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Distribution of Time of First Birth in Presence of Social Customs Regulating Physical Separation and Coital Frequency

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ABSTRACT

The interval between marriage and the first birth in India, particularly in rural areas, is much longer than what is observed in western countries. In eastern Uttar Pradesh, the mean interval is observed to be even longer, possibly due to traditional customs such as the female partner's visits to her parents in the early years of marriage and the smaller chance of coition because of the observance of rigid intercourse taboos. Thus the models to explain the length of the interval of marriage to first birth proposed by Western demographers, which assume that the period of cohabitation between marriage and first birth is uninterrupted, often do not describe the data satisfactorily when applied to rural India. In this paper a model to describe data on first birth interval is proposed that takes account of the distributions of timing and periods of physical separation and variation in fecundity with effective marriage duration.

1. INTRODUCTION

In societies where the incidence of premarital union and that of child marriages are negligible and women enter into active sexual union immediately after marriage, the onset of exposure to the risk of pregnancy is marked by marriage. As the beginning of sexual union starts after marriage, one can determine the exposure period between marriage and first birth exactly. This has for a long time encouraged demographers to study the period between marriage and first conception (or birth) for the estimation of fecundity. Owing to its importance to the study of human fertility, models explaining variation in time of first birth have been proposed under varying sets of assumptions. A

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review of the work up to 1972 is given in Sheps and Menken [1]. Critical reviews with more emphasis on recent work are given in Leridon [2] and Mode [3].

In some societies actual consummation following traditional marriage is delayed. For example, in most rural communities in India, the rite of marriage is restricted to the observation of certain rituals involved in the wedding, and with the exception of a very short visit just after this ceremony (absent in many cases when the bride is of tender age), the girl resides in her parents' home. It is only after the *Gauna* ceremony (hereafter termed return marriage, RM; see Jain [4] and Singh et al. [5]) that the bride moves in with the bridegroom and the partners enter into conjugal relations. Further, there is some gap between RM and the commencement of regular married life. After RM, brides usually stay for a short duration with their in-laws and then return to their parents for a considerable period of time. Following a third ceremony, they return to their in-laws. Brides for whom the ceremony could not be observed or who belong to communities where the ceremony does not exist customarily visit their parents on one occasion or the other during the early years of RM and stay for much longer periods. They also make frequent short visits to their parents in the next few years.

The models reviewed in [1–4] do not take into account this physical separation of marriage partners. Thus, they often fail to describe data on time from RM to first birth for women belonging to areas where sociocultural factors disturb the period of stable union and restrict sexual activity during the early years of cohabitation. For example, in India, Singh [6] used, for the first time, first birth interval data of women who were sampled from rural areas of Varanasi Tehsil (a subdivision of Varanasi district of Ultar Pradesh) to illustrate the application of his proposed model. To obtain a good fit he subtracted 6 months from the actual interval between RM and first birth on the basis of reasoning that conception is almost impossible during the first 6 months because of local customs. Later Pathak and Prasad [7], Pathak [8], and Nair [9, 10] extended Singh's model by making provision for adolescent sterility to explain the data.

Application of the above-mentioned models, even though on certain occasions it produces an acceptable fit, may make the analysis inappropriate, thus yielding an inadequate description of the observations in terms of faulty mathematical concepts. Therefore, the formulation of models accounting for the facts would give good agreement with the observed facts.

In this paper a probability distribution for time of first live birth is developed that is more appropriate for analyzing the time of first birth in the presence of social customs and taboos relating to physical separation and coital regulation. The application of the model is illustrated through real data.

2. CHANCE MECHANISM OF TIME OF FIRST BIRTH

Consider a woman who is fecund at the time of RM and is exposed to the risk of a visit to her parents. Her first live birth conception (l conception) may occur either before the first visit or following the return to in-laws after completing the visit.

At any time after RM a woman can be found in any of the following states:

- S_0: Cohabitation following RM is uninterrupted, and first l conception is yet to occur
- S_1: Staying with parents during first visit; and first l conception did not occur before first visit
- S_2: Completed the first visit and staying with in-laws, and first l conception yet to occur
- S_3: Occurrence of first l conception

Let us observe a woman from her RM until the occurrence of first l conception. The events "occurrence of first l conception in uninterrupted cohabitation" and "first visit to parents" may be considered as two competing risks.

We denote

- T_1: Time from RM to visit to parents,
- T_2: Time from RM to first l conception in uninterrupted cohabitation.

and introduce a random index I,

$$I = \begin{cases} 0 & \text{if } T_1 < T_2 \text{ (visit occurs before first } l \text{ conception)}, \\ 1 & \text{if } T_1 \geq T_2 \text{ (first } l \text{ conception occurs before visit)}. \end{cases}$$

For a woman whose first l conception did not occur before the time of the first visit (i.e., $I = 0$), define

- T_3: Period of stay with parents during first visit,
- T_4: Time between return to in-laws between first visit to parents and occurrence of first l conception.

It is observed empirically that the risk of first visit to parents following RM was not related to whether the female was of parity 0 or pregnant following her first l conception or had attained parity 1 at the time of visit. For women who visited parents before the occurrence of the first l conception, the period of stay with parents during the first visit (i.e., T_3) is also observed to be independent of the time of visit to parents (i.e., T_1).

The hazard of the first l conception at time t elapsing since RM for a woman in uninterrupted cohabitation (i.e., state S_0) may be different from that of a woman who completed her visit prior to time t (i.e., state S_2). However, empirically it is seen that the hazards in both situations are more or less the same.

