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Approximate reasoning approach to pattern recognition

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Abstract

Approximate reasoning approach to pattern recognition consists of linguistic rules tied together by means of two concepts: fuzzy implications and a compositional rule of inference. In this paper, first we study the applicability of different interpretations of fuzzy implication to pattern recognition problem and subsequently compare their performances over a set of synthetic data. Finally, we use the most applicable (according to our study) interpretations of implication for vowel recognition of three different Indian languages and we obtain very promising results.

Keywords: Approximate reasoning; Pattern recognition; Vowel recognition

1. Introduction

In real world, recognition and classification problems are faced with fuzziness that is connected with diverse facets of cognitive activity of the human being. An origin of sources of fuzziness is related to labels expressed in feature space as well as to labels of classes taken into account in classification procedures. Though a lot of scientific effort has already been dedicated to pattern recognition problems, especially classification procedures, still pattern recognition is confronted with a continuous challenge coming from a human being who can perform lot of extremely complex classification tasks by some sort of mental reasoning which cannot be represented, in straightforward way, through computer algorithm. But, fuzzy set provides a plausible tool for modelling and mimicking cognitive processes of the human reasoning, especially those concerning recognition aspects. Approximate reasoning, proposed by Zadeh is one such tool. So far we have seen many successful applications of the method of approximate reasoning [5, 10] to the design of fuzzy logic controllers in different fields [3]. But there is no such systematic study about the application of the method of approximate reasoning to the pattern recognition problems. Hence, in this paper, first of all we apply the method of approximate reasoning (under different interpretations of fuzzy implications and max–min compositional rule of inference) to classify a set of synthetic data and subsequently compare the performance (in terms of recognition score) of the method of approximate reasoning under different interpretations of implication. Finally, we use the most applicable (according to our study) method of approximate reasoning under *Mamdani/Gödelian/Gougen* interpretation of implication to the problem of vowel recognition of three different Indian languages and we obtain very promising results.

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2. Method of approximate reasoning

We shall consider the following form of inference [10]:

$$\begin{array}{l} \text{premise 1: If } X \text{ is } A \text{ then } y \text{ is } B \\ \text{premise 2: } X \text{ is } A' \\ \hline \text{Consequence: } Y \text{ is } B' \end{array}, \qquad (1)$$

where A, A' are fuzzy sets in U and B, B' are fuzzy sets in V. The consequence B' is deduced from premise 1 and premise 2 by taking the max–min composition $\circ$ of the fuzzy set A' and the fuzzy relation $A \to B$ obtained from the fuzzy implication "if A then B". That means, we get,

$$B' = A' \circ (A \to B),$$

$$\mu_{B'}(v) = \bigvee_u \{\mu_{A'}(u) \wedge \mu_{A \to B}(u, v)\}$$

If the fuzzy set A' is a singleton u_0, i.e. $\mu_{A'}(u_0) = 1$ and $\mu_{A'}(u) = 0$ for $u \neq u_0$, the consequence B' is simplified as

$$\begin{aligned} \mu_{B'}(v) &= \bigvee_u \{\mu_{A'}(u) \wedge \mu_{A \to B}(u, v)\} \\ &= \bigvee_{u(\neq u_0)} \{0 \wedge \mu_{A \to B}(u, v)\} \vee \{1 \wedge \mu_{A \to B}(u_0, v)\} \\ &= \mu_{A \to B}(u_0, v). \end{aligned} \qquad (2)$$

If the fuzzy implication $A \to B$ is represented by the direct product $A \times B$ of fuzzy sets A and B as in the case of Mamdani's method [5], B' is given as

$$\mu_{B'}(v) = \mu_A(u_0) \wedge \mu_B(v) \text{ at } A \to B = A \times B.$$

In Table 1 [6], for simplicity of demonstration, we list some selected interpretations of fuzzy implications $A \to B$ which will be used in the approximate reasoning approach to pattern recognition. It should be noted that the composition operator (e.g. max–min) is uniquely related with the way in which the individual rules are combined. That means, the max-t composition is linked with the implication operator induced by the t-norm [7]. In this sense, the list of interpretations of Table 1 should be infinite. But, our major objective is to establish the

Table 1
Some interpretations of Fuzzy implications $\mu_{A \to B}(u_0, v) = \mu_A(u_0) \to \mu_B(v)$ [6]

R_c: $\mu_A(u_0) \wedge \mu_B(v)$	Mamdani
R_p: $\mu_A(u_0) \cdot \mu_B(v)$	Larsen
R_{bp}: $0 \vee [\mu_A(u_0) + \mu_B(v) - 1]$	bounded product
R_{dp}: $\begin{cases} \mu_A(u_0), & \mu_B(v) = 1 \\ \mu_B(v), & \mu_A(u_0) = 1 \\ 0, & \mu_A(u_0)\mu_B(v) < 1 \end{cases}$	drastic product
R_a: $1 \wedge [1 - \mu_A(u_0) + \mu_B(v)]$	Zadeh's arithmetic rule
R_m: $[\mu_A(u_0) \wedge \mu_B(v)] \vee [1 - \mu_A(u_0)]$	Zadeh's maximum rule
R_b: $[1 - \mu_A(u_0)] \vee \mu_B(v)$	Boolean implication
R_s: $\begin{cases} 1, & \mu_A(u_0) \preceq \mu_B(v) \\ 0, & \mu_A(u_0) > \mu_B(v) \end{cases}$	Standard sequence
R_g: $\begin{cases} 1, & \mu_A(u_0) \preceq \mu_B(v) \\ \mu_B(v), & \mu_A(u_0) > \mu_B(v) \end{cases}$	Gödelian logic
R_Δ: $\begin{cases} 1, & \mu_A(u_0) \preceq \mu_B(v) \\ \mu_B(v)/\mu_A(u_0), & \mu_A(u_0) > \mu_B(v) \end{cases}$	Gougen logic
R^*: $1 - \mu_A(u_0) + \mu_A(u_0) \cdot \mu_B(v)$	Bandler logic
$R_\#$: $[1 - \mu_A(u_0) \vee \mu_B(v)] \wedge [\mu_A(u_0) \vee 1 - \mu_A(u_0)] \wedge [\mu_B(v) \vee 1 - \mu_B(v)]$	Bandler logic

effectiveness of the method of approximate reasoning in the field of pattern recognition. Hence, we just suitably picked up few interpretations of fuzzy implications in Table 1.

Now, we consider the following form of inference [5] in which a fuzzy conditional proposition "**if** ... **then** ..." contains two fuzzy propositions "X is A" and "Y is B" combined using the connective "and".

$$\begin{array}{l} \text{Premise 1: If } X \text{ is } A \text{ and } Y \text{ is } B \text{ then } Z \text{ is } C \\ \text{premise 2: } X \text{ is } A' \text{ and } Y \text{ is } B' \\ \hline \text{Consequence: } Z \text{ is } C' \end{array} \qquad (3)$$

where A, A' are fuzzy sets in U; B, B' are fuzzy sets in V and C, C' are fuzzy sets in W.

The consequence C' can be deduced from Premise 1 and Premise 2 by taking the max–min composition $\circ$ of a fuzzy set (A' and B') in $U \times V$ and a fuzzy relation (A and B) $\to C$ in $U \times V \times W$. That means, we get

$$C' = (A' \text{ and } B') \circ [(A \text{ and } B) \to C],$$

$$\mu_{C'}(w) = \bigvee_{u,v} \{[\mu_{A'}(u) \wedge \mu_{B'}(v)] \wedge [(\mu_A(u) \wedge \mu_B(v)) \to \mu_C(w)]\}. \qquad (4)$$

In the case of Mamdani's method R_c in Table 1, the fuzzy implication $[(A \text{ and } B) \to C]$ is translated into $\mu_A(u) \wedge \mu_B(v) \wedge \mu_C(w)$ by virtue of $a \to b = a \wedge b$. Thus, the consequence C' is given as

$$\mu_{C'}(w) = \bigvee_{u,v} \{[\mu_{A'}(u) \wedge \mu_{B'}(v)] \wedge [\mu_A(u) \wedge \mu_B(v) \wedge \mu_C(w)]\} \qquad (5)$$

Let $R_c(A, B; C) = (A \text{ and } B) \to C$, $R_c(A; C) = A \to C$ and $R_c(B; C) = B \to C$ be fuzzy implications by Mamdani's method R_c. Then, the consequence C' of Eq. (5) is reduced to

$$\mu_{C'}(w) = \bigvee_u \left\{\mu_{A'}(u) \wedge \mu_A(u) \wedge \mu_C(w) \wedge \bigvee_v [\mu_{B'}(v) \wedge \mu_B(v) \wedge \mu_C(w)]\right\}$$

$$= \bigvee_u \{\mu_{A'}(u) \wedge \mu_A(u) \wedge \mu_C(w) \wedge \mu_{B' \circ R_c(B;C)}(w)\}$$

$$= \mu_{A' \circ R_c(A;C)}(w) \wedge \mu_{B' \circ R_c(B;C)}(w).$$

Therefore, the consequence $C' = (A' \text{ and } B') \circ R_c(A, B; C)$ can be obtained as the intersection of $A' \circ R_c(A; C)$ and $B' \circ R_c(B; C)$ for Mamdani's implication R_c. That means, we get

$$C' = (A' \text{ and } B') \circ R_c(A, B; C)$$
$$= [A' \circ R_c(A; c)] \cap [B' \circ R_c(B; C)].$$

Similarly, we can have

$$(A' \text{ and } B') \circ [(A \text{ and } B) \to C]$$
$$= [A' \circ (A \to C)] \cap [B' \circ (B \to C)]\} \qquad (6)$$

for the fuzzy implications R_p, R_{bp} and R_{dp} in Table 1.

Note that R_a, R_b, R^*, R_s, R_g and R_Δ in Table 1, for which the equality $(a \wedge b) \to c = (a \to c) \wedge (b \to c)$ holds, satisfy the following [5],

$$(A' \text{ and } B') \circ [(A \text{ and } B) \to C]$$
$$= [A' \circ (A \to C)] \cup [B' \circ (B \to C)] \qquad (7)$$

If the fuzzy sets A' and B' are singletons in (3), i.e., $A' = u_0$ and $B' = v_0$, the consequence C' of (4) is represented as

$$\mu_{C'}(w) = \bigvee_{\substack{u(\neq u_0) \\ or\,(v \neq v_0)}} \{0 \wedge [\mu_A(u) \wedge \mu_B(v) \to \mu_C(w)]\} \wedge \{1 \wedge [(\mu_A(u_0) \wedge \mu_B(v_0)) \to \mu_C(w)]\}$$
$$= [\mu_A(u_0) \wedge \mu_B(v_0)] \to \mu_C(w). \qquad (8)$$

For example, in the case of R_c and R_a, we have consequences C' at $A' = u_0$ and $B' = v_0$ as follows:

$$R_c\!: \ [\mu_A(u_0) \wedge \mu_B(v_0)] \wedge \mu_C(w), \qquad (9)$$

$$R_a\!: \ 1 \wedge [1 - (\mu_A(u_0) \wedge \mu_B(v_0)) + \mu_C(w)]. \qquad (10)$$

Similar results can be obtained from other fuzzy implications in Table 1.

In the above discussion, the operation $\wedge$ (= min) is used as the meaning of "and". It is possible to introduce other operations, say, algebraic product $\bullet$ and more generally t-norms as "and".

As a generalized form of approximate reasoning we shall consider approximate reasoning with several fuzzy conditional proposition combined with "else".

Premise 1: If X is A_1 and Y is B_1 then Z is C_1 else
Premise 2: If X is A_2 and Y is B_2 then Z is C_2 else
$\vdots$ (11)
Premise n: If X is A_n and Y is B_n then Z is C_n
Premise $n+1$: If X is A' and Y is B'

Consequence: Z is C'.

If we interpret "else" as union (U) which is valid for the interpretations of fuzzy implications R_c, R_p, R_{bp} and R_{dp} in Table 1, we can deduce the consequences C' (refer Eq. (6)) as

$$\begin{aligned} C' &= (A' \text{ and } B') \circ [((A_1 \text{ and } B_1) \to C_1) \cup \cdots \cup ((A_n \text{ and } B_n) \to C_n)] \\ &= [(A' \circ A_1 \to C_1) \cap (B' \circ B_1 \to C_1)] \\ &\quad \cup \cdots \cup [(A' \circ A_n \to C_n) \cap (B' \circ B_n \to C_n)] \\ &= C'_1 \cup C'_2 \cup \cdots \cup C'_n, \end{aligned} \tag{12}$$

whereas for the interpretations of fuzzy implications R_a, R_m, R_b, R^*, $R_{\#}$ and R_Δ in Table 1, "else" in (11) is interpreted as intersection ($\cap$). Thus, the consequences C' for these fuzzy implications are defined as

$$\begin{aligned} C' &= (A' \text{ and } B') \circ [((A_1 \text{ and } B_1) \to C_1) \cap \cdots \cap ((A_n \text{ and } B_n) \to C_n)] \\ &\subseteq [(A' \circ A_1 \to C_1) \cup (B' \circ B_1 \to C_1)] \\ &\quad \cap \cdots \cap [(A' \circ A_n \to C_n) \cup (B' \circ B_n \to C_n)]. \end{aligned}$$

It is noted that the consequence C' is not equal to but contained in the intersection of fuzzy inference results $[(A' \circ A_i \to C_i) \cup (B' \circ B_i \to C_i)]\ \forall i$. However, for simplicity of calculation C' will be represented as

$$C' = C'_1 \cap C'_2 \cap \cdots \cap C'_n. \tag{13}$$

3. Statement of the problem

There are two main approaches to pattern recognition, namely the decision theoretic and the syntactic. Since, the approximate reasoning approach to pattern recognition is similar to the decision theoretic method of pattern recognition, we will first briefly describe the basic concept of the decision theoretic approach to pattern recognition and then try to establish the similarity between the decision theoretic approach and the approximate reasoning approach to pattern recognition.

