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A note on the measurement of mobility

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Abstract

This paper shows that the minimum discrimination information statistic suggested by Kullback can be regarded as a measure of mobility. We also show the use of the Kullback measure for testing certain statistical hypotheses concerning mobility.

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1. Introduction

Indices of inequality are summary statistics of the dispersion of incomes at a particular point in time. Even if such indices are computed for a number of successive periods, by their very nature they will ignore many features of the time path of incomes which are of interest. As time progresses we observe changes in relative incomes observed in any given period. Indices of mobility are meant to measure the magnitude of these changes.

Dardanoni (1993) considered the interesting problem of ranking mobility matrices in a simple Markov chain model of social mobility. He also investigated the question of coherence of different mobility indices with the derived ordering. While Dardanoni's analysis is an important dimension of mobility study, another interesting dimension would be to test the significances of some statistical hypotheses concerning mobility. In this paper we show that Kullback's (1959) minimum discrimination information statistic, which can be regarded as a measure of mobility, can be used for this purpose.

2. The proposed measure

We consider a discrete Markov chain of income mobility and assume that there are n income classes. Let p_{ij}^t be the probability that an individual in state i at time t will be in state j

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at time $(t+1)$. Clearly, $p^t_{ij} \geq 0$ for all i, j and t; and for each t and i, $\sum^n_{j=1} p^t_{ij} = 1$. Suppose Π^t_i stands for the proportion of total population at time t belonging to class i. Then $\Pi^{t+1} = \Pi^t p^t$, where $p^t = (p^t_{ij})_{n \times n}$ is the transition probability matrix and $\Pi^j (j = t, t+1)$ is the vector $(\Pi^j_1, \Pi^j_2, \ldots, \Pi^j_n)$.

Kullback's (1959) minimum discrimination information statistic is defined as

$$K = \sum_{i=1}^{n} \Pi^t_i \log\left(\frac{\Pi^t_i}{\Pi^{t+1}_i}\right). \qquad (1)$$

Clearly, K is a measure of divergence between the population share vectors Π^t and Π^{t+1}. K is always non-negative. It takes on the minimum value of zero if and only if $\Pi^t_i = \Pi^{t+1}_i$ for all i. The necessary and sufficient condition for this to happen is that the transition matrix p^t is an identity matrix. Shorrocks (1978) argued that the mobility structure represented by the identity matrix should display as much immobility as any other transition matrix. Thus, K attains its lower bound of zero in the case of perfect immobility (the identity transition matrix). For any other structure K is positive. K is maximized when p^t has identical rows, that is, the probability of moving to any class is independent of that originally occupied. Prais (1955) identified such a structure as displaying maximum mobility, that is, as the case representing perfect mobility. In view of this discussion it is evident that we can regard K as a measure of mobility. Note that K also satisfies symmetry in pairs (Π^t_i, Π^{t+1}_i) and continuity in its $2n$ arguments.

We may note here that the index K does not meet the monotonicity property suggested by Shorrocks (1978). A mobility index I defined on the set of transition matrices is said to be monotone if for any two transition matrices p^t and q^t of the same order, where $p^t_{ij} \geq q^t_{ij}$ for all $i \neq j$, with $>$ for at least $i \neq j$, $I(p^t) > I(q^t)$. Shorrocks, however, notes the incompatibility of this property with perfect mobility postulate. He argues that if we wish to retain monotonicity at the cost of perfect mobility property:

'we may lose insight of any objective notion of maximum mobility and have to rely instead on whatever one specific measure tells us is the most mobile structure. There is another reason for being reluctant to abandon the perfect mobility condition. Interest in mobility is not only concerned with movement but also predictability – the extent to which future positions are dictated by the current place in the distribution (Shorrocks, 1978, p. 1016).

Monotonicity and perfect mobility can be regarded as distinguishing these separate aspects. Dardanoni (1993) shows that while his ordering is coherent with perfect mobility and perfect immobility assumptions, it does not imply monotonicity.

We may be interested in examining the significance of the following hypotheses:

H_{OM}: Society exhibits perfect mobility.

H_{OI}: Society exhibits perfect immobility.

To test the above hypotheses, we need to consider some test statistics.

Let us now arrange the successive pairs of observations of the occurrences in the periods t and $t+1$ of the Markov chain in the form of a two-way contingency table, with period t of the pair as the row category and period $(t+1)$ of the pair as the column category. Then the number of entries in the ith row and the jth column represents the number of persons a_{ij} who were in class i during period t but are in class j during period $(t+1)$. Under H_{OM} each row of the transition matrix will be of the form $(\alpha_1^t, \alpha_2^t, \ldots, \alpha_n^t)$, where the α_i^ts are given, $\alpha_i^t > 0$ for all i and $\sum_{i=1}^n \alpha_i^t = 1$. In order to test this null hypothesis against the alternative hypothesis that society does not display perfect mobility, the appropriate minimum discrimination information statistic is

$$2\hat{K} = 2\left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\log a_{ij} - \sum_{i=1}^{n} a_i \log(a_i\alpha_i^t)\right], \tag{2}$$

where $a_i = \sum_{j=1}^n a_{ij} \cdot 2\hat{K}$ in (2) is distributed asymptotically chi-square with $n(n-1)$ degrees of freedom under the null hypothesis of perfect mobility (see Kullback et al., 1962). If the probability of exceeding the computed value of $2\hat{K}$ is smaller according to the chi-square distribution for $n(n-1)$ degrees of freedom, then the null hypothesis is rejected.

To test the significance of H_{OI}, we use the minimum discrimination statistic

$$2\hat{K}_1 = 2\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\log\frac{a_{ij}}{a_i p_{ij}^t} \tag{3}$$

Note that $2\hat{K}$ in (2) is a particular case of $2\hat{K}_1$ in (3) under the specification that $p_{ij}^t = \alpha_i^t$ for all $i, j = 1, 2, \ldots, n$. Observe also that in (2) it has been assumed that all the p_{ij}^t (that is, α_i^t) values are positive. Kullback et al. show that if the null hypothesis specifies that there are c instances for which $p_{ij}^t = 0$, then $2\hat{K}_1$ follows the asymptotic chi-square with $n(n-1) - c$ degrees of freedom. Now, under H_{OI}, $n(n-1)$ of the p_{ij}^t values are zero. Consequently, under the null hypothesis of perfect immobility the test criterion $2\hat{K}_1$ in (3) is distributed asymptotically chi-squares with no degrees of freedom. Hence we can look at the significance of H_{OI}. (Note that in Kullback et al., $a_{ij}\log p_{ij}^t$ has been assumed to take the value of zero whenever $p_{ij}^t = 0$. The null hypothesis is obviously rejected if $a_{ij} > 0$ in any such case.)

We may wish to test the significance of the other hypotheses also. For instance, the following null hypotheses might be of interest: (a) $p_{ij}^t = 1/n$ for all $i, j = 1, 2, \ldots, n$, and (b) $p_{ij}^t = 0$ if $i = j$ and $p_{ij}^t > 0$, $i \neq j$. Hypothesis (a) is a particular case of H_{OM}. Hypothesis (b) means that a person who was in class i during period t is in a different class during period $(t+1)$. In this case we again use the test criterion $2\hat{K}_1$ and the degrees of freedom now become $n^2 - 2n$.

3. Conclusion

Mobility is a many-faceted phenomenon. (See Chakravarty, 1990, Ch. 9, for a discussion on alternative concepts of mobility.) We hope by interpreting the minimum discrimination information statistic as a measure of mobility and using it for testing certain hypotheses, we have further widened the notion of mobility.

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