3. MODEL

Let us consider a cohort of women of the same age at RM and with the same duration of effective marital life, say exactly T' years. For a woman the time of first birth would be available if it occurred in the first T' years of RM, and otherwise the time of first birth is known to be more than T' years. Define $T = T' - g$, where g is the length of gestation associated with a live birth. In this section a probability distribution for time between RM and first live birth conception is derived under the following assumptions:

(1) Females consist of two groups—those who are exposed to the risk of a visit to parents at RM and those who are not exposed to this risk. Let their respective proportions be p and $1-p$, $0 \leq p < 1$.

(2) Let α, $0 < \alpha \leq 1$, be the probability that a woman is fecund at the time of RM and remains fecund until the occurrence of her first l conception. Then $1-\alpha$ is the probability that the woman is primarily sterile.

(3) Given that a woman is living with her husband at time t and has not experienced an l conception, the probability that an l conception will occur during $(t, t+dt)$ is $\lambda(t)\,dt + o(dt)$; $\lambda(t) > 0$ for $t > 0$.

(4) Given that a woman belonging to the first group did not experience an l conception until time t, the probability that her first visit to her parents takes place during $(t, t+dt)$ is $h(t)\,dt + o(dt)$; $h(t) > 0$ for $t > 0$.

(5) A woman cannot conceive during her stay with her parents.

(6) The probability density function (p.d.f.) of the period of stay with parents for a woman who visits prior to the occurrence of her first l conception is $f_3(t)$.

(7) Both the time of first l conception during uninterrupted cohabitation (i.e., T_2) and the period of stay with parents during the first visit (i.e., T_3) are independent of the time of first visit to parents (i.e., T_1).

Under the above assumptions, T_0, the time of first l conception, is given by

$$T_0 = \begin{cases} T_1 + T_3 + T_4 & \text{if } I = 0, \\ T_2 & \text{if } I = 1. \end{cases}$$

Define as $S(t)$ the probability that a woman is in state S_0 before or at time t (i.e., neither first l conception nor visit to parents occurred before or at time t); that is,

$$S(t) = P[T_1 > t, T_2 > t],$$

$$-\left.\frac{\delta \log S(t)}{\partial T_1}\right|_{T_1 = t, T_2 = t} = h(t),$$

$$-\left.\frac{\delta \log S(t)}{\delta T_2}\right|_{T_1 = t, T_2 = t} = \lambda(t),$$

where $\lambda(t)$ and $h(t)$ are the crude hazard rates of first l conception and first visit to parents, respectively.

Since T_1 and T_2 are independent to each other,

$$S(t) = P[T_1 > t, T_2 > t] = P[T_1 > t]P[T_2 > t].$$

The marginal p.d.f.'s of T_1 and T_2, say $f_1(t)$ and $f_2(t)$, respectively, and the conditional density of T_4 given $T_1 = t$ and $T_1 + T_3 = \tau$ (which depends only on τ), say $f_4(t/\tau)$, are

$$f_1(t) = h(t)\exp\left[-\int_0^t h(u)\,du\right], \qquad t > 0;$$

$$f_2(t) = \lambda(t)\exp\left[-\int_0^t \lambda(u)\,du\right], \qquad t > 0;$$

$$f_4(t/\tau) = \lambda(t)\exp\left[-\int_\tau^t \lambda(u)\,du\right], \qquad t > \tau > 0.$$

Let $F_1(t)$ and $F_2(t)$ be the distribution functions corresponding to p.d.f.'s $f_1(t)$ and $f_2(t)$, respectively.

Denoting by

$$U_1(t) = P[T_2 \le t, T_1 \ge T_2]$$

the probability that the first l conception to a woman occurs on or before time t if it precedes the first visit to her parents, and by

$$U_2(t) = P[T_1 + T_3 + T_4 \le t, T_1 < T_2]$$

the probability that the first l conception to a woman occurs on or before time t if it occurs after her return to her in-laws upon completing the first stay with her parents, the probability that the first l conception to a woman who is exposed to the risk of a visit to her parents occurs on or before time t can be expressed as

$$\begin{aligned} P[T_0 \le t] &= P[T_2 \le t, T_1 \ge T_2] + P[T_1 + T_3 + T_4 \le t, T_1 < T_2] \\ &= U_1(t) + U_2(t). \end{aligned} \tag{1}$$

Now

$$U_1(t) = \int_0^t \lambda(u) S(u)\, du = \int_0^t [1 - F_1(u)] f_2(u)\, du \tag{2}$$

and

$$\begin{aligned} U_2(t) &= \int_0^t P[z < T_0 < z + dz, I = 0] \\ &= \int_{z=0}^{t} \int_{x=0}^{z} \int_{y=0}^{z-x} P[x < T_1 < x + dx, T_1 < T_2] \\ &\quad \times P[y < T_3 < y + dy | T_1 = x, T_1 < T_2] \\ &\quad \times P\{z - x - y < T_4 < z + dz - x - y | T_1 + T_3 \\ &\qquad\qquad = x + y, T_1 = x, T_1 < T_2\} \\ &= \int_{z=0}^{t} \int_{x=0}^{z} \int_{y=0}^{z-x} h(x) S(x) f_3(y) f_4(z/(x+y))\, dy dx dz \\ &= \int_{z=0}^{t} \int_{x=0}^{z} \int_{y=0}^{z-x} f_1(x) f_3(y) f_4(z/(x+y))\, dy dx dz. \end{aligned} \tag{3}$$