Under decision theoretic approach, each pattern is represented by a vector of features. The feature space is divided into a number of regions, each of which represents a prototype pattern or a cluster of patterns. A decision function maps the given patterns to previously determined regions.

In the approximate reasoning approach to pattern recognition each element of the feature vector is represented by the fuzzy linguistic variable instead of a real number. For instance, suppose we have a (2×1) feature vector $F = (F_1, F_2)^T$, T is transpose where F_1 is the first formant frequency of a speech signal and F_2 is the second formant frequency. In the decision theoretic approach to pattern recognition, F_1 and F_2 are two features and are represented by, say, 800 Hz and 550 Hz. Whereas in the approximate reasoning approach to pattern recognition F_1 and F_2 are represented by the fuzzy

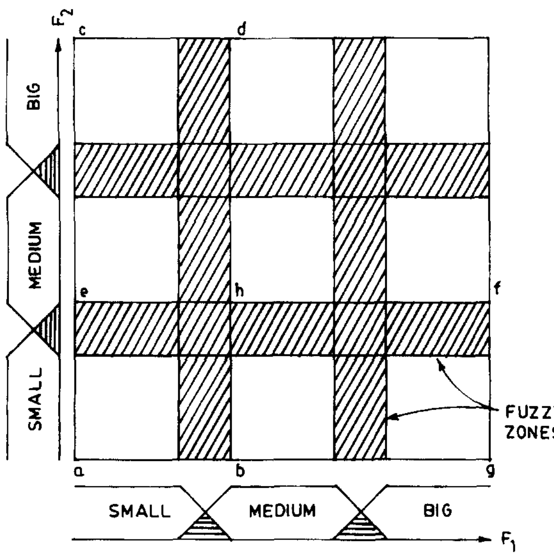


Fig. 1. Fuzzy partitioning of pattern space.

linguistic variables, e.g., F_1 is **small** and F_2 is **medium**. The elements of the feature vector which are represented by fuzzy linguistic variables are characterized by their membership functions which fuzzily partition the feature space as shown in Fig. 1. These elements of the feature vector actually constitute the antecedent part of the fuzzy implication. The consequent part of the fuzzy implication represents the possibility of occurrence of each class on the fuzzily partitioned feature space. Thus, fuzzy **If–Then** rules are generated which map the given patterns to previously determined regions.

The process of recognition is usually divided into two steps, learning and classification which sometimes overlap. The main stages in the learning process are feature extraction, feature selection, clustering and determination of the appropriate fuzzy **If–Then** rules which will constitute a decision function.

The main stages in classification are: extraction of the selected set of features, application of **If–Then** rules and decision making based on the results of the application of the **If–Then** rules. In the classification process, first an unknown pattern is presented to the system, then a set of predetermined features are extracted from the pattern. Finally, a set of **If–Then** rules determines the possibility of occurrence of each class on the feature space.

Fig. 2 schematically represents the approximate reasoning approach to pattern recognition.

4. Formulation of the problem

First of all, at the learning stage, we fuzzify the selected features by fitting any standard distribution like triangular/trapezoidal/bell shaped or any non-standard distribution as shown in the present case (see Fig. 4(a)–(e)). Depending upon the nature of the shape of the distribution we fuzzily partition the feature space and generate the **If–Then** rules to classify the patterns. After the initial generation of the **If–Then** rules, we test the validity of the rules by classifying some known patterns. If we get satisfactory classification we proceed further; otherwise, we tune the rules by changing the shape of the

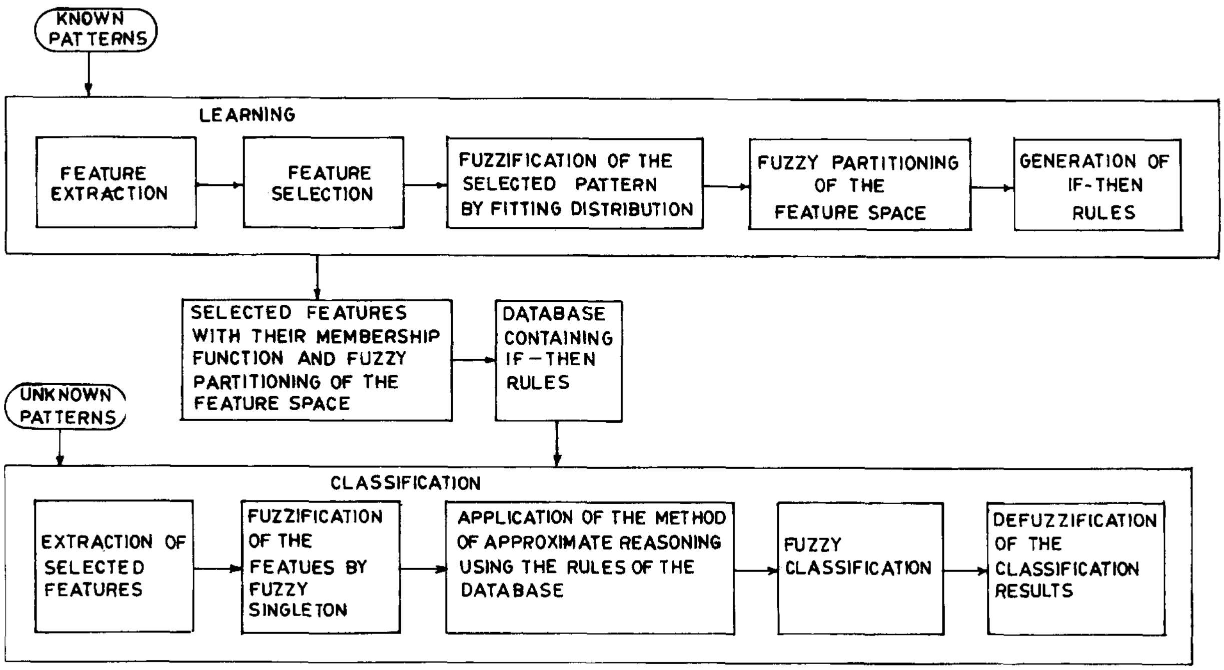


Fig. 2. Schematic representation of approximate reasoning approach to pattern recognition.

distribution. As off-line generation of fuzzy **If–Then** rules for pattern classification basically deals with a static situation, the tuning of the grade of the membership functions of the antecedent part and the consequent part does not take a very long time which is a very common phenomenon for tuning fuzzy control rules of the dynamical system. In the present situation, tuning of the consequent part of the rule is primarily guided by the seed points of the clusters of the features and that of the antecedent part of the rule is guided by the error generated in the classification. The possibility of occurrence of each class of the fuzzily partitioned feature space is calculated as follows.

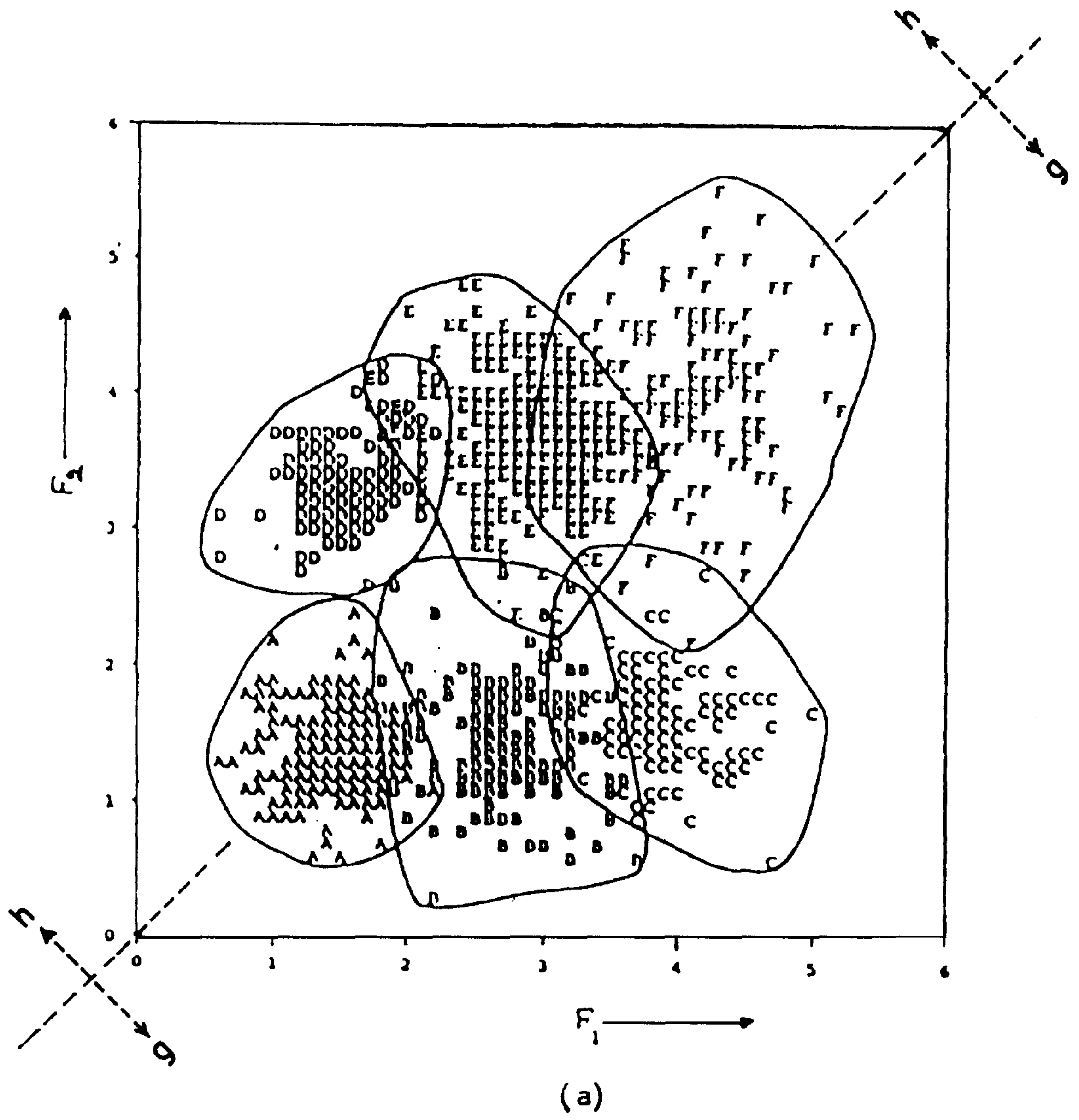
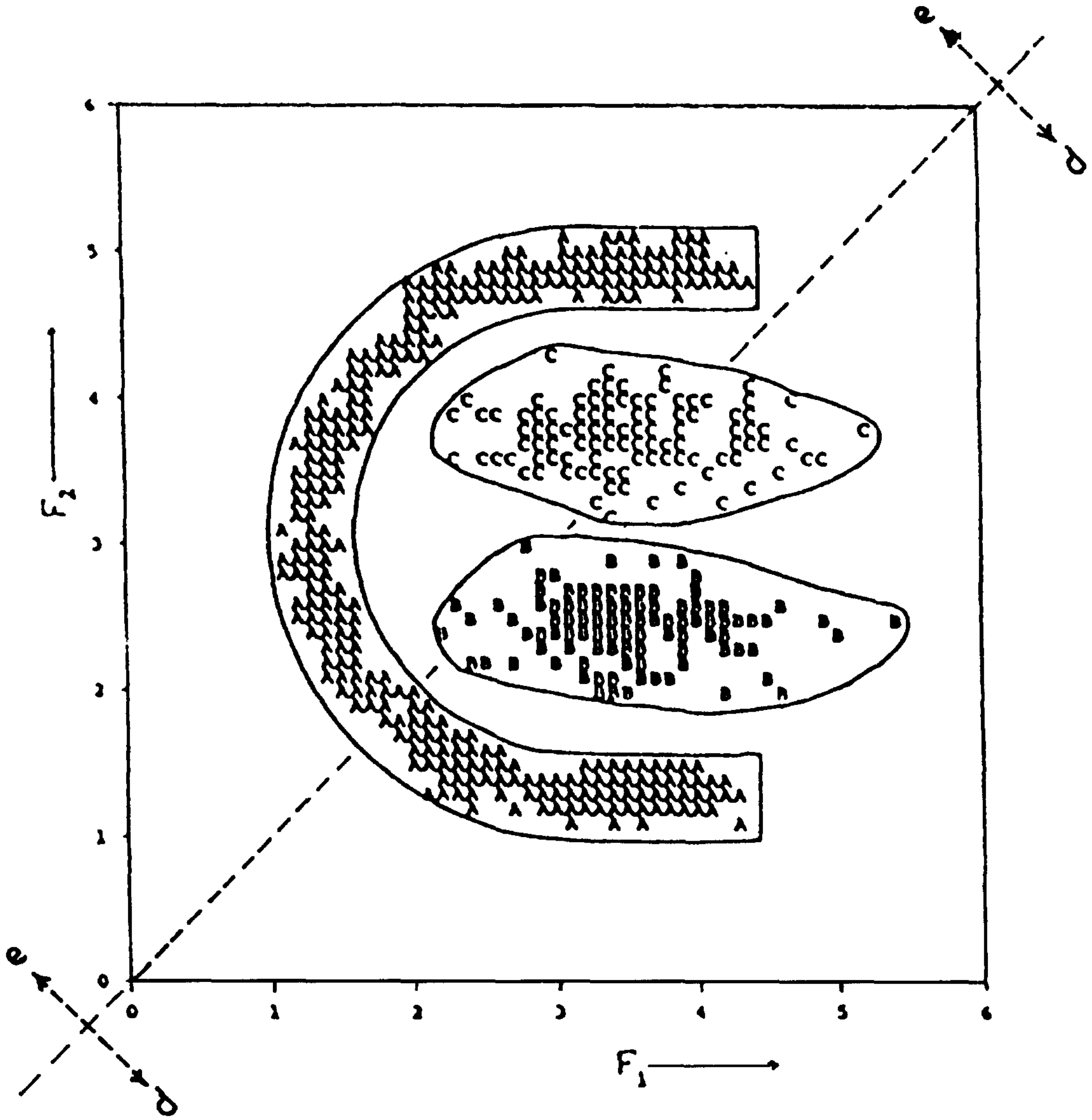


