently, it is intuitively obvious why the Command Economy dominates the Market Economy in welfare even at the same rate of growth $g^c = g^{mt^c}$. In fact, there is no value of t at which the Market Economy dominates the Command Economy. While this is true by definition (of a Command Economy), a regular proof is supplied below for the sake of completeness.

Proposition 7. *The Command Economy attains a higher welfare than the Market Economy for all values of the tax rate.*

Proof: Note from Eq. (41) that for the demand and the supply rates to be equal to a common value g, the Command Economy chooses the ratio (K_g/K_y) as

$$\left(\frac{g}{B}\right)^{(1/\beta)}\left(\frac{g\sigma+\rho}{\alpha A}\right)^{1/(1-\alpha)},$$

whereas, for the Market Economy, Eq. (23) requires the *necessary* ratio (K_g/K_y) corresponding to the pair (g,t) to be

$$\left(\frac{g}{B}\right)^{(1/\beta)}\left(\frac{g\sigma+\rho}{(1-t)A\alpha}\right)^{1/(1-\alpha)}.$$

These are unequal for $t \neq 0$. On the other hand, at $t=0$, Eq. (12) implies that the RHS of Eq. (41) is violated by the Market Economy. Hence, Eq. (41) is violated by the market solution for all values of t. Since Eq. (41) is a necessary condition for optimality of the growth path and the latter is unique, the result follows. QED

What, however, makes g^{m} greater than g^{c}? To answer this question, let us compute the consumption paths associated with g^{c} and $g^{mt^{c}}$. From Eq. (34) and the fact that in the Command Economy

$$C+\dot{K}=A(\phi^{c}K)^{\alpha}G^{1-\alpha}.$$

we can easily show that

$$C(\theta)^{c}=\left\{\frac{\phi^{c}}{\alpha}\rho+\left(\frac{\phi^{c}}{\alpha}\sigma-1\right)g^{c}\right\}K(0)\mathrm{e}^{g^{c}\theta}, \tag{42}$$

where $C(\cdot)^{c}$ stands for consumption undertaken by the Command Economy. From Proposition 5, we have $(1-\alpha)/\alpha>(1-\phi^{c})/\phi^{c}$. This means $\phi^{c}/\alpha>1$. Comparing Eqs. (25) and (42) now, it is clear that $C(\theta)^{c}>C(\theta)^{mt}$ when $t=t^{c}$. In other words, even at the same rate of growth, the Command Economy enjoys a higher level of consumption than the Market Economy at each point of time if they start from the same value of $K(0)$. To make up for the shortfall in the consumption level therefore, the Market Economy tries to raise its welfare by maximizing the rate of growth, i.e., it chooses $t=0$ and grows at the rate $g^{m}>g^{mt^{c}}=g^{c}$. Although $g^{m}>g^{c}$, however, the negative *level* effect dominates the positive *growth* effect as Proposition 7 demonstrates. This happens because the latter attains *any* rate of growth inefficiently.

6. Conclusion

The paper attempted to develop a theoretical model of growth involving a nonrival infrastructural input supplied on the basis of a per unit charge to a competitive sector producing a consumable and accumulable private good. The agents (symbolized by an infinitely lived representative Household) were seen to prefer a policy of maximal growth.

In most growth models, the rates of growth of the crucial variables chosen by the Command Economy are higher than the ones derived from private optimization. The divergence, which represents a failure of the First Fundamental Theorem of Welfare Economics (as applied to a dynamic context), arises mostly from the existence of positive externalities generated for the economy by private investment activities. The Command Economy accumulates at a higher rate than the free system given its ability to internalize the externality.

In the present paper also, there is a difference between the two solutions, caused by a failure on the part of the Market Economy to attain allocative efficiency both with respect to the rival as well as the nonrival factors of production. As opposed to standard models, however, the Market Economy grows faster than the Command Economy, though the growth rate attained by the latter obviously dominates in welfare.

The failure of markets in the presence of nonrival commodities is a well established fact in the static context. This paper shows that the result has a dynamic counterpart also in its implication for achievable steady growth rates. Moreover, it manifests itself in the form of an *apparently* efficient behaviour, viz., a high growth rate. A study of the transitional dynamics for the Market Economy (see Appendix A) reveals moreover that the economy, when left to itself, is likely to end up with this steady state.