The probability that the first l conception to a woman who is not exposed to the risk of a visit to her parents occurs on or before time t is

$$U_0(t) = 1 - \exp\left[- \int_0^t \lambda(u)\, du \right]. \tag{4}$$

Since p is the proportion of women who are exposed to the risk of a visit to parents and $1 - \alpha$ is the proportion of women who are primarily sterile, the probability that the first l conception occurs on or before time t to a woman selected at random from the population is

$$P[T_0 \le t] = \alpha F(t), \qquad 0 < t \le T, \tag{5}$$

where

$$F(t) = (1-p)U_0(t) + p[U_1(t) + U_2(t)], \tag{6}$$

and the probability that the time of first l conception exceeds T is

$$\begin{aligned} P[T_0 > t] &= (1-\alpha) + \alpha[1 - F(T)] \\ &= 1 - \alpha F(T). \end{aligned} \tag{7}$$

4. APPLICATION

The data used for the present study were taken from the survey "Effects of Socio-cultural Factors on Determinants of Fertility in Eastern Uttar Pradesh (Rural)" carried out by the Centre of Population Studies, Banaras Hindu University, India, during 1987–1989 under the financial support of the Indian Council of Medical Research (ICMR), New Delhi.

In rural areas, religion, particularly among Hindus, is an important determinant of occupation, education, and socioeconomic status in the community. Further, caste beliefs exert a strong and stable influence on human behavior, social habits, customs, and practices. Therefore, religion/caste was taken as a characteristic for the study of the effects of sociocultural factors on various components of fertility. In eastern Pradesh, about 90% of the households are Hindus, distributed among about 35 castes. These were roughly subdivided into groups of upper, middle, and scheduled (Sc/st) castes. Muslims were taken as a single group.

The survey was carried out in the rural areas of the districts, namely, Varanasi, Ghazipur, and Azamgarh in eastern Uttar Pradesh.

In accordance with the objectives of the survey, information was collected on various items from about 350 households belonging to each religion/caste group for each selected district. Information was collected from households using a quota sampling procedure. If, however, a village was included in the sample, information was collected from all the households of that village irrespective of caste. There were 4448 households in the sample, with an least one eligible couple from each household.

Currently married women below 50 years of age (hereafter referred to as *eligible women*) who were normal residents of the sample households were interviewed, and for each such woman, in addition to other information, data included order of most recent marriage, age at RM, if she is married once particulars of first visit to parents after RM—1) not exposed to the risk of visit to parents (i.e., neither yet visited parents and nor would visit them in future), or 2) not yet visited parents would visit them in future or likely to visit, or 3) already had visited. In the event of having already visited, the duration of cohabitation before first visit to parents (interval from RM to either first visit to parents or reference date of survey, whichever is shorter) and its termination status, and period of stay with parents (interval from time of visit to either time of return to in-laws or survey, whichever is shorter) and its termination status (whether returned to in-laws after first visit). Also, time of first birth, etc. were recorded.

Sample households in five villages where a study entitled "Breast-Feeding and Its Effect on Fertility" was conducted were also covered by the present study. However, information on time of visit to parents and period of stay with parents was not collected from women of these villages. Therefore these women were excluded from the present study.

The present study took account of currently married females married only once. Females having an age at RM of 13 years or less or 20 years or more were excluded from the analysis, being few in number. Females whose conception occurred during their stay with their parents were also excluded.

Religion/caste and age at RM for women who had already visited or were likely to visit parents, the duration of cohabitation before first visit to parents and its termination status, and, for those who visited, period of stay with parents and its termination status, were used to obtain the mean duration of cohabitation before first visit to parents and the mean period of stay with them during first visit by using a life table technique.

It was observed that the mean duration of cohabitation between RM and first visit to parents was about 14 months in upper caste Hindus, whereas it was in the neighborhood of 1–3 months in the remaining three religion/caste groups.

For those females who visited their parental home before the occurrence of the first *l* conception, the mean duration of stay with parents was found to be about 13 months for upper caste Hindus, which was around 2–5 months longer than for any of the remaining three religion/caste groups. Therefore, on the basis of these results, females were classified into two broad groups:

Group I: Hindu upper castes
Group II: Hindu middle and scheduled castes and Muslims

Table 1 presents the distribution of women whose first *l* conception occurred during the first 7 years of return marriage according to the time of first *l* conception and the number of women who did not

TABLE 1

Distribution of Women Whose Effective Marital Duration Was 7 yr or More on Reference Date of Survey According to Time of First *l* Conception and Number of Women Who Did Not Conceive During that Period by Age at RM for Groups I and II

Interval from RM to first *l* conception (yr)	Religion/caste group								
	Group I			Group II			All		
	14	15–16	17–19	14	15–16	17–19	14	15–16	17–19
0.0–1.0	12	81	88	35	124	64	47	205	152
1.0–2.0	22	78	81	72	181	115	94	259	196
2.0–3.0	6	36	46	52	200	97	58	236	143
3.0–4.0	12	50	30	49	133	75	61	183	105
4.0–5.0	11	35	26	37	100	55	48	135	81
5.0–6.25	17	25	20	20	61	22	37	86	42
≥ 6.25	29	72	54	37	115	77	66	187	131
Total	109	377	345	302	914	505	411	1291	850

conceive during the aforesaid period by age at RM for each of the two groups.

The table below presents the proportion of women exposed to the risk of visit to parents (p) by age at RM for four religion/caste groups.