Fig. 3(a).

The class memberships have been chosen as the ratio between individual population and total population of a particular cluster. For example, if we write the rule as "if F_1 is **small** and F_2 is **small** then 0.8/a + 0.2/b + 0.1/c + 0.01/d", then it means that the possibility of occurrence of class "a" is the highest, then class "b" and so on. It is to be mentioned in this context, that we wanted to map the entire feature space, hence classes which do not appear in the figures evolved, e.g. in Fig. 3(a), we have considered classes like 'g' and 'h', where 'g' is class representation for the empty portion at the bottom right corner of the feature space and 'h' is the class representation for the empty portion at the top left corner of the same feature space. The idea remains same for all the figures, i.e., extra two



(b)

Fig. 3(b).

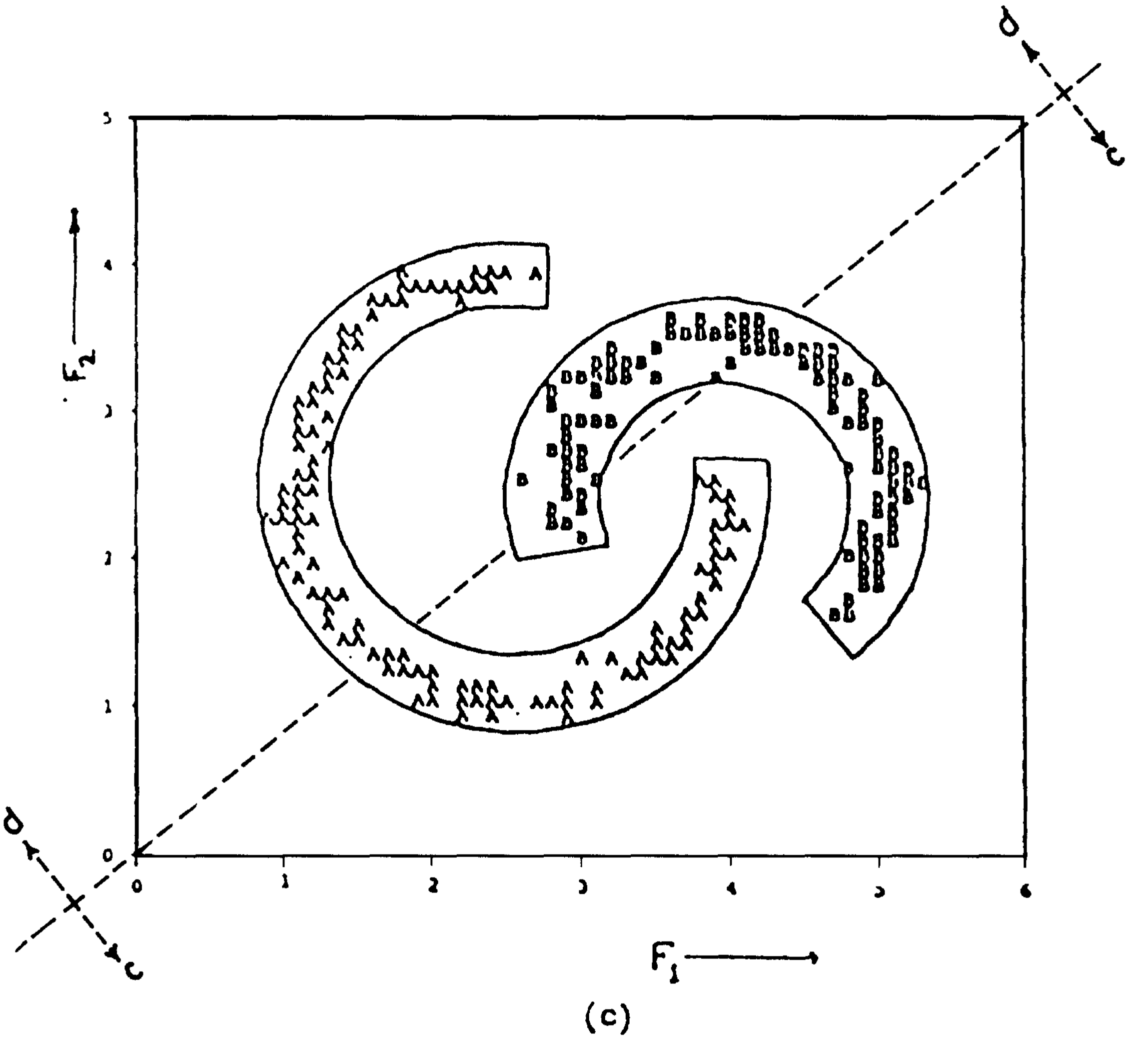


Fig. 3(c).

classes which do not appear in the figures appear in the rules.

At the classification stage, the selected features are fuzzified using the concept of fuzzy singleton as discussed in Section 2. The classification process is performed using Eqs. (12) and (13). The classification results produce the possibility of occurrence of each pattern at different fuzzily partitioned region. At the time of taking nonfuzzy decision out of this fuzzy classification (i.e. defuzzification) we can go by selecting class having highest possibility value. In case of a tie situation, which normally occurs for the patterns lying in the fuzzy zone (see Fig. 1), we have to state the equal possibility of a pattern to belong to both of the classes. And such a conclusion is quite natural which normally does not exist in conventional classification approach. In some cases, patterns in the fuzzy zones are classified with "*almost equal*" possibility of occurrence at more than one class. If such situation is treated as tie situation mentioned above, we have to select an appropriate threshold which entirely depends on the need of the problem.

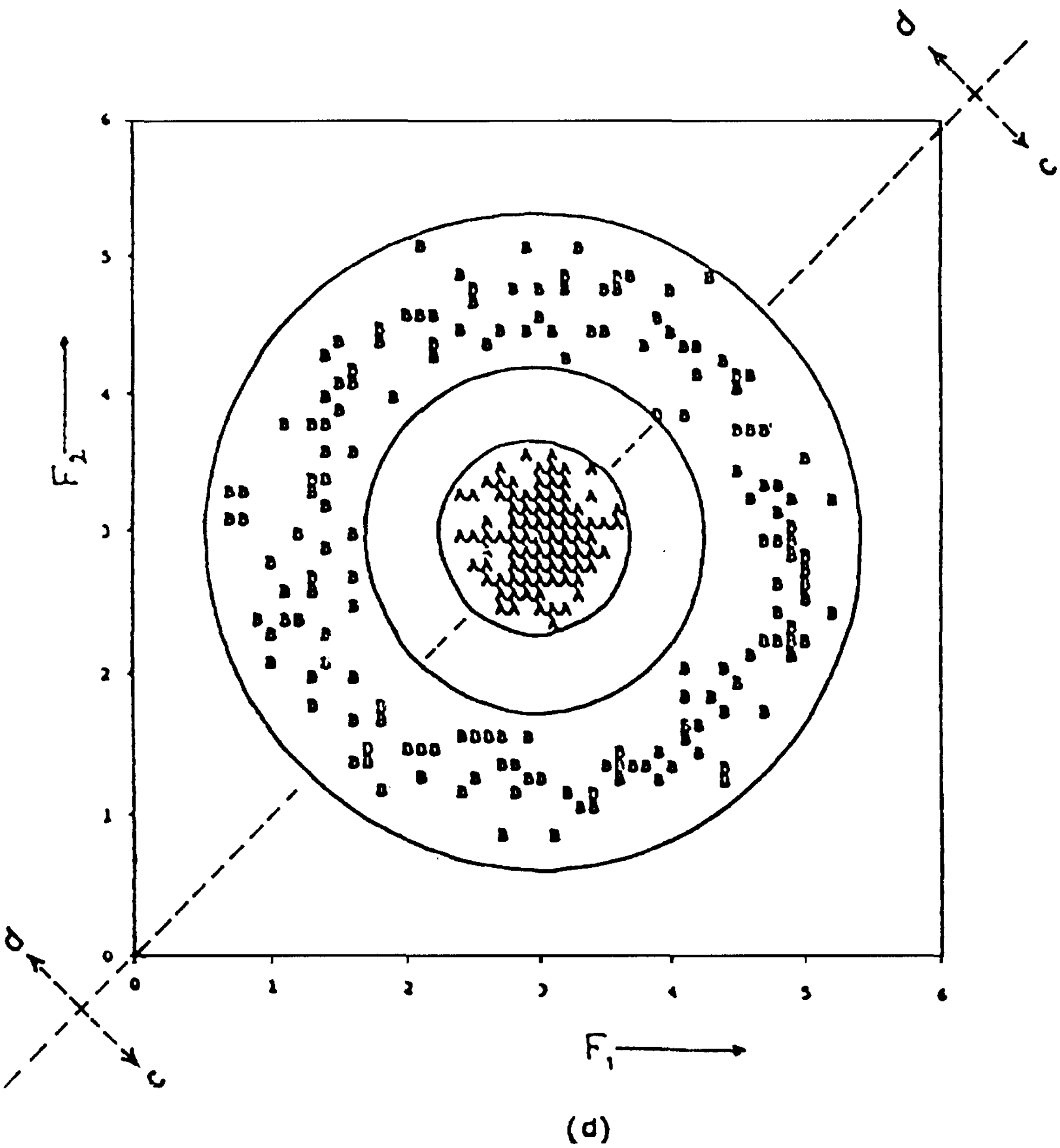


Fig. 3(d).

We have tested our proposed scheme on a set of synthetic data and some real-life vowel recognition problem which are discussed in the following sections.

5. Numerical examples

To test the effectiveness of the proposed scheme as shown in Fig. 2, simulations were run for

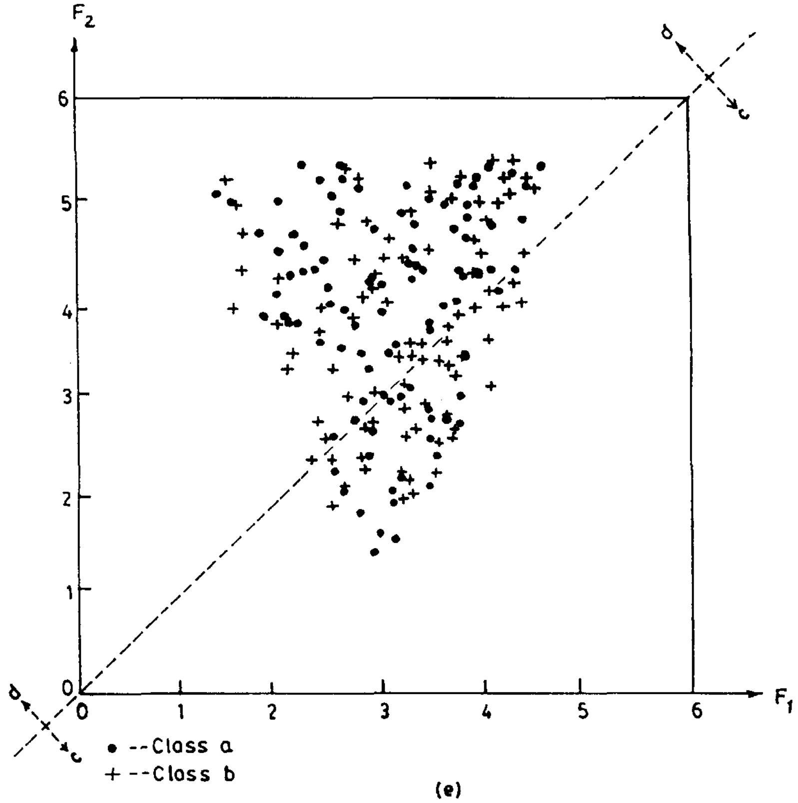


Fig. 3(e).

