A NOTE ON WAVES DUE TO ROLLING OF A PARTIALLY IMMERSED NEARLY VERTICAL PLATE*

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Abstract. A rigid, nearly vertical, partially immersed wide plate is constrained to rotate about a horizontal axis through it. The waves from small rolling oscillations of the plate are studied. Expressions for the first-order corrections to the amplitudes of the wave motion so set at large distances on the right and left sides of the plate are obtained by the use of Green's integral theorem. Assuming a Fourier expansion of a function related to the shape of the plate, these corrections are calculated explicitly. Considering some particular explicit forms for the shape function, numerical calculations are performed.

Key words. rolling oscillation, nearly vertical plate, amplitude at infinity

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1. Introduction. Within the framework of linearised theory of water waves, only a limited number of problems admit of exact solutions. One such problem is concerned with oscillation of a partially immersed thin vertical plate due to rolling about a horizontal axis in its plane. The problem was tackled initially by Ursell [7] who used Havelock's expansion of water wave potential to obtain the wave amplitude at infinity, and also the explicit form of the velocity potential at any point in the fluid region can be found from his analysis. Later, Evans [2] computed the wave amplitude at infinity produced by the general motion of a partially immersed thin flexible plate by a simple application of Green's integral theorem in a very interesting manner.

Problems associated with nearly vertical barriers were first tackled by Shaw [6], who considered the diffraction of water waves by a partially immersed nearly vertical plate and used Green's integral theorem to reduce the problem to the solution of a singular integral equation whose solution was then obtained up to first order by a perturbational technique. The first-order corrections to the reflection and transmission coefficients were also obtained. Recently, Mandal and Chakrabarti [4], used a simplified perturbational technique (different from Shaw's [6]) along with the application of Green's theorem with Evans's type of idea, to determine these corrections in a much simpler way. Very recently, Mandal and Kundu [5] also used the method of Shaw [6] as well as that of [4] for the problem of diffraction by a submerged nearly vertical plate.

In the present paper we generalize the problem of rolling of a partially immersed vertical plate considered in [7] to the problem of rolling of a partially immersed nearly vertical plate. Because of the curved nature of the plate, the amplitudes of the wave motion at large distances on the two sides of the plate could not be the same. The first-order corrections to these wave amplitudes are obtained here by employing a perturbational analysis similar to that used in [4] and [5] along with the exploitation of Evans's idea of utilizing a suitable application of Green's integral theorem. Considering some particular forms for the shape of the curved plate, the amplitude of the wave motion at large distances from the plate on its two sides are calculated numerically and compared with vertical plate results.

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2. Statement and formulation of the problem. We consider a nearly vertical thin plate $x = \varepsilon c(y)$, $0 \leqq y \leqq a$ (where $\varepsilon \ll 1$ and $c(y)$ is a continuous bounded function) extending from above the free surface of an incompressible inviscid fluid occupying the region $y \geqq 0$ with $y = 0$ as the mean free surface. The plate is hinged at $(\varepsilon c(b), b)$ and is forced to perform small simple harmonic oscillations of amplitude $\theta = \mathrm{Re}\,(\theta_0\, e^{-i\sigma t})$ about its nearly vertical mean position, σ being the frequency of oscillation. We assume the motion to be irrotational and is described by the velocity potential $\mathrm{Re}\,\{\varphi(x, y)\, e^{-i\sigma t}\}$. Then φ satisfies

$$\nabla^2 \varphi = 0 \quad \text{in the fluid region,} \tag{2.1}$$

the linearised free surface condition

$$K\varphi + \frac{\partial \varphi}{\partial y} = 0 \quad \text{on } y = 0, \quad x \neq 0, \tag{2.2}$$

where $K = \sigma^2/g$, g being the acceleration due to gravity, the condition on the plate (see Appendix A for derivation of the condition)

$$\frac{\partial \varphi}{\partial n} = i\sigma\theta_0(y - b) \quad \text{on } x = \varepsilon c(y), \quad 0 < y < a, \tag{2.3}$$

$\partial/\partial n$ being the normal derivative on the curved plate. If A and B denote the amplitudes of the wave motion set up by rolling oscillations of the plate at large distances on its two sides, then

$$\varphi \sim \begin{cases} A\, e^{-Ky+iKx} & \text{as } x \to \infty, \\ B\, e^{-Ky-iKx} & \text{as } x \to -\infty. \end{cases} \tag{2.4}$$