Age at RM (yr)	Religion			
	Hindu			
	Caste			
	Upper	Middle	Sc/st	Muslim
14	0.99	0.99	0.99	0.98
15–16	0.99	0.98	0.97	0.96
17–19	0.99	0.97	0.96	0.92

From this table it can be seen that in Hindu caste groups the proportion did not depend on age at RM. However, for Muslims it ranged between 0.98 and 0.92 for ages at RM of 14 and 17–19, respectively. Therefore, it was taken to be 0.99 and 0.97 for group I and group II, respectively. The proportion for group II was obtained as the weighted average of proportions of three religion/caste groups, the weights being the number of women in each of these groups. Similarly, we obtained the weighted average of proportions of all four religion/caste groups combined as 0.97.

In each religion/caste group the mean duration of cohabitation between RM and first visit to parents was found to be more or less the same for women having an age at RM of 14 years and above (not tabulated here). Thus, for each religion/caste group, the life table estimates of the proportion of women who had not yet visited parents for the first time by month t elapsed since RM, say $D_1(t)$, for different values of t were obtained for women of 14 or older at RM and are presented in Table 2. If it is assumed that $f_1(t)$ is distributed uniformly in the interval (t_i, t_{i+1}), $i = 0, 1, \ldots$, then the estimate of $f_1(t)$ is obtained as

$$\hat{f}_1(t) = \frac{D_1(t_i) - D_1(t_{i+1})}{t_{i+1} - t_i}, \qquad t_i \le t < t_{i+1}.$$

Further, for women who visited parents before the occurrence of their first l conception, the life table estimates of the proportion of women still staying with parents during first visit by month t elapsed since time of visit to parents, say $D_2(t)$, for different values of t were obtained according to age at RM and religion/caste group and are presented in Table 3. Under the assumption that $f_3(t)$ is distributed uniformly in the

TABLE 2

Among Women Who Are Exposed to the Risk of Visit to Parents, Proportion Still Staying with In-laws by Months Elapsed Since Return Marriage

Months since RM	Religion			
	Hindu caste			
	Upper	Middle	Sc/st[a]	Muslim
0	1.00	1.00	1.00	1.00
1	0.96	0.53	0.41	0.47
2	0.92	0.44	0.30	0.41
3	0.89	0.39	0.25	0.37
6	0.81	0.27	0.15	0.30
9	0.71	0.17	0.09	0.26
12	0.65	0.15	0.08	0.23
18	0.43	0.05	0.02	0.10
24	0.35	0.04	0.02	0.08
30	0.19	0.03	0.01	0.04
36	0.17	0.02	0.01	0.04
48	0.09	0.01	0.01	0.02
60	0.06	0.01	0.01	0.01

[a]Sc/st =. Scheduled castes/scheduled tribes.

TABLE 3

Among Women Who Visited Parents Before Occurrence of First l Conception, Proportion Still Staying with Parents by Months Elapsed Since Time of First Visit to Parents by Age at RM

Age at RM (yr)	Months since first visit										
	0	3	6	9	12	18	24	30	36	48	60
Upper caste Hindu											
14	1.00	.91	.86	.79	.73	.45	.35	.20	.13	.02	.02
15–16	1.00	.88	.83	.69	.60	.33	.22	.12	.07	.01	.01
17–19	1.00	.88	.81	.63	.56	.31	.20	.11	.07	.01	.01
Middle caste Hindu											
14	1.00	.92	.86	.71	.65	.20	.15	.09	.07	.02	.02
15–16	1.00	.91	.81	.65	.61	.20	.14	.08	.06	.01	.00
17–19	1.00	.92	.83	.66	.63	.17	.12	.05	.05	.00	.00
Scheduled cast Hindu											
14	1.00	.81	.70	.49	.44	.05	.02	.02	.02	.00	.00
15–16	1.00	.77	.61	.43	.41	.05	.04	.02	.02	.00	.00
17–19	1.00	.73	.54	.29	.27	.03	.02	.01	.01	.00	.00
Muslim											
14	1.00	.85	.72	.54	.51	.21	.20	.09	.07	.05	.05
15–16	1.00	.85	.66	.52	.47	.17	.13	.04	.03	.01	.01
17–19	1.00	.85	.69	.50	.44	.17	.12	.02	.02	.00	.00

interval (t_i, t_{i+1}), $i = 0, 1, \ldots,$ the estimate of $f_3(t)$ is obtained as

$$\hat{f}_3(t) = \frac{D_2(t_i) - D_2(t_{i+1})}{t_{i+1} - t_i}, \qquad t_i \le t < t_{i+1}.$$

5. ILLUSTRATION OF MODELS

The model may be used to estimate the fertility parameters where the interval between RM and the first l conception is uninterrupted or is interrupted due to the bride visiting her parental home.

When $p = 0$ (i.e., females would never visit parents following RM) and $\lambda(t) = \lambda$ for $t \in 0, T$, that is, $\lambda(t)$ is constant over time and the same for all females, then the distribution function $F(t)$ in Equation (6) reduces to a displaced exponential distribution and is given by (Model 1)

$$F(t) = 1 - \exp[-\lambda t], \qquad 0 < t \le T. \tag{8}$$

When $p=1$, $F_1(t)=\{^{0 \ \text{if } t<0}_{1 \ \text{if } t\geq 0}$ (i.e., all the females visit parents immediately after RM) and $F_3(3)$ (the distribution function corresponding to $F_3(t)$), given by

$$F_3(t)=\begin{bmatrix} 0 & \text{if } t<c, \\ 1 & \text{if } t\geq c \end{bmatrix}$$