Figs. 3(a)–(e). The data sets of Figs. 3(a)–(e) are synthetic. The rules and the distribution patterns are shown in Fig. 4(a)–(e). The recognition scores are found to be quite satisfactory in all the cases. In Table 2 and Table 3, which are self-explanatory in nature, we have produced a comparative study on the performance of the proposed scheme under different interpretations of implications stated in Table 1. At the time of calculating the recognition score, we have considered exhaustive data for each figure. For further improvement in the recognition score, we consider classification at the fuzzy zone and obtain significantly good results. But, for brevity of representation, we do not mention them on separate tables. Instead, we apply this experience of improvement in recognition score under fuzzy-zone classification on real data considered in Section 6 where we explicitly mention the improved scores.

6. Applications

After achieving satisfactory results on synthetic set of data, we picked up Mamdani's interpretation of implication, implication using Gödelian logic and implication using Gougen logic for vowel recognition problems of three Indian languages namely Telegu, Assamese and Bengali. In the following subsections, we will briefly discuss the experimental procedures for extraction of vowels of each language and the recognition procedures using the method of approximate reasoning.

6.1. *Experiment I with Telegu vowel*

A number of discrete phonetically balanced speech samples for the Telegu vowels in *CNC* (*Consonant–Vowel nucleus–Consonant*) form were

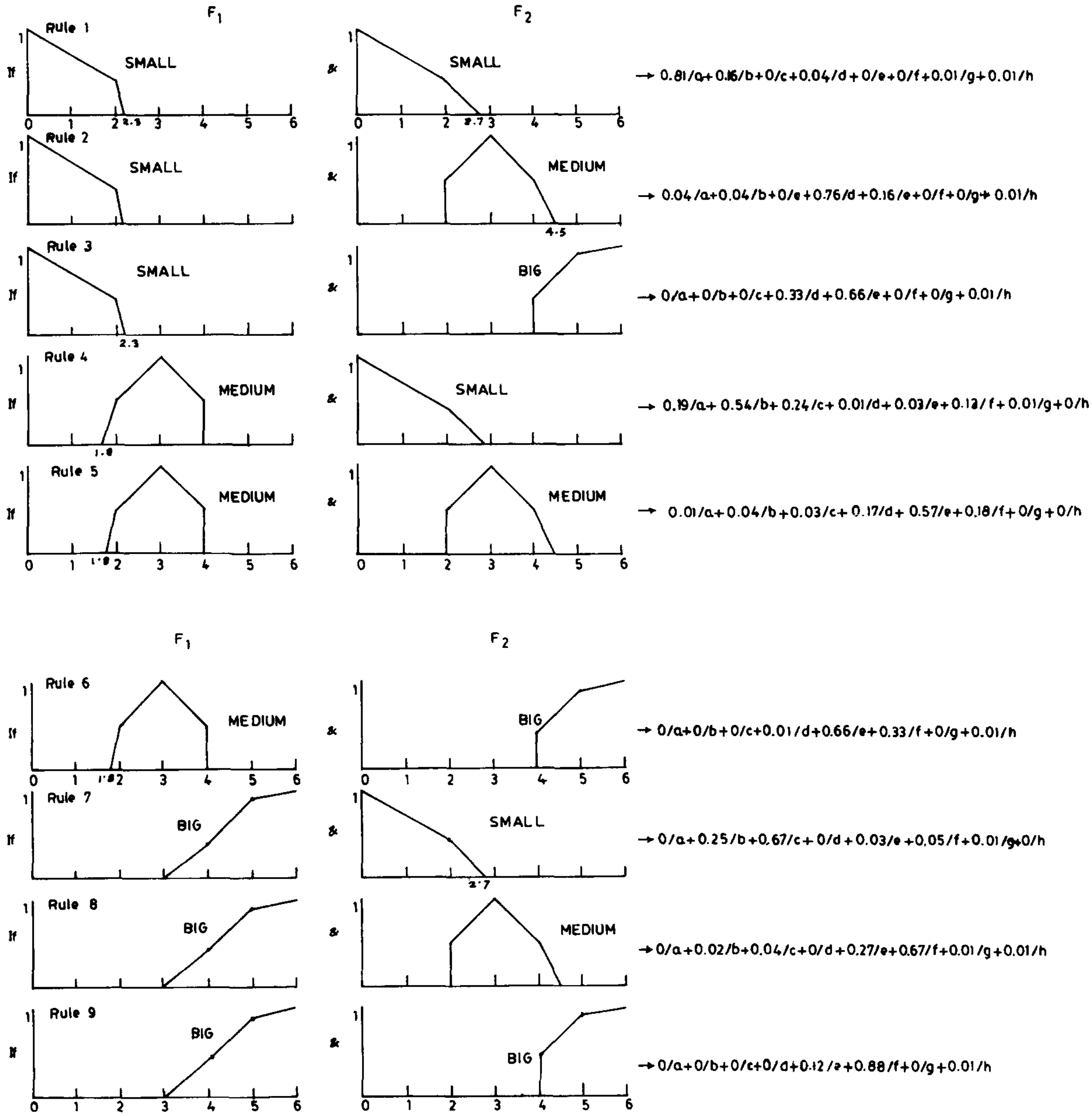
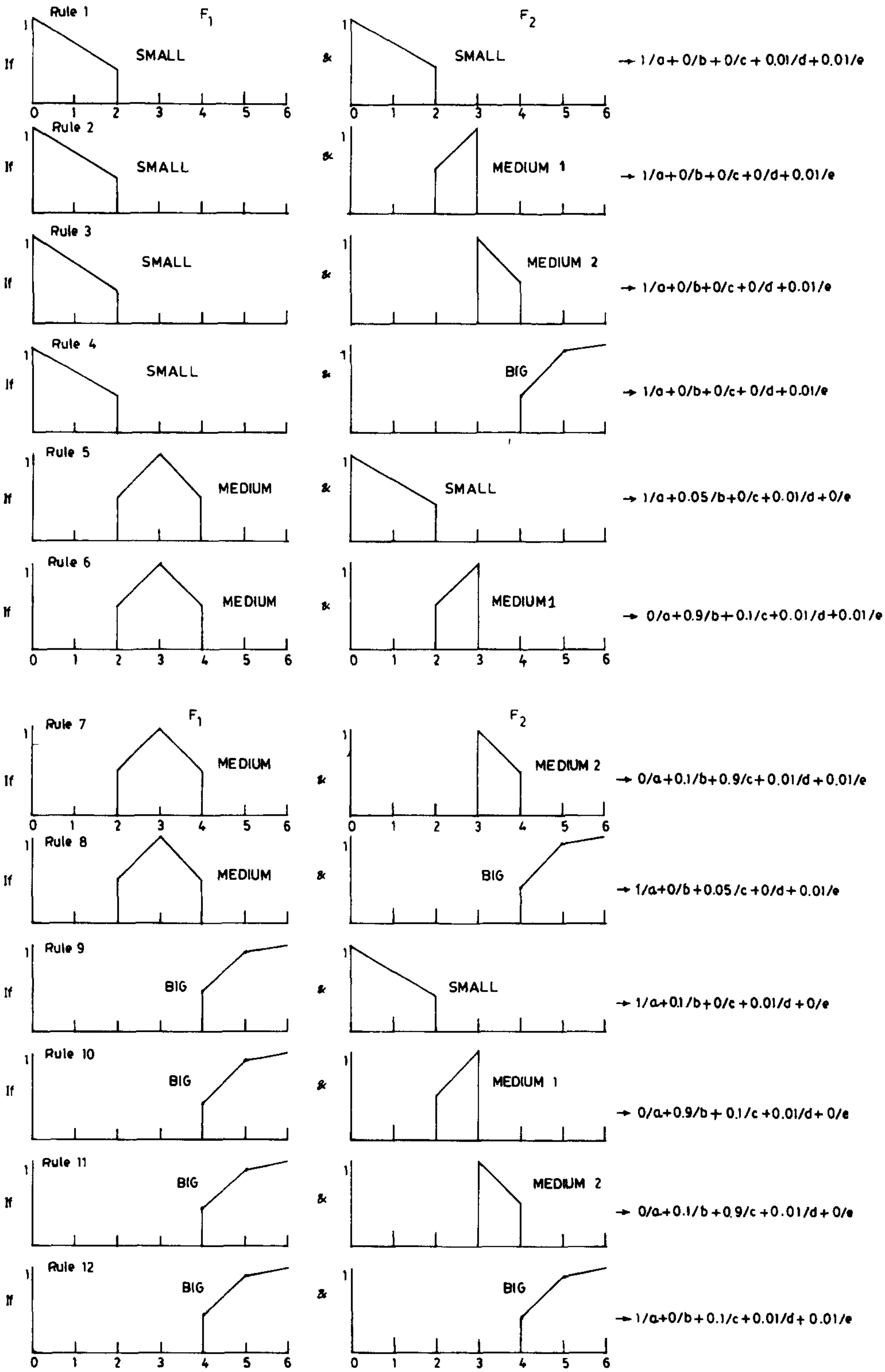


Fig. 4(a). Fuzzy If–Then rules for Fig. 3(a).

selected. CNC combination is taken because the form of consonants connected to a vowel is responsible for influencing the role and quality of vowels. These speech units were recorded by five informants on an AKAI-type recorder. The spectrographic analysis has been done on Kay Sonograph Model 7029-A which is a very standard audio frequency spectrum analyser that produces a permanent record of the spectra of any complex waveform in the range of 5 Hz to 16 kHz. For the present study of


F1
F2
Rule 1 If SMALL & SMALL → 1/a+0/b+0/c+0.01/d+0.01/e
Rule 2 If SMALL & MEDIUM 1 → 1/a+0/b+0/c+0/d+0.01/e
Rule 3 If SMALL & MEDIUM 2 → 1/a+0/b+0/c+0/d+0.01/e
Rule 4 If SMALL & BIG → 1/a+0/b+0/c+0/d+0.01/e
Rule 5 If MEDIUM & SMALL → 1/a+0.05/b+0/c+0.01/d+0/e
Rule 6 If MEDIUM & MEDIUM1 → 0/a+0.9/b+0.1/c+0.01/d+0.01/e
Rule 7 If MEDIUM & MEDIUM 2 → 0/a+0.1/b+0.9/c+0.01/d+0.01/e
Rule 8 If MEDIUM & BIG → 1/a+0/b+0.05/c+0/d+0.01/e
Rule 9 If BIG & SMALL → 1/a+0.1/b+0/c+0.01/d+0/e
Rule 10 If BIG & MEDIUM 1 → 0/a+0.9/b+0.1/c+0.01/d+0/e
Rule 11 If BIG & MEDIUM 2 → 0/a+0.1/b+0.9/c+0.01/d+0/e
Rule 12 If BIG & BIG → 1/a+0/b+0.1/c+0.01/d+0.01/e
0 1 2 3 4 5 6