The model developed in this paper is rather aggregative. Hence, it may not be a true representation of a full-fledged multisectoral economy. As such, one has to be careful in interpreting the behaviour of real economies as instances of the results proved here. Nevertheless, at the cost of possibly spurious generalizations, it is tempting to visualize the Market Economy of our paper as an *intermediate* step in the transition from a fully planned to a fully privatized system. In this context, the lesson to be learnt is that along the transformation path there may occur welfare costs, in spite of apparent gains in terms of rates of growth. In other words, a pseudo-system cannot be an acceptable alternative to a fully market oriented economy.

On the other hand, a fully State controlled economy cannot be the preferred alternative either. We are of the view that the first best represented by the planned system is operationally vacuous, since the degree of centralization it demands gives rise to high *social* costs. At the same time, unless appropriate pricing strategies can be devised for crucial sectors, such as infrastructure in our example, an economy may continue to perform suboptimally even if markets function in remaining areas. [20] Whether privatized economies are automatically able to arrive at such optimal pricing rules is not clear to us at this stage. As such, developing

[20] An example of such a pricing rule for the present model may be found in Dasgupta (1998), which constructs agent specific prices that sustain the Command Economy steady state path as a Lindahl equilibrium.

economies engaged in recasting the public provision of infrastructure in the mould of a privately supplied good may be faced with a dilemma.

Acknowledgements

The author benefited from the detailed comments of two anonymous referees. He also wishes to acknowledge comments received from Sukanta Bhattacharya, Debajyoti Chakrabarty, Fumio Ohtake, Mihir Rahshit, Takamasa Shirai and *most particularly*, Kazuo Mino. The final version of the paper was completed during the author's one year visit to the Faculty of Economics, Hokkaido University, Sapporo, Japan, in 1998. Research facilities enjoyed in this connection are gratefully recalled.

Appendix A. Transitional dynamics for the Market Economy

The optimal profit tax rate for the Market Economy was shown to be zero. Consequently, the discussion of transitional dynamics will be restricted to the case where $t=0$.

Define

$$k=\frac{K}{G} \text{ and } c=\frac{C}{K}.$$

Using Eqs. (16), (9), (2), (6) and (7),

$$\frac{\dot{k}}{k}=A\alpha^{\alpha}k^{\alpha-1}-B\big((1-\alpha)k\big)^{\beta}-c. \tag{A.1}$$

Again, from Eqs. (17), (9), (16) and (4),

$$\frac{\dot{c}}{c}=\frac{1-\sigma}{\sigma}A\alpha^{\alpha}k^{\alpha-1}-\frac{\rho}{\sigma}+c. \tag{A.2}$$

This system of differential equations describes the transitional dynamics of the Market economy. To study its properties, we shall first establish the existence of a unique equilibrium or stationary state for this system. The latter would simultaneously solve

$$c=A\alpha^{\alpha}k^{\alpha-1}-B\big((1-\alpha)k\big)^{\beta} \tag{A.3}$$

and

$$c=-\left(\frac{1-\sigma}{\sigma}\right)A\alpha^{\alpha}k^{\alpha-1}+\frac{\rho}{\sigma}. \tag{A.4}$$

Eliminating c, the stationary value of k is a solution to

$$\frac{1}{\sigma}A\alpha^{\alpha}k^{\alpha-1}=B((1-\alpha)k)^{\beta}+\frac{\rho}{\sigma}.$$

It is easy to see that this equation has a unique positive solution k^*. The corresponding unique value for c^* is found from either Eq. (A.3) or Eq. (A.4). The sign of c^* is discussed below.

In order to study the local stability of the dynamic system, we may rewrite Eqs. (A.1) and (A.2) as

$$\dot{k}=\left(A\alpha^{\alpha}k^{\alpha-1}-B((1-\alpha)k)^{\beta}-c\right)k$$
$$\dot{c}=\left(\frac{1-\sigma}{\sigma}A\alpha^{\alpha}k^{\alpha-1}-\frac{\rho}{\sigma}+c\right)c.$$

Linear approximation around (k^*,c^*) gives

$$\dot{k}=\left(A\alpha^{1+\alpha}(k^*)^{\alpha-1}-B(1+\beta)((1-\alpha)k^*)^{\beta}-c^*\right)(k-k^*)$$
$$-k^*(c-c^*)$$
$$\dot{c}=\left\{[(1-\sigma)/\sigma]A(\alpha-1)\alpha^{\alpha}(k^*)^{\alpha-2}c^*\right\}(k-k^*)$$
$$-\{[(1-\sigma)/\sigma]A\alpha^{\alpha}(k^*\}^{\alpha-1}-(\rho/\sigma)+2c^*)(c-c^*). \qquad (A.5)$$