(i.e., females stay with their parents for c units of time), and $\lambda(t)=\lambda$ for $t\in(c,T)$ and is the same for all females, then the distribution function $F(t)$ in Equation (6) reduces to Singh's [6] model and is given by (Model 2)

$$F(t)=1-\exp[-\lambda(t-c)], \qquad c<t\leq T. \tag{9}$$

If $0<p<1$ and the risk of first l conception is assumed to be constant until the occurrence of the first l conception, that is, $\lambda(t)=\lambda$ for $t\in(0,T)$ and is the same for all females, then the distribution function $F(t)$ in Equation (6) reduces to (Model 3)

$$F(t)=(1-p)[1-e^{-\lambda t}]+p\left[\int_0^t[1-F_1(x)]\,\lambda e^{-\lambda x}\,dx + \int_{z=0}^{t}\int_{x=0}^{z}\int_{y=0}^{z-x} f_1(x)f_3(y)\,\lambda e^{-\lambda(z-y)}\,dy\,dx\,dz\right]. \tag{10}$$

The truncated forms of the models given by Equations (8)–(10) were applied to the observed distributions of females given in Table 1. A scoring method was used to obtain the maximum likelihood (m.l.)

TABLE 4

Values of χ^2 and λ Obtained by Fitting the Models in the Distributions Given in Table 1[a]

Age at RM (yr)	Religion/caste group	Model 1		Model 2		Model 3	
		χ^2	λ	χ^2	λ	χ^2	λ
14	Group I	9.9	0.035	9.1	0.122	14.7	0.125
	Group II	29.2	0.148	6.9	0.263	10.7	0.307
	All	27.3	0.121	4.5	0.229	21.9	0.301
15–16	Group I	10.5	0.251	22.9	0.380	18.5	0.401
	Group II	74.3	0.163	22.8	0.280	11.7	0.316
	All	54.4	0.186	16.4	0.307	15.3	0.348
17–19	Group I	3.9	0.344	13.3	0.500	14.8	0.493
	Group II	53.3	0.194	15.5	0.321	16.7	0.350
	All	38.4	0.252	8.4	0.389	19.4	0.426

[a] d.f. = 3.

estimates of λ. The chi-square values and the estimates of λ are presented in Table 4. All three models give a very poor fit to all sets of data except for a few cases. Taking m.l. estimates of λ obtained from truncated forms of the above distributions as pilot values, the m.l. estimates of α were obtained from the untruncated forms of these models. The values of α (Table 5) were found to be greater than 1 in almost all cases. Hence the models discussed above did not fit the present sets of data. The estimates of α were also found to be inconsistent.

Using the actuarial method, the estimate of instantaneous risk of the first l conception showed that the risk during the first year of RM was much smaller than estimated for later segments of effective marital duration. After 1 year, the risks were observed to be more or less the same for different segments of effective marital duration (tables not presented here).

We therefore assume that

$$\lambda(t) = \begin{cases} \lambda & \text{for } 0 < t < 1, \\ \rho\lambda & \text{for } t \geq 1. \end{cases} \tag{11}$$

The distribution function of the time of first l conception, when p and density functions $f_1(t)$ and $f_3(t)$ are known, involves three unknown parameters— λ, ρ, and α. Maximum likelihood estimates of parameters λ, ρ, and α were obtained using the scoring method (see Appendix). In estimating the parameters, p was taken to be 0.99 for group I and 0.97 for group II. The model was applied to the observed distributions of females given in Table 1.

TABLE 5

Estimates of α Corresponding to Table 4

Age at RM (yr)	Religion/ caste group	Model 1	Model 2	Model 3
14	Group I	3.724	1.455	1.087
	Group II	1.457	1.126	1.086
	All	1.581	1.146	1.037
15–16	Group I	1.023	0.911	0.966
	Group II	1.370	1.093	1.104
	All	1.243	1.032	1.040
17–19	Group I	0.955	0.894	0.914
	Group II	1.026	1.007	1.092
	All	1.067	0.947	0.959

Tables 6 and 7 present the observed and expected distributions of females according to the time of first l conception and age at RM, chi-square values, estimates of parameters, variances of estimators, and correlation coefficients between the estimators for groups I and II, respectively.

The distribution is described satisfactorily for all the data sets. In both the groups, the estimated risk of the first l conception during the first year of RM (i.e., $\hat{\lambda}$) as well as the risk in succeeding years of RM (i.e., $\hat{\rho}\hat{\lambda}$) were observed to be high for females having an age at RM of 17–19 years compared to females having an age at RM of 14 or 15–16 years. The smaller value of risk for younger females may be attributed to adolescent subfecundity, strict traditional coitus regulation, and other

TABLE 6

Observed and Expected Distribution of Women According to Time of First l Conception with Marital Duration 7 Years or More on Reference Date of Survey by Age at RM for Group I

Interval from RM to first l conception (yr)	Age at RM (yr)					
	14		15–16		17–19	
	Obs freq	Exp freq	Obs freq	Exp freq	Obs freq	Exp freq
0.0–1.0	12	12.0	81	81.5	88	89.9
1.0–2.0	22	14.7	78	66.1	81	70.4
2.0–3.0	6	12.7	36	52.0	46	50.9
3.0–4.0	12	13.0	50	46.4	30	38.3
4.0–5.0	11	15.0	35	35.9	26	26.5
5.0–6.25	17	12.6	25	23.1	20	25.7
≥ 6.25	29	29.0	72	72.0	54	53.4
Total	109	109.0	377	377.0	345	345.0
χ^2	9.8		7.5		5.1	
d.f.	3		3		3	
λ	0.133		0.318		0.443	
ρ	1.959		1.667		1.543	
α	—*		0.849		0.848	
$V(\lambda)$	0.003		0.001		0.002	
$V(\rho)$	0.525		0.074		0.063	
$V(\alpha)$	—		0.002		0.001	
Corr(λ, ρ)	−0.245		−0.443		−0.371	
Corr(λ, α)	—		−0.141		−0.004	
Corr(ρ, α)	—		−0.693		−0.777	

*Estimate of α was just a little greater than 1. Therefore, estimates of λ and ρ were obtained by assuming α as unity.