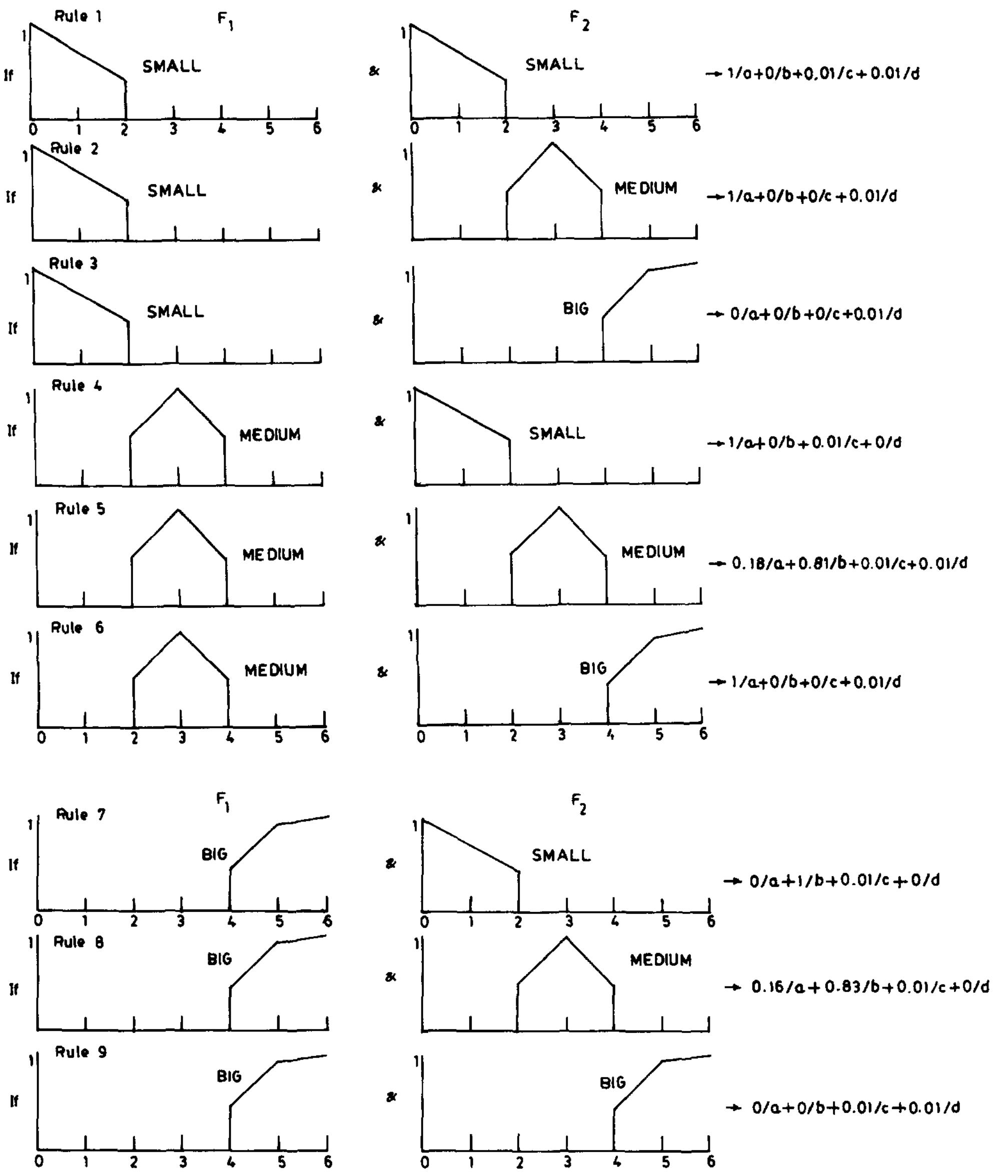


Fig. 4(c). Fuzzy If–Then rules for Fig. 3(c).

vowel, the spectrographic display of frequency vs. time (see Fig. 5) has been done for 800 Telegu words uttered by three male informants in the age group of 25 to 30 years chosen from 15 educated persons.

◄———

Fig. 4(b). Fuzzy If–Then rules for Fig. 3(b).

The total bandwidth of the system is 80 Hz to 8 kHz with a resolution of 300 Hz.

The experiment deals with the formant frequencies at the steady state of the Telegu vowels and their variations in different consonantal context and for different speakers. The average positions (see Table 4) of different Telegu vowels with respect

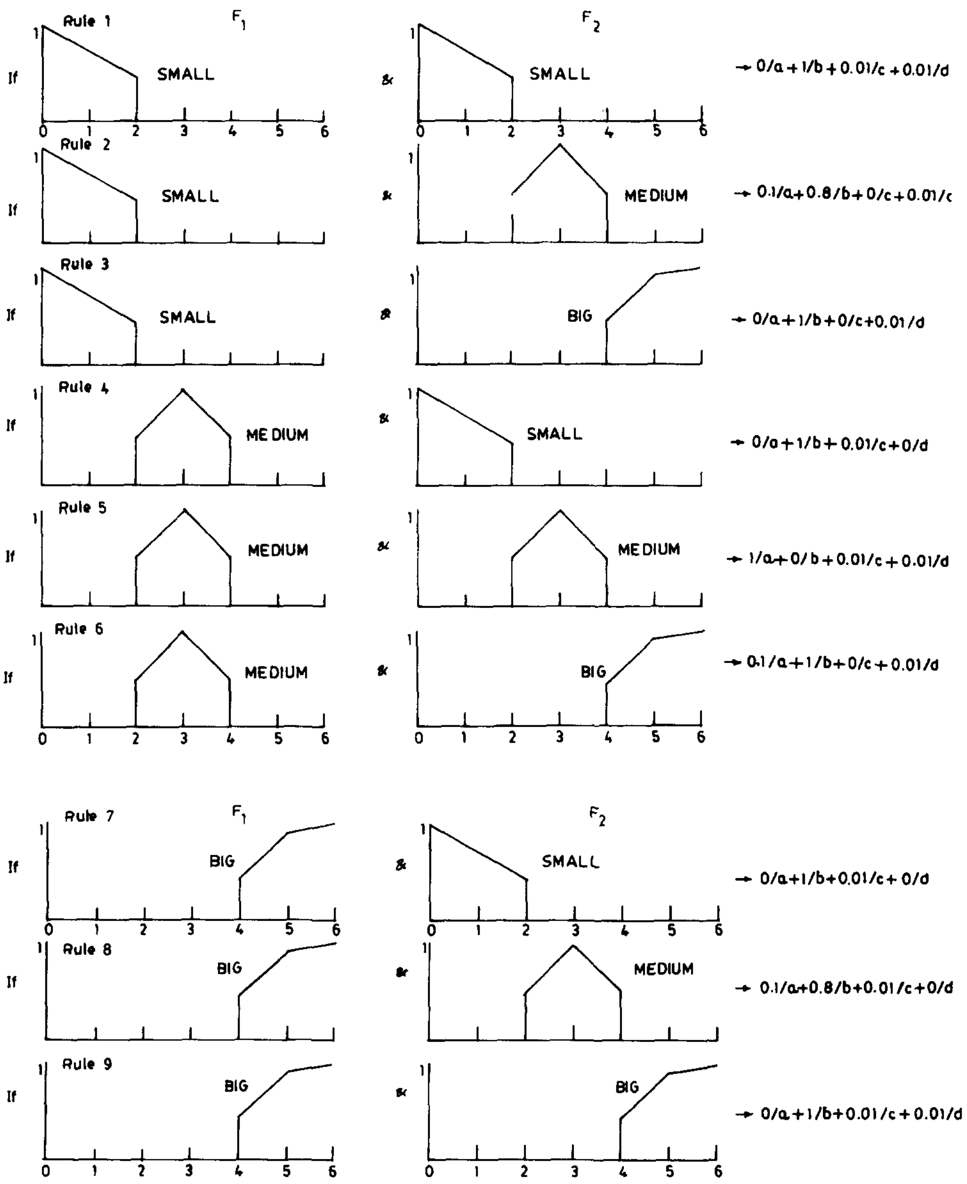


Fig. 4(d). Fuzzy If–Then rules for Fig. 3(d).

to cardinal vowels and their distribution in F_1–F_2 frequency planes are considered (see Fig. 6).

6.1.1. Segmentation procedures

The purpose of segmentation is to determine the vowel boundaries and the boundary of the steady state. For the determination of vowel boundaries the segmentation procedure should satisfactorily solve the problems of determination of vowel boundary in relation to stops, fricatives, affricates, laterals in voiced/unvoiced, aspirated/unaspirated as well as their combined manners. The boundaries in these three different situations need to be defined first.

1. *Vowel in combination with stop consonants*

The unaspired stop in initial position appears as spike in the spectrogram and this energy of plosion

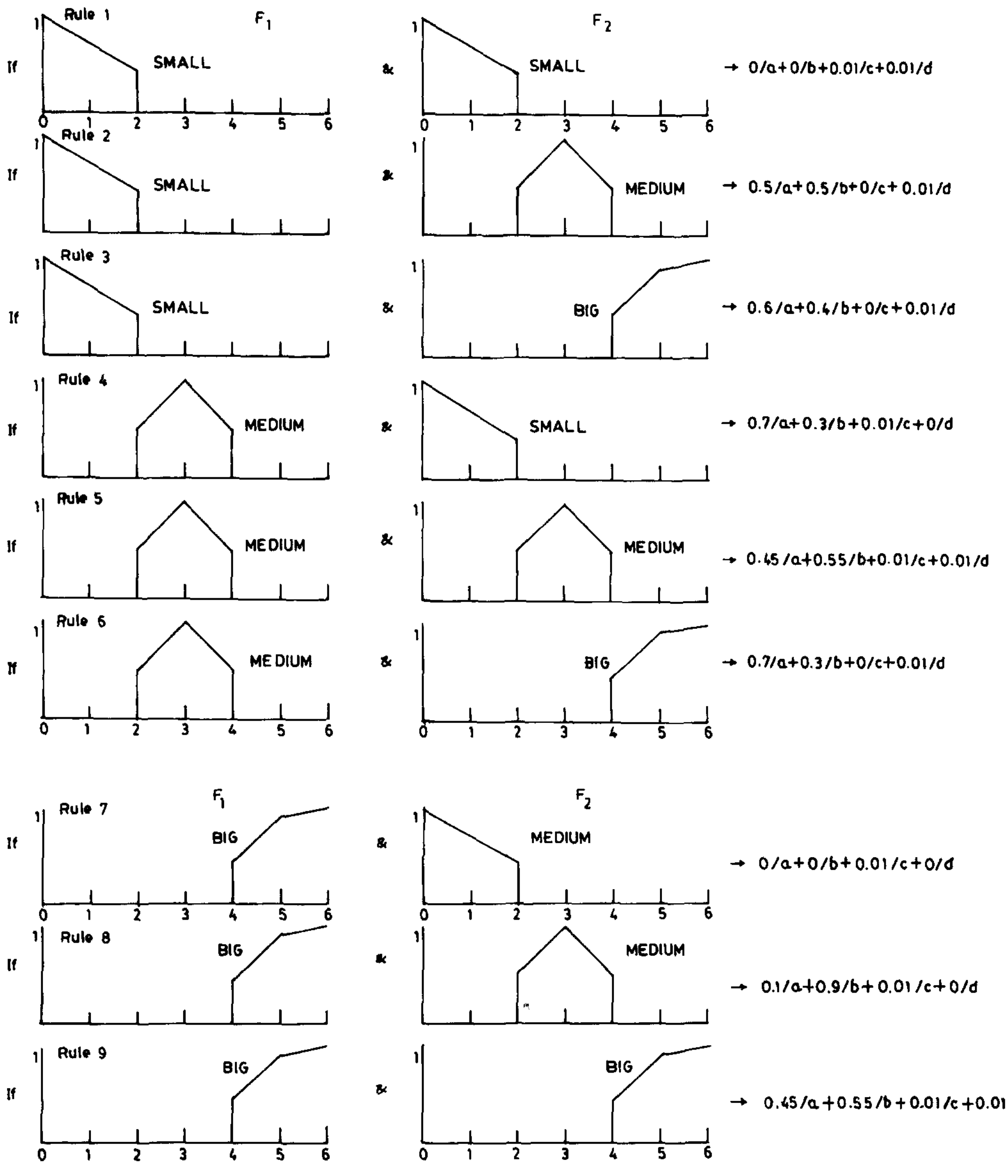


Fig. 4(e). Fuzzy If–Then rules for Fig. 3(e).

will clearly demarcate stop consonant and the adjacent vowel formants (Fig. 5). For aspirated stop in the initial position the onset of voicing is taken to be the boundary.

For the stop consonant in the final position, the absence of all vowel formants with the start of occlusion period of the consonant will indicate the termination of the vowel (Fig. 5).