Using Eqs. (A.3) and (A.4), the relevant determinant simplifies to

$$(\alpha-(1+\beta))g^{\mathrm{m}}c^*+(\alpha-1)\frac{\rho}{\sigma}<0,$$

where $g^{\mathrm{m}}=g^{\mathrm{d}}=g^{\mathrm{s}}=B((1-\alpha)k^*)^{\beta}$. The sign of the determinant establishes that the equilibrium is a saddle point. In other words, for each possible choice of the initial value $k(0)$ of k in a small neighbourhood of k^*, $\exists$ a choice of the initial value of c in a corresponding small neighbourhood of c^*, such that the system (A.5) converges to (k^*,c^*).

The phase diagrams below, pertaining to the two cases $\sigma>1$ and $\sigma<1$, reveal the dynamics more clearly.

Case 1: $\sigma>1$

Subtracting Eq. (A.4) from Eq. (A.3), it is easily seen that the difference is strictly positive for $k\cong 0$. Further, along Eq. (A.3), c is monotone decreasing in k, $c\to\infty$ as $k\to 0$ and $c\to-\infty$ as $k\to\infty$. Similarly, from Eq. (A.4), c is monotone decreasing in k, $c\to\infty$ as $k\to 0$ and $c\to\rho/\sigma$ as $k\to\infty$. Clearly, $c^*>\rho/\sigma>0$. The stable path is indicated by the thick arrowheads. See Fig. 5.

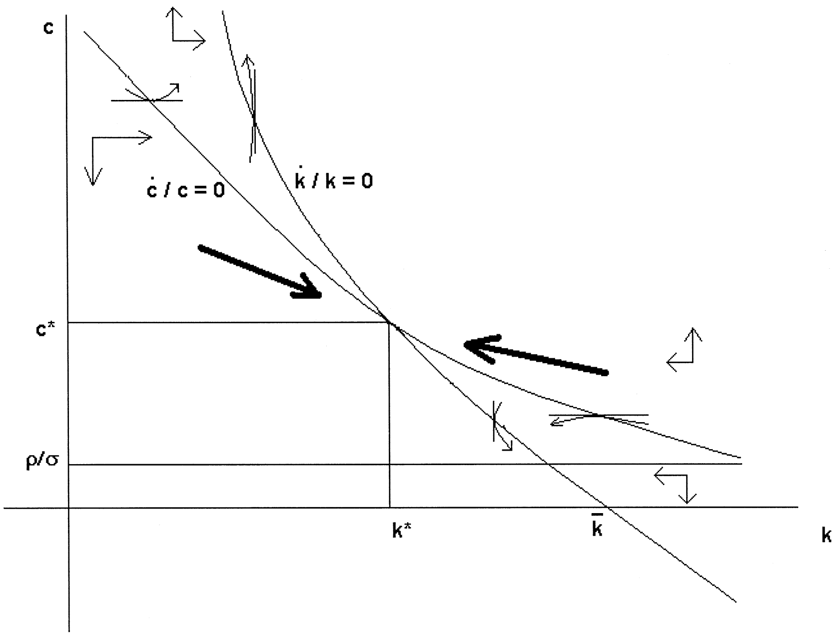

Fig. 5. Transitional dynamics (Case $\sigma > 1$).

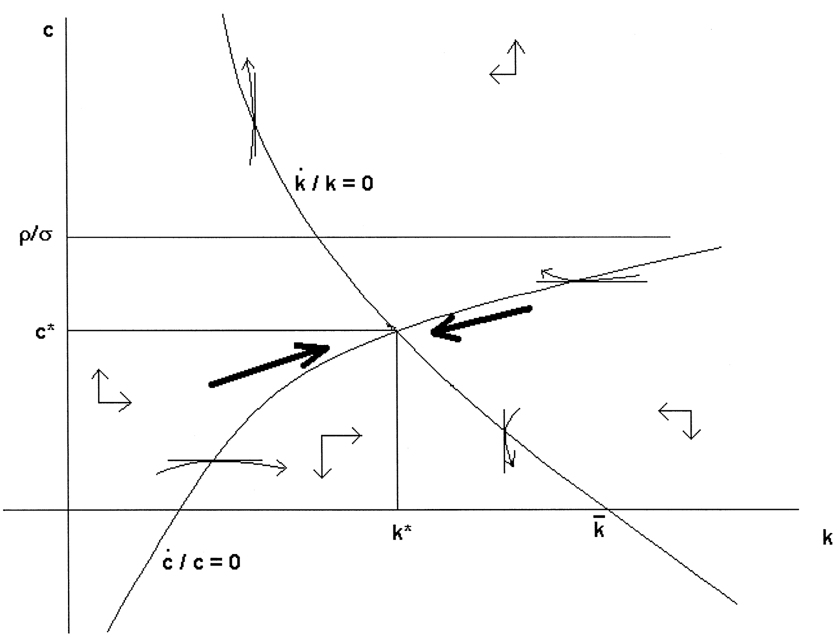

Fig. 6. Transitional dynamics (Case $\sigma < 1$).