TABLE 7

Observed and Expected Distributions of Women According to Time of First l Conception with Marital Duration 7 yr or More on Reference Date of Survey by Age at RM for Group II

Interval from RM to first l conception (yr)	Age at RM (yr)					
	14		15–16		17–19	
	Obs freq	Exp freq	Obs freq	Exp freq	Obs freq	Exp freq
0.0–1.0	35	35.5	124	125.3	64	68.5
1.0–2.0	72	65.0	181	187.3	115	109.8
2.0–3.0	52	65.2	200	191.1	97	106.0
3.0–4.0	49	43.7	133	133.3	75	67.3
4.0–5.0	37	32.5	100	97.1	55	46.0
5.0–6.25	20	23.1	61	64.7	22	30.5
≥ 6.25	37	37.0	115	115.2	77	76.9
Total	302	302.0	914	914.0	505	505.0
χ^2	5.2		0.9		6.3	
d.f.	3		3		3	
λ	0.255		0.272		0.296	
ρ	1.537		1.369		1.439	
α	—*		—*		0.958	
$V(\lambda)$	0.001		0.001		0.001	
$V(\rho)$	0.136		0.035		0.075	
$V(\alpha)$	—		—		0.007	
$\mathrm{Cor}(\lambda, \rho)$	−0.264		−0.216		−0.159	
$\mathrm{Cor}(\lambda, \alpha)$	—		—		−0.254	
$\mathrm{Cor}(\rho, \alpha)$	—		—		−0.889	

*α is taken as 1.

factors. On the other hand, the sexual relation is expected to become weaker for females married at ages 15 years and above, and this might be the reason for higher values of $\hat{\lambda}$ for ages at RM of 15–16 and 17–19 years than for those of 14 years.

In group I, the risks of the first l conception increased with increase in age at RM in succeeding years, but in group II the risks were almost the same in succeeding years for ages at RM of 14 and 15–16 years. This risk was slightly higher for age at RM of 17–19 years. Among females having an age at RM of 14 years, the estimates of λ and $\rho\lambda$ were observed to be lower for group I than for group II. The opposite was the case for the other two age groups (i.e., 15–16 and 17–19 years).

Table 8 presents the observed and expected distributions of women (combining all religion/caste groups) according to the time of first l

TABLE 8

Observed and Expected Distributions of Women According to Time of First l Conception with Marital Duration 7 yr or More on Reference Date of Survey by Age at RM (All Religion/Caste Groups Combined)

Interval from RM to first l conception (yr)	Age at RM (yr)					
	14		15–16		17–19	
	Obs freq	Exp freq	Obs freq	Exp freq	Obs freq	Exp freq
0.0–1.0	47	46.7	205	205.0	152	151.5
1.0–2.0	94	79.6	259	256.2	196	184.0
2.0–3.0	58	78.8	236	244.0	143	157.9
3.0–4.0	61	58.2	183	176.7	105	106.9
4.0–5.0	48	47.0	135	136.0	81	75.4
5.0–6.25	37	35.3	86	85.1	42	41.1
≥ 6.25	66	65.4	187	188.0	131	133.2
Total	411	411.0	1291	1291.0	850	850.0
χ^2	8.3		0.5		2.7	
d.f.	3		3		3	
λ	0.236		0.339		0.363	
ρ	1.834		1.404		1.510	
α	0.919		0.935		0.876	
$V(\lambda)$	0.001		0.001		0.001	
$V(\rho)$	0.131		0.021		0.042	
$V(\alpha)$	0.007		0.002		0.002	
Corr(λ, ρ)	−0.375		−0.176		−0.139	
Corr(λ, α)	−0.216		−0.280		−0.175	
Corr(ρ, α)	−0.796		−0.865		−0.913	

conception with marital duration 7 years or more on the reference date of survey. It could be seen that as the age at RM increased, the risk increased in the first year but in succeeding years was almost the same for all age at RM groups.

It is difficult to identify the inability of a couple to procreate, because sterility is suspected only if a woman is unable to conceive during a sufficiently long period after marriage. Estimates of the proportion of primarily sterile couples $(1 - \alpha)$ were obtained according to age at RM. These proportions were 0.081, 0.065, and 0.124 for age at RM of 14, 15–16, and 17–19 years, respectively. Many factors were responsible for the slight increase in this proportion for age at RM 17–19 years. This may be due to the fact that increasing age at RM may make conception difficult or a significant proportion of husbands of females migrate to urban areas for higher studies or in search of work, delaying conception for a considerable period.

About 6% of couples were reported to be primarily sterile based on the data of the present survey and others in the same locality [5, 11, 12]. Thus, the estimates obtained for females of age at RM 15–16 years may be considered to be closure to the true value.