Also φ satisfies the edge condition that

$$r^{1/2}\nabla\varphi \quad \text{is bounded as } r \to 0, \tag{2.5}$$

where r is the distance from the lower edge of the plate, and the bottom condition that

$$\varphi, \nabla\varphi \to 0 \quad \text{as } y \to \infty. \tag{2.6}$$

3. The method of solution. The boundary condition (2.3) can be expressed approximately to the first order of ε in the form (cf. Shaw [6])

$$\frac{\partial \varphi}{\partial x} - \varepsilon \frac{d}{dy}\left\{c(y)\frac{\partial \varphi}{\partial y}\right\} = i\sigma\theta_0(y - b) \quad \text{on } x = \pm 0, \quad 0 < y < a. \tag{3.1}$$

This form of boundary condition suggests that we may assume the following perturbational expansion, in terms of the small parameter ε, for the unknown quantities φ, $A(\varepsilon)$, and $B(\varepsilon)$ as

$$\begin{gathered} \varphi(x, y, \varepsilon) = \varphi_0(x, y) + \varepsilon\varphi_1(x, y) + O(\varepsilon^2), \\ A = A_0 + \varepsilon A_1 + O(\varepsilon^2), \qquad B = B_0 + \varepsilon B_1 + O(\varepsilon^2). \end{gathered} \tag{3.2}$$

Here A_0 and B_0 are the amplitudes (complex) at infinity of the wave motion set up by the rolling oscillations of a partially immersed vertical plate on its two sides, so that $A_0 = -B_0$ (cf. Evans [2]). Thus A_1 and B_1 are the first-order corrections to the wave amplitudes at large distances from the nearly vertical plate at its two sides. We content ourselves with the calculations of A_1 and B_1. Substituting (3.2) in the basic partial differential equation (2.1) and the boundary conditions (2.2), (3.1), (2.4)–(2.6), we find after equating the coefficients of identical powers of ε from both sides of the results thus derived, that the functions φ_0 and φ_1 are the solutions of the following two independent boundary value problems.

(BVPI) The function φ_0 satisfies

$$\nabla^2\varphi_0=0 \quad \text{in } y>0,$$

$$K\varphi_0+\frac{\partial\varphi_0}{\partial y}=0 \quad \text{on } y=0, \quad |x|>0,$$

$$\frac{\partial\varphi_0}{\partial x}=i\sigma\theta_0(y-b) \quad \text{on } x=\pm 0, \quad 0<y<a,$$

$$\varphi_0\sim\begin{cases}A_0\, e^{-Ky+iKx} & \text{as } x\to\infty,\\ -A_0\, e^{-Ky-iKx} & \text{as } x\to-\infty,\end{cases}$$

$$r^{1/2}\nabla\varphi_0 \quad \text{is bounded as } r=\{x^2+(y-a)^2\}^{1/2}\to 0,$$

$$\varphi_0,\ \nabla\varphi_0\to 0 \quad \text{as } y\to\infty.$$

(BVPII) The function $\varphi_1(x, y)$ satisfies

$$\nabla^2\varphi_1=0 \quad \text{in } y>0,$$

$$K\varphi_1+\frac{\partial\varphi_1}{\partial y}=0 \quad \text{on } y=0, \quad |x|>0,$$

$$\frac{\partial\varphi_1}{\partial x}(\pm 0, y)=\frac{d}{dy}\left\{c(y)\frac{\partial\varphi_0}{\partial y}(\pm 0, y)\right\}, \qquad 0<y<a,$$

$$\varphi_1\sim\begin{cases}A_1\, e^{-Ky+iKx} & \text{as } x\to\infty,\\ B_1\, e^{-Ky-iKx} & \text{as } x\to-\infty,\end{cases}$$

$$r^{1/2}\nabla\varphi_1 \quad \text{is bounded as } r=\{x^2+(y-a)^2\}^{1/2}\to 0,$$

$$\varphi_1,\ \nabla\varphi_1\to 0 \quad \text{as } y\to\infty.$$

For (BVPI), we note that $\varphi_0(x, y)$ is the velocity potential of the motion set up by small rolling oscillations of a thin vertical partially immersed plate about a line in its plane through the point $(0, b)$, and its solution was obtained in [7] (although the explicit form was not given there). It is given by (see Appendix B for its derivation)

$$\varphi_0(x,y)=\begin{cases}A_0\, e^{-Ky+iKx}+\displaystyle\int_0^\infty \chi(k)(k\cos ky-K\sin ky)\, e^{-kx}\, dk, & x>0,\\ -A_0\, e^{-Ky-iKx}-\displaystyle\int_0^\infty \chi(k)(k\cos ky-K\sin ky)\, e^{kx}\, dk, & x<0,\end{cases} \tag{3.3}$$