2. *Vowel in combination with fricatives and affricates*

The fricatives with a wide-band continuous energy spectrum of low intensity can be easily segmented from the narrow band of much stronger intensity of vowel formants. The line separating these two distinctly separate spectral distributions is the vowel boundary for fricatives in the initial

Table 2
Recognition scores (in %) where "and" of the antecedent part of the rule is interpreted as "min"

	Fig. 3(a)						Fig. 3(b)			Overall score	
Type of operator	a	b	c	d	e	f	a	b	c	Fig. 3(a)	Fig. 3(b)
R_c	89.6	89.7	35.4	78.6	99.3	58.2	99.6	100	92	78.5	98.1
R_p	93.9	86.3	36.7	87.6	98.6	61.7	99.6	100	92	80.5	98.1
R_{bp}	93.9	81.2	46.8	87.6	98.6	71.3	100	94.8	79	82.4	94.6
R_{dp}	89.6	89.7	35.4	78.6	99.3	58.2	99.6	100	92	78.5	98.1
R_a	89.6	88.8	35.4	80.8	98.6	62.6	99.6	100	92	79.3	98.1
R_m	95.6	65.8	22.7	49.4	78.8	33.9	100	82.4	73	61.2	90.9
R_b	89.6	78.6	35.4	78.6	99.3	58.2	99.6	100	92	76.6	98.1
R_s	100	27.4	34.1	66.3	35.1	52.2	100	93.8	79	52	94.4
R_g	99.1	83.7	35.4	92.1	94.7	75.6	100	100	92	82.9	98.3
R_Δ	89.6	88.8	35.4	80.9	97.4	70.4	100	100	92	80.4	98.3
R^*	89.6	89.7	35.4	79.7	98.6	60	99.6	100	92	78.8	98.1
$R_\#$	95.6	65.8	22.8	49.4	78.8	33.9	100	82.5	73	61.2	90.9

	Fig. 3(c)		Fig. 3(d)		Fig. 3(e)		Overall score		
Type of operator	a	b	a	b	a	b	Fig. 3(c)	Fig. 3(d)	Fig. 3(e)
R_c	79.8	100	100	98.1	53.1	69.1	89.3	98.8	60.6
R_p	79.8	100	100	98.1	47.8	39.2	89.3	98.8	43.8
R_{bp}	82.4	100	100	98.1	55.3	64.3	90.7	98.8	59.5
R_{dp}	79.8	100	100	98.1	50	72.6	89.3	98.8	60.6
R_a	82.4	100	100	98.1	55.3	64.2	90.7	98.8	59.5
R_m	85.9	86.1	100	71.1	57.4	54.8	86.1	81.2	56.1
R_b	79.8	100	100	98.1	53.2	64.3	89.3	98.8	60.6
R_s	82.4	71.3	100	91.4	89.4	14.3	77.2	94.4	53.9
R_g	80.7	100	100	98.1	55.3	64.3	89.8	98.8	59.6
R_Δ	80.7	100	100	98.1	55.3	64.3	89.8	98.8	59.6
R^*	79.8	100	100	98.1	52.1	67.8	89.3	98.8	59.5
$R_\#$	86	86.1	100	71.2	36.2	54.8	86.1	81.2	45

and also in the final position. The line of demarcation for affricates in initial and final position is same as in fricative and stop consonants, respectively.

3. *Vowel in combination with liquids*

The segmentation problem becomes more complex for the liquid and vowel combination. The liquids possess a formant-like structure very similar to vowel formants and thus create real confusion. But, careful observation will reveal that the formant structure of liquids is less intense with a much lesser degree of transition. These characteristic differences are used for determining the vowel boundary in this case.

6.1.2. Measurement procedure

The steady state of the vowel is that part on the record in which all formants lie parallel to the time axis (Fig. 5). The transition is depicted by the inclined formant patterns (Fig. 5).

The exact point of inflection is difficult to locate in the records. This can be done very satisfactorily by tracing the central line for each formant band. Once these points are located for all available formants, the steady state of the vowel is taken to be the shortest horizontal span for all the formants.

The formant frequencies are measured from the baseline (i.e. zero-line) of the spectogram to the central line of formant bands where the formant is

Table 3
Recognition scores (in %) where "and" of the antecedent part of the rule is interpreted as "algebraic product"

Type of operator	Fig. 3(a) a	b	c	d	e	f	Fig. 3(b) a	b	c	Overall score Fig. 3(a)	Overall score Fig. 3(b)
R_c	89.7	89.7	32.9	80.9	98.6	55.6	99.7	100	92	77.9	98.1
R_p	93.9	86.3	35.4	87.6	98.6	60	99.7	100	92	80	98.1
R_{bp}	95.7	54.7	21.5	84.3	91.4	66.1	100	94.8	79	72.1	68.6
R_{dp}	89.7	89.8	35.4	78.7	99.3	58.3	99.7	100	92	78.6	98.1
R_a	89.7	89.8	32.9	80.9	98.7	64.3	99.7	100	92	79.5	98.1
R_m	100	39.3	3.7	21.3	61.6	15.6	100	52.6	38	44.2	77.6
R_b	95.7	53.8	20.3	78.7	90.8	51.3	99.7	100	92	68.4	98.1
R_s	98.3	54.7	43.1	74.2	57.6	56.5	100	93.8	79	63.3	94.4
R_g	98.3	86.3	32.9	91.1	97.4	72.2	100	100	92	82.7	98.4
R_Δ	89.7	89.8	32.9	80.9	98.1	66.1	100	100	92	79.7	98.4
R^*	89.7	89.8	35.4	79.8	98.7	60	99.7	100	92	78.9	89.3
$R_\#$	100	39.3	3.8	21.4	61.6	15.7	100	52.6	38	44.3	77.7

Type of operator	Fig. 3(c) a	b	Fig. 3(d) a	b	Fig. 3(e) a	b	Overall score Fig. 3(c)	Overall score Fig. 3(d)	Overall score Fig. 3(e)
R_c	79.8	100	100	98.1	65.9	54.7	89.3	98.8	60.7
R_p	79.8	100	100	98.1	52.1	67.9	89.3	98.8	59.6
R_{bp}	82.5	100	100	98.1	66	52.4	90.7	98.8	59.6
R_{dp}	79.8	100	100	98.1	50	72.7	89.3	98.8	60.7
R_a	82.5	100	100	98.1	66	51.2	90.7	98.8	59
R_m	100	71.3	100	36.2	69.2	41.6	86.5	58.4	56.2
R_b	79.8	100	100	98.1	66	53.6	89.3	98.8	60.2
R_s	82.5	85.2	100	94.5	87.2	19.1	83.7	96.4	55.1
R_g	80.7	100	100	98.2	66	52.4	89.8	98.8	59.6
R_Δ	80.7	100	100	98.2	66	52.4	89.8	98.8	59.6
R^*	79.8	100	100	98.2	52.2	67.9	89.3	98.8	59.6
$R_\#$	100	71.3	100	36.2	19.2	41.7	86.6	58.4	29.8

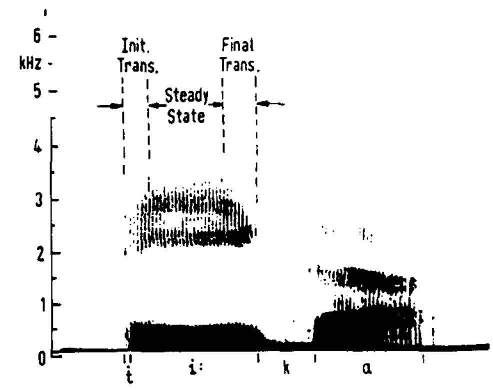


Fig. 5. Spectrogram of the Telegu word [ti:ka].

Table 4
Average formant frequencies of Telegu vowels

Phonetic symbol	F_1 (Hz)	F_2 (Hz)	F_3 (Hz)
/∂/	606	1473	2420
/a:/	710	1240	2400
/i⊢/	365	2116	2757
/i:/	325	2260	2836
/u⊣/	370	1066	2500
/u:/	348	923	2543
/e⊢/	517	1796	2633
/e:/	470	1883	2657
/o⊣/	476	1133	2630
/o:/	486	1000	2540
/ae/	575	1744	2700

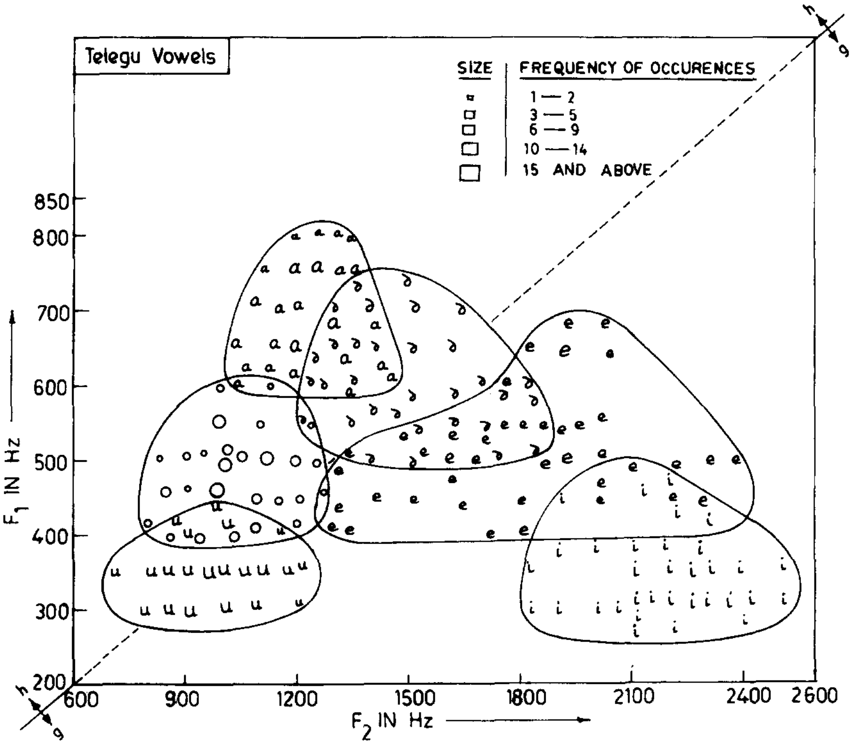


Fig. 6. Telegu vowels in the F_1–F_2 plane.

in a steady state. The scale used for this measurement is derived from the calibrated tune of 500 Hz recorded on each and every spectrogram. One small division of the scale is equal to 20 Hz. A rechecking of 5% samples revealed that formant frequencies have been recorded within an accuracy of 10 Hz. For every 50 spectrograms two fine marker recordings, one at the beginning and one at the end were taken. The scale for duration was done by taking an average of these two recordings. However, throughout the whole recording non-cognizable differences between these two recordings were observed. In a few cases, for particularly fast informants, it has been noted that the vowel hardly reaches a stable state. In such cases, the congruence of on-glide and off-glide has been taken as the steady state.

6.1.3. Recognition of Telegu vowels

The present investigation has been carried out with the Telegu vowels (listed in Table 4) both short and long. It is well known that the first three formant frequencies carry most of the information regarding the vowel quality. But for all practical purposes of vowel recognition, we can use the first two formant frequencies, i.e. F_1 and F_2.

For recognition of Telegu vowels the F_1–F_2 distribution of Fig. 6 is considered. The rules and the distribution patterns are shown in Fig. 7. The recognition score with Mamdani's law of implication (where the connective "and" of the antecedent part of the rule is interpreted as "min") is given in Table 5. It is obvious from Table 5 that when we consider the classification of vowels at the overlap fuzzy zone the recognition score has been increased. The recognition score using other operators are also similar to the one mentioned in Table 5 and hence omitted here.

6.2. Experiment II with Bengali vowels

This experiment has been conducted on a sample of carefully selected 350 commonly spoken Bengali

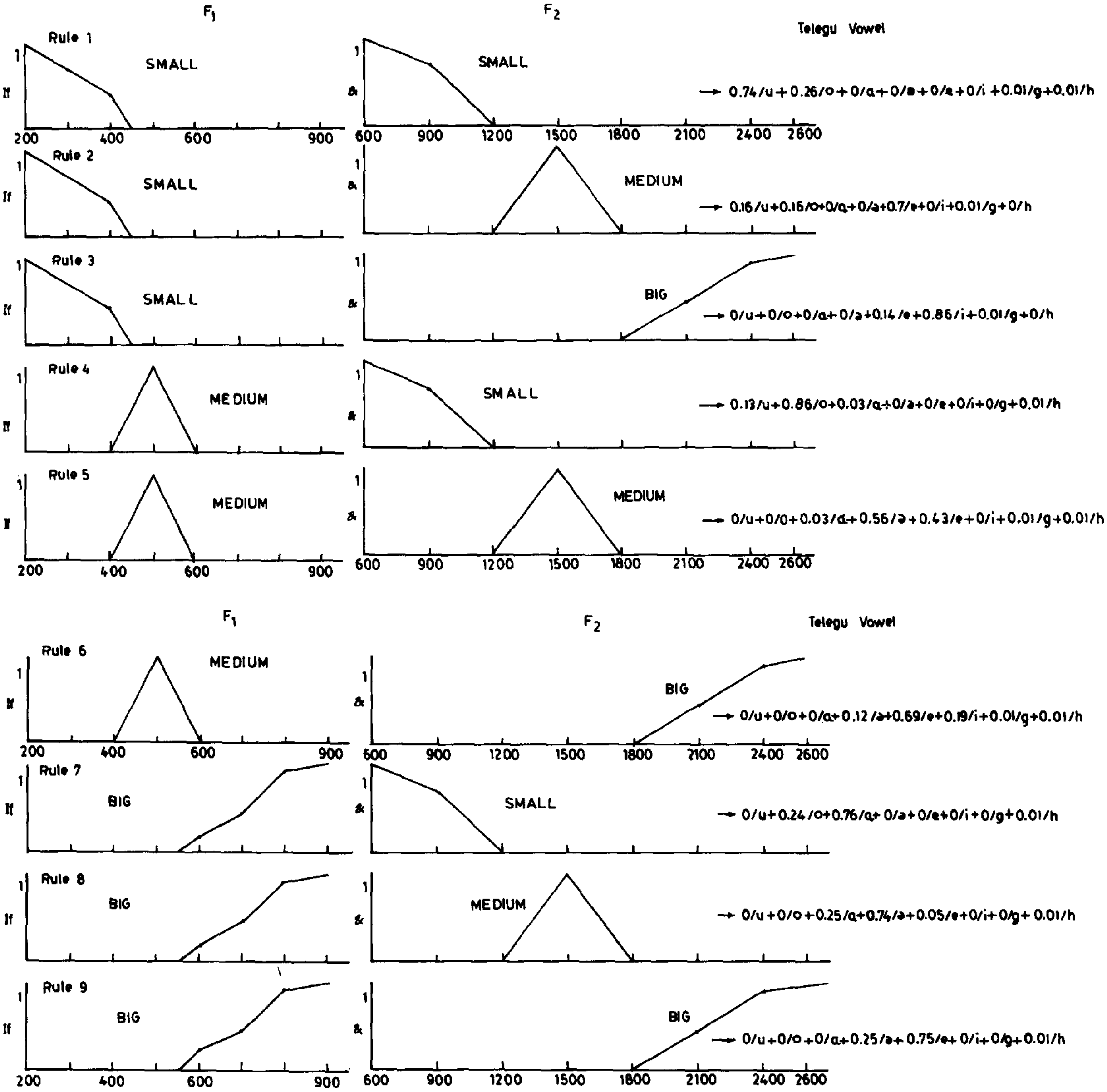


Fig. 7. Fuzzy If Then rules for Telegu vowels.