APPENDIX: ESTIMATION

The risk of the first l conception was assumed to be of the form

$$\lambda(t)=\begin{cases}\lambda & \text{for } 0<t<a,\\ \rho\lambda & \text{for } t\geq a.\end{cases} \tag{A1}$$

Under this assumption, for $t<a$, expressions for $U_0(t)$, $U_1(t)$, and $U_2(t)$ in Equation (6) reduce to

$$U_0(t)=1-e^{-\lambda t}, \tag{A2}$$

$$U_1(t)=\int_0^t\{1-F_1(x)\}\lambda e^{-\lambda x}\,dx, \tag{A3}$$

$$U_2(t)=\int_{z=0}^{t}\int_{x=0}^{z}\int_{y=0}^{z-x}f_1(x)f_3(y)\,\lambda e^{-\lambda(z-y)}\,dy\,dx\,dz, \tag{A4}$$

and for $t\geq a$,

$$U_0(t)=1-\exp\{-\lambda\{a+\rho(t-a)\}\}, \tag{A5}$$

$$U_1(t)=\int_a^t[1-F_1(x)]\,\rho\lambda\exp\{-\lambda[a+\rho(x-a)]\}\,dx + \int_0^a\{1-F_1(x)\}\lambda e^{-\lambda x}dx \tag{A6}$$

$$\begin{aligned}U_2(t)=&\int_{z=a}^{t}\int_{x=0}^{a}f_1(x)\,\rho\lambda\\ &\times\left[\int_{y=0}^{a-x}f_3(y)\exp\{-\lambda[(a-y)+\rho(z-a)]\}\,dy\right.\\ &\left.+\int_{y=a-x}^{z-x}f_3(y)\exp\{-\lambda[x+\rho(z-x-y)]\}\,dy\right]dx\,dz\\ &+\int_{z=a}^{t}\int_{x=a}^{z}\int_{y=0}^{z-x}f_1(x)f_3(y)\,\rho\lambda\\ &\times\exp\{-\lambda[a+\rho(z-a-y)]\}\,dy\,dx\,dz\\ &+\int_{z=0}^{a}\int_{x=0}^{z}\int_{y=0}^{z-x}f_1(x)f_3(y)\,\lambda e^{-\lambda(z-y)}dy\,dx\,dz.\end{aligned} \tag{A7}$$

A procedure to obtain maximum likelihood (m.l.) estimates of the parameters λ, ρ, and α involved in the distribution of time of first l conception [Equations (5) and (7)] and their variance–covariance matrix for known values of parameters a and p and known density functions $f_1(t)$ and $f_3(t)$ is presented below for grouped data.

Let the range of the first l conception be partitioned into $k+1$ intervals $(t_0, t_1], (t_1, t_2], \ldots, (t_k, t_{k+1}]$, where $t_0 = 0$, $t_k = T$, $t_{k+1} = \infty$.

The probability that the time of a woman's first l conception falls in the interval $(t_{j-1}, t_j]$ is

$$P_j = \begin{cases} \alpha\left[F(t_j) - F(t_{j-1})\right] & j = 1,2,\ldots,k, \\ 1 - \alpha F(T) & j = k+1, \end{cases} \tag{A8}$$

where

$$F(t_j) = (1-p)U_0(t_j) + p\left[U_1(t_j) + U_2(t_j)\right].$$

In a sample of N women with effective marital duration T' years or more, let $n_1, n_2, \ldots, n_k$ be the number of women whose first l conception occurs during the intervals $1,2,\ldots,k$, respectively, such that $\Sigma_{l=1}^{k} n_i = n$. Either the remaining $n_{k+1} = N - n$ women are primarily sterile or their time of first l conception is more than T years $(T = T' - g)$. The observed number of women classified in this manner follows a multinomial distribution given by

$$L = \frac{N!}{\prod_{j=1}^{k+1} n_j!} \prod_{j=1}^{k+1} P_j^{n_j}. \tag{A9}$$

The m.l. estimates of the parameters are solutions to the equations

$$\sum_j \frac{n_j}{P_j} \frac{\delta P_j}{\delta \theta_u} = 0, \qquad u = 1,2,3, \tag{A10}$$

where $\theta_1 = \lambda$, $\theta_2 = \rho$, and $\theta_3 = \alpha$.

In order to calculate $\delta P_j / \delta \theta_u$, $u = 1,2,3$, we need the $\delta F(t_j)/\delta \theta_u$. For $u = 1,2$,

$$\frac{\delta P_j}{\delta \theta_u} = \begin{cases} \alpha \dfrac{\delta}{\delta \theta_u}\left[F(t_j) - F(t_{j-1})\right], & j = 1,2,\ldots,k, \\ -\alpha \dfrac{\delta}{\delta \theta_u} F(T) & j = k+1 \end{cases} \tag{A11}$$

and

$$\frac{\delta P_j}{\delta \theta_3} = \begin{cases} F(t_j) - F(t_{j-1}), & j = 1,2,\ldots,k, \\ -F(T) & j = k+1, \end{cases} \tag{A12}$$

where, for $u = 1,2$,

$$\frac{\delta F(t_j)}{\delta\theta_u} = (1-p)\frac{\delta}{\delta\theta_u}U_0(t_j) + p\left[\frac{\delta}{\delta\theta_u}U_1(t_j) + \frac{\delta}{\delta\theta_u}U_2(t_j)\right]. \quad \text{(A13)}$$

For $t < a$,

$$\frac{\delta U_0(t)}{\delta\lambda} = te^{-\lambda t} = t[1 - U_0(t)], \quad \text{(A14)}$$

$$\frac{\delta U_1(t)}{\delta\lambda} = \int_0^t [1 - F_1(x)](1-\lambda x)e^{-\lambda x}\,dx, \quad \text{(A15)}$$

$$\frac{\delta U_2(t)}{\delta\lambda} = \int_{z=0}^{t}\int_{x=0}^{z}\int_{y=0}^{z-x} f_1(x)f_3(y)[1-\lambda(z-y)]e^{-\lambda(z-y)}\,dy\,dx\,dz, \quad \text{(A16)}$$

$$\frac{\delta U_0(t)}{\delta\rho} = \frac{\delta U_1(t)}{\delta\rho} = \frac{\delta U_2(t)}{\delta\rho} = 0. \quad \text{(A17)}$$