Case 2: $\sigma < 1$

The situation is depicted in Fig. 6.

The intersection of the two curves occurs in the positive orthant. To see this, suppose that the reverse is the case. Let $\bar{k}$ denote the value of k for which $c = 0$ in Eq. (A.3). By assumption, $k^* > \bar{k}$. Substituting this value in Eq. (A.4),

$$-\frac{1-\sigma}{\sigma}B\left((1-\alpha)\bar{k}\right)^{\beta} + \frac{\rho}{\sigma} < 0.$$

From Eqs. (2), (6) and (7), the expression $B((1-\alpha)\bar{k})^{\beta}$ represents the value of g^{s} corresponding to $\bar{k}$. Since $k^* > \bar{k}$, it follows that $g^{\mathrm{m}} > B((1-\alpha)\bar{k})^{\beta}$, where $g^{\mathrm{m}} = (B(1-\alpha)k^*)^{\beta}$. Then

$$-\frac{1-\sigma}{\sigma}g^{\mathrm{m}} + \frac{\rho}{\sigma} < -\frac{1-\sigma}{\sigma}B\left((1-\alpha)\bar{k}\right)^{\beta} + \frac{\rho}{\sigma} < 0,$$

or,

$$\rho + (\sigma - 1)g^{\mathrm{m}} < 0,$$

which violates Eq. (20).

References

Aghion, P., Howitt, P., 1992. A model of growth through creative destruction. Econometrica 60, 323–351.

Aghion, P., Howitt, P., 1998. Endogenous Growth Theory. The MIT Press, Cambridge, MA.

Alesina, A., Rodrik, D., 1994. Distributive politics and economic growth. Quarterly Journal Economics 109 (2), 465–490.

Barro, R., 1990. Government spending in a simple model of endogenous growth. Journal of Political Economy 98 (5), S103–S125.

Cass, D., 1965. Optimum growth in an aggregative model of capital accumulation. Review of Economic Studies 32, 233–240.

Dasgupta, D., 1998. Lindahl Pricing, Nonrival Infrastructure and Endogenous Growth. Hokkaido University, Sapporo (mimeographed).

Futagami, K., Morita, Y., Shibata, A., 1993. Dynamic analysis of an endogenous growth model with public capital. Scandinavian Journal of Economics 95 (4), 607–625.

Lucas, R., 1988. On the mechanics of economic development. Journal of Monetary Economics 22 (1), 3–42.

Lucas, R., 1990. Supply side economics: an analytical review. Oxford Economic Papers 42, 293–316.

Phelps, E.S., 1961. The golden rule of accumulation: a fable for growthmen. American Economic Review 51, 638–643.

Rafzin, A., 1972. Investment in human capital and economic growth. Metroeconomica 24, 102–116.

Rebelo, S., 1991. Long-run policy analysis and long-run growth. Journal of Political Economy 99, 501–521.

Romer, P., 1986. Increasing returns and long-run growth. Journal of Political Economy 94 (5), 1002–1037.

Romer, P., 1990. Endogenous technical change. Journal of Political Economy 98 (5), S71–S102.

Samuelson, P.A., 1954. The pure theory of public expenditure. Review of Economics and Statistics 36, 387–389.

Solow, R.M., 1956. A contribution to the theory of economic growth. Quarterly Journal of Economics 70 (1), 65–94.

The India Infrastructure Report, 1996. NCAER, New Delhi.

Truong, T.P., 1993. Dynamics of Public Infrastructure and Industrial Productivity: The Role of Non-Rival Production Factor. School of Economics Discussion Paper# 93/21, The University of New South Wales.

Truong, T.P., 1993. Dynamics of Public Infrastructure and Industrial Productivity and Profitability: A Comment. School of Economics Discussion Paper# 93/22, The University of New South Wales.

Uzawa, H., 1965. Optimum technical change in an aggregative model of economic growth. International Economic Review 6, 19–31.

World Bank, 1994. Development report.