Table 5
Recognition scores (in %) of the Telegu vowels where "and" of the antecedent part of the rule is interpreted as "min"

Hard partitioning	Fuzzy partitioning
60.2	78

words so that the vowels can be studied in all possible *Consonant-Nucleus-Consonant* contexts. These words were uttered by 10 male and 10 female educated and phonetically conscious informants drawn from linguists, professors of literature and dons of performing arts. Of all these records, data for 3 male informants has been chosen on the basis of good and clear spectrographs. The age of the informants varied between 30 to 55 years. The spectrographic analysis has been on Kay Sonograph model 7029-A in the bandwidth of 80 Hz to 8 kHz using a resolution of 300 Hz. The acoustic data, namely the first four formant frequencies and the duration are derived from the spectrographs. As the vowels are embedded in various consonantal contexts in the multisyllabic words, appropriate segmentation procedures were adopted to accurately fix both the transitions and the steady state of vowels. The details of the segmentation procedure and measurement procedure are same as stated earlier. Table 6 represents the average formant frequencies for first three formants.

Table 6
Average formant frequencies of Bengali vowels

Phonetic symbol	F_1 (Hz)	F_2 (Hz)	F_3 (Hz)
/u/	327	935	2198
/o/	438	1015	2308
/ə/	626	1095	2391
/ɐ/	695	1326	2424
/ae/	681	1663	2320
/e/	374	1935	2410
/i/	304	2095	2565

For recognition of Bengali vowels the F_1–F_2 distribution of Fig. 8 is considered. The rules and the distribution patterns are shown in Fig. 9. The recognition score with Mamdani's law of implication (where the connective "and" of the antecedent part of the rule is interpreted as "min") is given in Table 7. It is obvious from Table 7 that when we consider the classification of vowels at the overlap fuzzy zone, the recognition score has significantly improved. The recognition score using other

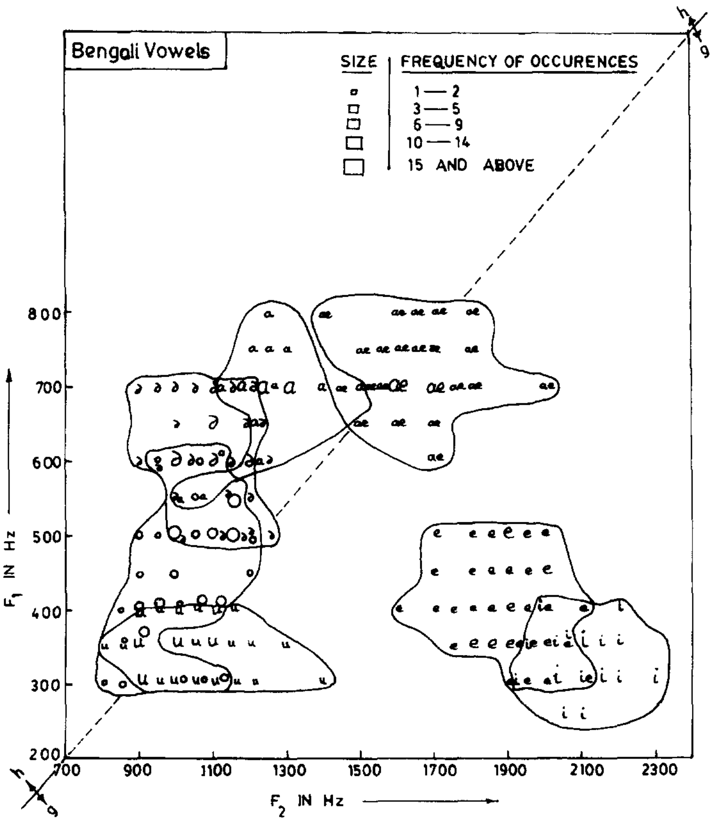


Fig. 8. Bengali vowels in the F_1–F_2 plane.

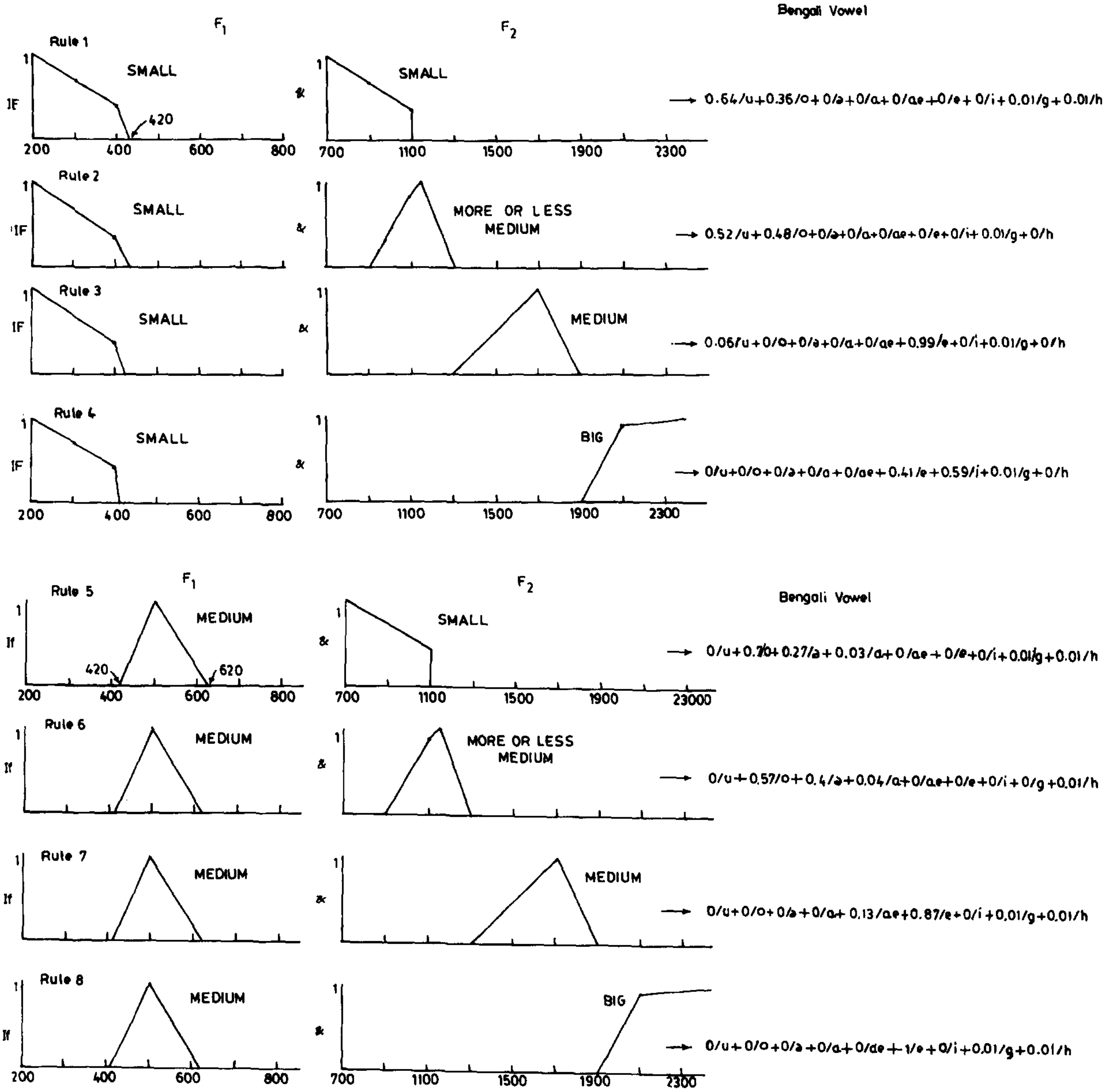


Fig. 9. Fuzzy If–Then rules for Bengali vowels.

operators are also similar to the one mentioned in Table 7 and hence omitted here.

6.3. Experiment III with Assamese vowels

This experiment has been conducted on a sample of carefully selected 300 commonly spoken Assamese words so that the vowels can be studied in all possible *Consonant–Vowel–Consonant* (*CVC*) contexts. These words were uttered by one male highly educated and phonetically conscious informant. The age of informant is about 55 years. The spectrographic analysis has been done on digital sonograph model DSP 5500 in the bandwidth

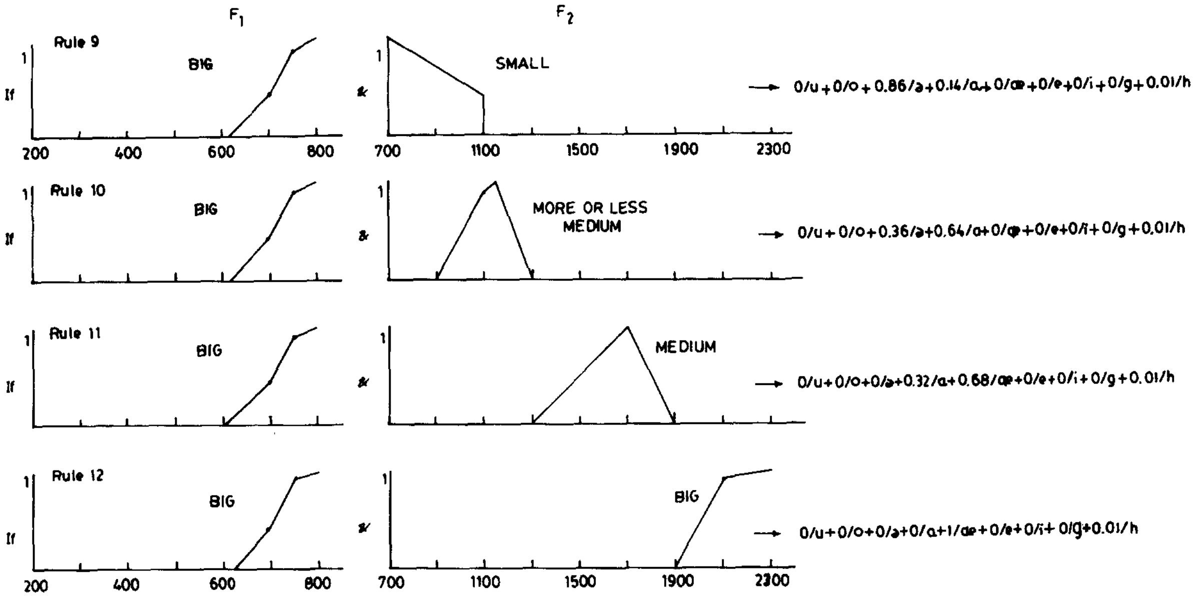


Fig. 9. (Contd.)

Table 7
Recognition scores (in %) of the Bengali vowels where "and" of the antecedent part of the rule is represented as "min"

Hard partitioning	Fuzzy partitioning
62	96.4

Table 8
Average formant frequencies of Assamese vowels

Phonetic symbol	F_1 (Hz)	F_2 (Hz)	F_3 (Hz)
/u/	320	820	2880
/ū/	380	800	2800
/o/	430	840	3020
/ə/	620	1290	2830
/a/	820	1530	2780
/æ/	680	1860	2760
/e/	460	2140	2720
/i/	320	2380	2770

of 80 Hz to 8 kHz using a resolution of 300 Hz. The first three formant frequencies of the vowels were measured at the section taken at central portion of the steady state. The details of the segmentation and measurement procedures are same as stated earlier. Table 8 represents the average frequencies for the first three formants of Assamese vowels.