For $t \geq a$,

$$\begin{aligned}\frac{\delta U_0(t)}{\delta\lambda} &= \exp\{-\lambda[a+\rho(t-a)]\}[a+\rho(t-a)] \\ &= [a+\rho(t-a)][1-U_0(t)], \quad \text{(A18)}\end{aligned}$$

$$\begin{aligned}\frac{\delta U_1(t)}{\delta\lambda} &= \int_a^t [1-F_1(x)]\,\rho\{1-\lambda[a+\rho(x-a)]\} \\ &\quad \times\exp\{-\lambda[a+\rho(x-a)]\}\,dx \\ &\quad + \int_0^a [1-F_1(x)](1-\lambda x)e^{-\lambda x}dx, \quad \text{(A19)}\end{aligned}$$

$$\begin{aligned}\frac{\delta U_2(t)}{\delta\lambda} = \int_{z=a}^{t}\int_{x=0}^{a} \rho f_1(x)\Bigg[&\int_{y=0}^{a-x} f_3(y)\{1-\lambda[(a-y)+\rho(z-a)]\} \\ &\times\exp\{-\lambda[(a-y)+\rho(z-a)]\}\,dy \\ &+\int_{y=a-x}^{z-x} f_3(y)\{1-\lambda[x+\rho(z-x-y)]\} \\ &\times\exp\{-\lambda[x+\rho(z-x-y)]\}\,dy\Bigg]dx\,dz \\ +\int_{z=a}^{t}\int_{x=a}^{z}\int_{y=0}^{z-x} &\rho f_1(x)f_3(y)\{1-\lambda[a+\rho(z-a-y)]\} \\ \times\exp\{-\lambda[a+&\rho(z-a-y)]\}\,dy\,dx\,dz\end{aligned}$$

$$+\int_{z=0}^{a}\int_{x=0}^{z}\int_{y=0}^{z-x} f_1(x)f_3(y)[1-\lambda(z-y)]xe^{-\lambda(z-y)}dy\,dx\,dz, \tag{A20}$$

$$\frac{\delta U_0(t)}{\delta\rho} = e^{-\lambda[a+\rho(t-a)]}\lambda(t-a) = \lambda(t-a)[1-U_0(t)], \tag{A21}$$

$$\frac{\delta U_1(t)}{\delta\rho} = \int_a^t [1-F_1(x)]\,\lambda[1-\rho\lambda(x-a)]e^{-\lambda\{a+\rho(x-a)\}}dx, \tag{A22}$$

$$\begin{aligned}\frac{\delta U_2(t)}{\delta\rho} = \int_{z=a}^{t}\int_{x=0}^{a}\lambda f_1(x)\Bigg[\int_{y=0}^{a-x} & f_3(y)[1-\lambda\rho(z-a)] \\ & \times\exp\{-\lambda[(a-y)+\rho(z-a)]\}dy \\ & +\int_{y=a-x}^{z-x} f_3(y)[1-\rho\lambda(z-x-y)] \\ & \times\exp\{-\lambda[x+\rho(z-x-y)]\}dy\Bigg]dx\,dz \\ +\int_{z=a}^{t}\int_{x=a}^{z}\int_{y=0}^{z-x} & \lambda f_1(x)f_3(y)[1-\rho\lambda(z-a-y)] \\ & \times\exp\{-\lambda[a+\rho(z-a-y)]\}dy\,dx\,dz.\end{aligned} \tag{A23}$$

Equation (A10) does not provide explicit expressions of m.l. estimates. Hence m.l. estimates of the parameters have been computed by the scoring method.

The m.l. estimates were obtained by solving the matrix equation

$$I\delta\boldsymbol{\theta} = S,$$

where

$$I = \{I_{st}\}, \qquad s,t = 1,2,3,$$
$$\delta\boldsymbol{\theta}' = (\delta\theta_1, \delta\theta_2, \delta\theta_3),$$
$$S' = (S_1, S_2, S_3),$$

and

$$I_{st} = N\sum_{j=0}^{k+1}\frac{1}{P_j}\frac{\delta P_j}{\delta\theta_s}\frac{\delta P_j}{\delta\theta_t}$$
$$S_s = \sum_{j=0}^{k+1}\frac{n_j}{P_j}\frac{\delta P_j}{\delta\theta_s}.$$

The asymptotic variance–covariance matrix of the m.l. estimates is the inverse of information matrix I.

A computer program has been developed to obtain (i) m.l. estimates of parameters λ, ρ, and α for known values of parameters a and p and known density functions $f_1(t)$ and $f_3(t)$ and (ii) theoretical cell frequencies.

The evaluation of the integrals in the calculation of P_i and the variance–covariance matrix was done using the NAG mathematical subroutine library program. Expressions involving single or double integrals were directly evaluated using the NAG subroutine. For evaluation of triple integrals, say

$$\int_{z_1}^{z_2}\left[\int_{\xi_1(z)}^{\xi_2(z)}\int_{\psi(x,z)}^{\psi_2(x,z)} f(x,y,z)\,dy\,dx\right]dz,$$

the first two integrals, that is, the integral over y and the integral over x, were evaluated using the NAG mathematical subroutine for each value of z in the interval (z_1, z_2) and then the numerical quadrature formula was used to evaluate the third integral, that is, the integral over z.

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