For recognition of Assamese vowels the F_1–F_2 distribution of Fig. 10 is considered. The rules and the distribution patterns are shown in Fig. 11. The recognition scores under different laws of implication of Table 1 and under two different interpretations of the connective "and" of the antecedent part of the rules are listed in Table 9. From Table 10, it is obvious that when we consider the classification of vowels at the overlap fuzzy zone the recognition scores have improved.

7. Conclusion

In this paper, we have considered pattern recognition on $\mathbb{R}^2$. This approach can easily be extended for pattern recognition on $\mathbb{R}^n$ [8]. So far, people have experienced the success of the method of approximate reasoning for the design of the fuzzy logic controller. This paper further establishes the potentiality of the method of approximate reasoning for rule-based pattern recognition. In this

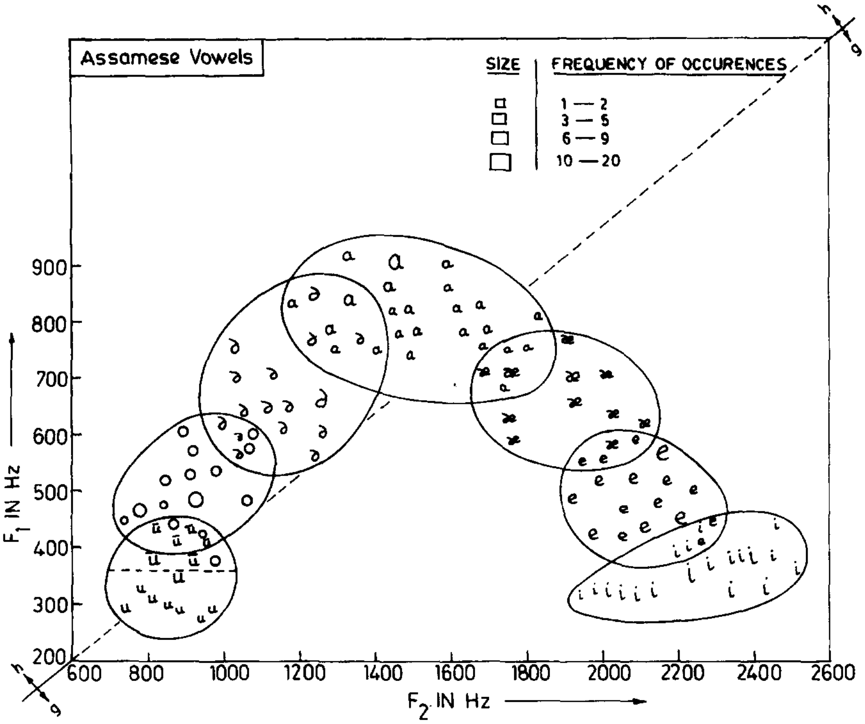


Fig. 10. Assamese vowels in the F_1–F_2 plane.

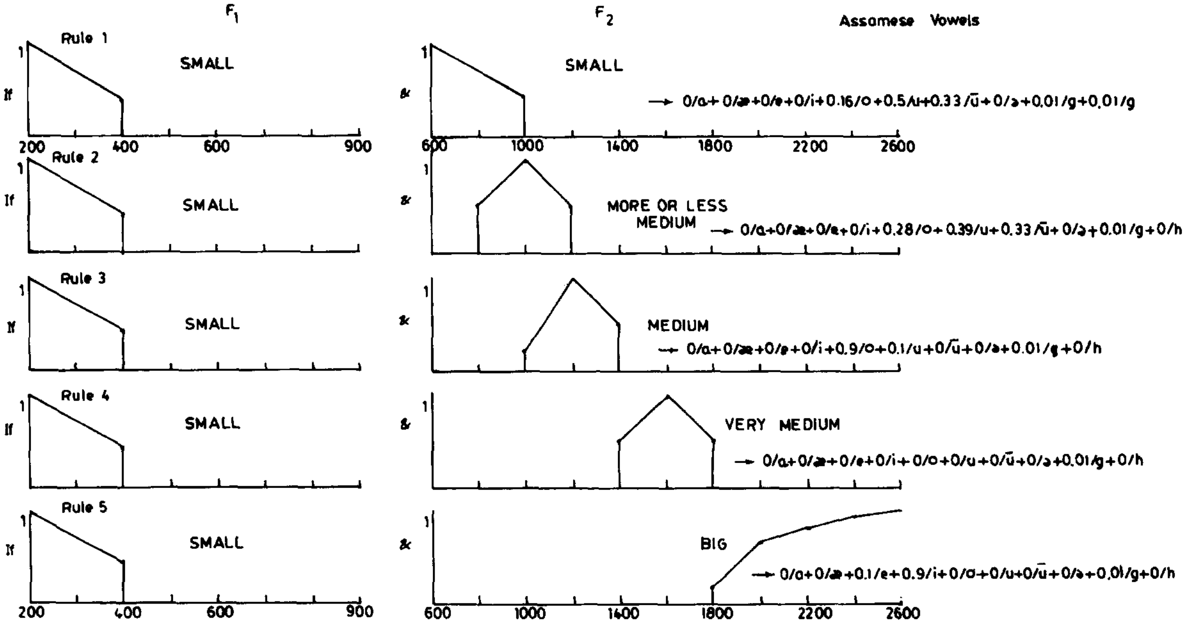


Fig. 11. Fuzzy If–Then rules for Assamese vowels.

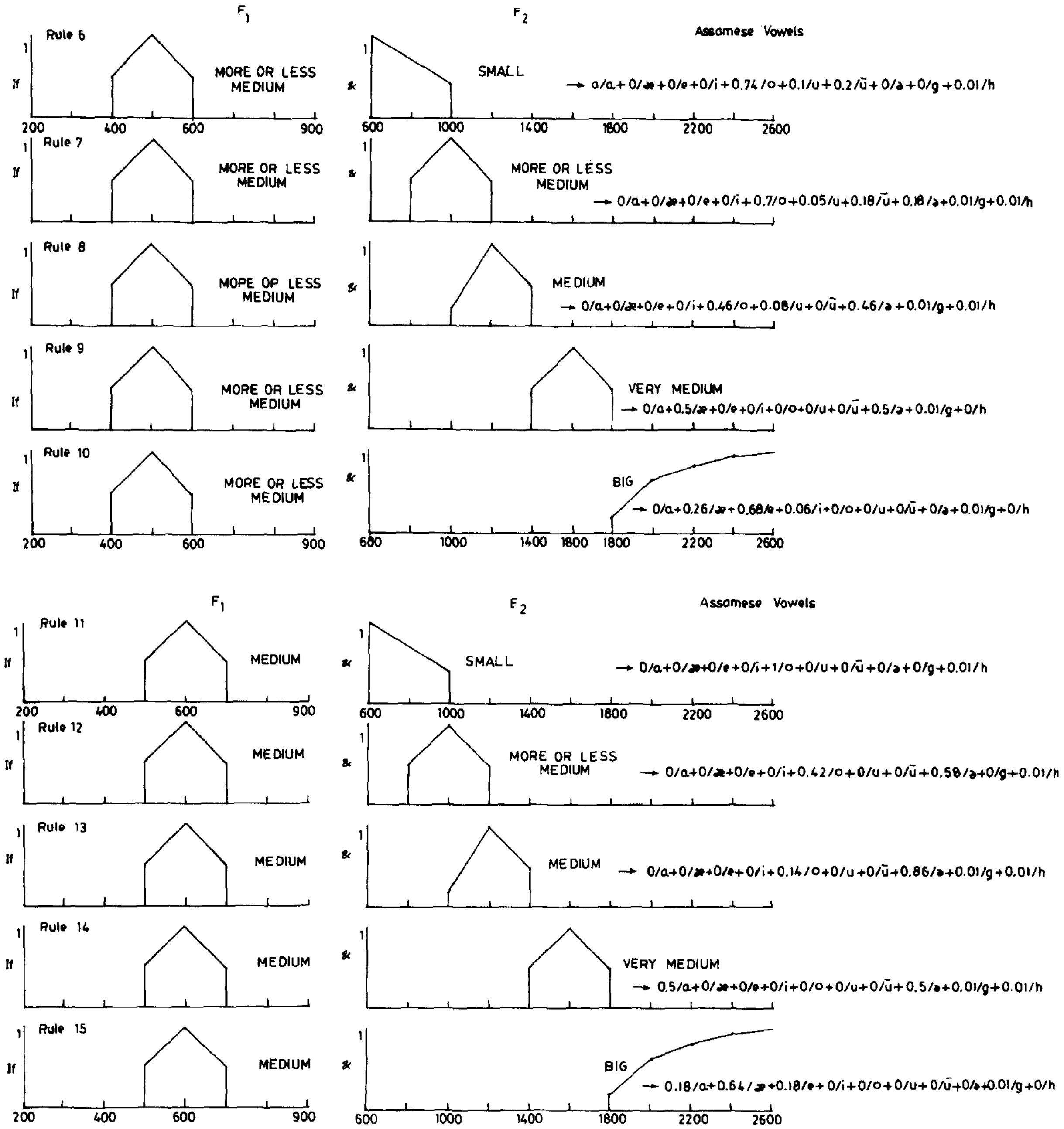


Fig. 11. (Contd.)

paper, we have considered several examples using synthetic data and real-life application to vowel recognition. In all these cases very satisfactory results have been achieved under different laws of fuzzy implications and the *max–min* composition. Instead of considering *max–min* composition, depending upon the need of the problem, we may also use other types of composition.

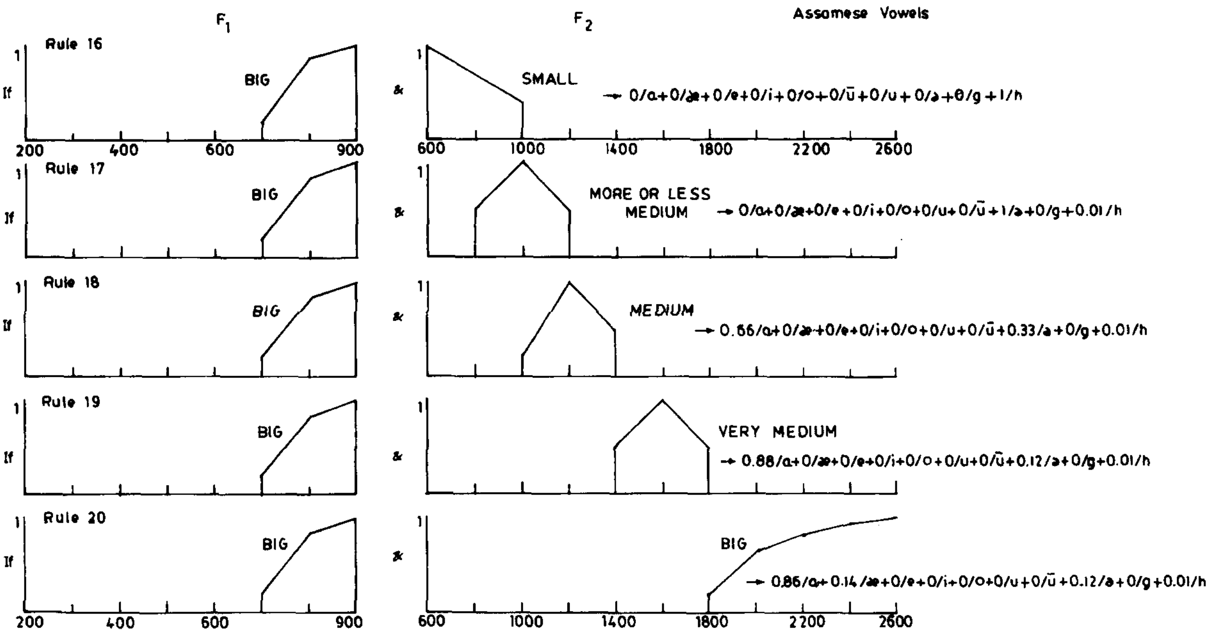


Fig. 11. (Contd.)

Table 9
Recognition scores (in %) of the Assamese vowels where "and" of the antecedent part of the rule is represented as "min" and then "algebraic product"

Type of operator	Min Hard partitioning	Algebraic product Hard partioning
R_c	78.4	76.7
R_p	81.8	81.8
R_{bp}	81.6	75.8
R_{dp}	76.7	76.6
R_a	75.8	75.8
R_m	63.8	50.8
R_b	75.8	69.8
R_s	45.6	59.5
R_g	73.3	75
R_Δ	74.13	74.13
R^*	75.8	77.6
$R_\#$	61.2	45.6

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Table 10
Recognition scores (in %) of the Assamese vowels were "and" of the antecedent part of the rule is represented as "min" and then "algebraic product"

Type of operator	Min Fuzzy partitioning	Algebraic product Fuzzy partioning
R_c	97.4	97.4
R_p	97.4	96
R_{bp}	97.4	95

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