where

$$\chi(k)=\frac{\sigma a\theta_0}{k^2+K^2}\left[(q+ip)aJ_1(ka)-\frac{iKa}{k}J_2(ka)\right.$$
$$\left.+\frac{i(Kb-1)}{k}\{J_1(ka)\mathbf{H}_0(ka)-\mathbf{H}_1(ka)J_0(ka)\}\right],$$

$$q=\frac{\pi}{\Delta_1 Ka}\left[\frac{1}{2}+\frac{Kb-1}{Ka}\{I_1(Ka)+\mathbf{L}_1(Ka)\}\right],$$

$$\Delta_1=K_1^2(Ka)+\pi^2 I_1^2(Ka),$$

$$p=\frac{1}{I_1(Ka)}\left[\frac{1}{2}I_2(Ka)+\frac{1-Kb}{Ka}\left\{I_1(Ka)\mathbf{L}_0(Ka)-\mathbf{L}_1(Ka)I_0(Ka)\right\}-\frac{q}{\pi}K_1(Ka)\right], \tag{3.4}$$

$$A_0=\sigma\theta_0 qa^2\{\pi I_1(Ka)-iK_1(Ka)\}. \tag{3.5}$$

Let $\Psi_0(x, y)$ be the solution of the diffraction problem for a partially immersed thin vertical barrier due to a normally incident train of surface water waves represented by $e^{-Ky+iKx}$. Then $\Psi_0(x, y)$ is given by [4]

$$\Psi_0(x, y) = \begin{cases} e^{-Ky+iKx} + R_0\, e^{-Ky-iKx} \\ \quad + \dfrac{1}{\Delta}\displaystyle\int_0^\infty \frac{J_1(ka)\, e^{kx}(k\cos ky - K\sin ky)}{k^2+K^2}\, dk & \text{for } x<0, \\ T_0\, e^{-Ky+iKx} \\ \quad - \dfrac{1}{\Delta}\displaystyle\int_0^\infty \frac{J_1(ka)\, e^{-kx}(k\cos ky - K\sin ky)}{k^2+K^2}\, dk & \text{for } x>0, \end{cases}$$

where

$$R_0 = \frac{\pi I_1(Ka)}{\Delta}, \qquad T_0 = \frac{iK_1(Ka)}{\Delta}, \tag{3.6}$$

$$\Delta = \pi I_1(Ka) + iK_1(Ka).$$

In order to obtain B_1 we utilize Evans's [2] idea along with the application of Green's theorem to the harmonic functions $\varphi_1(x, y)$ and $\Psi_0(x, y)$ in the region bounded by the lines

$$y=0,\quad 0<x\leqq X, \qquad x=X,\quad 0\leqq y\leqq Y, \qquad y=Y,\quad -X\leqq x\leqq X,$$

$$x=-X,\quad 0\leqq y\leqq Y, \qquad y=0,\quad -X\leqq x<0, \qquad x=0-,\quad 0\leqq y\leqq a,$$

$$x=0+,\quad 0<y<a \quad \text{with } X, Y>0,$$

and ultimately make both X, $Y\to\infty$. We find that

$$-iB_1 = \int_0^a \left\{ \Psi_0(+0, y)\frac{\partial\varphi_1}{\partial x}(+0, y) - \Psi_0(-0, y)\frac{\partial\varphi_1}{\partial x}(-0, y)\right\} dy. \tag{3.7}$$

Now it is known that (cf. [4])

$$\Psi_0(+0, y) = \begin{cases} e^{-Ky}\{1\pm M(y)\} & \text{for } y<a, \\ e^{-Ky} & \text{for } y\geqq a, \end{cases}$$

where

$$M(y) = \frac{1}{\Delta a}\int_a^y \frac{t\, e^{Kt}}{(a^2-t^2)^{1/2}}\, dt.$$

Using this in (3.7) we find

$$-iB_1 = \int_0^a e^{-Ky}\frac{d}{dy}\left[c(y)\frac{\partial}{\partial y}\{\varphi_0(+0, y) - \varphi_0(-0, y)\}\right] dy$$

$$+ \int_0^a e^{-KY} M(y)\frac{d}{dy}\left[c(y)\frac{\partial}{\partial y}\{\varphi_0(+0, y) + \varphi_0(-0, y)\}\right] dy.$$

But from (3.3) we find that

$$\varphi_0(+0, y) = -\varphi_0(-0, y) \quad \text{for all } y; \tag{3.8}$$

then the second term in the right side vanishes so that B_1 is given by

$$B_1 = -i\int_0^a e^{-Ky}\frac{d}{dy}\left[c(y)\frac{\partial}{\partial y}\{\varphi_0(+0, y) - \varphi_0(-0, y)\}\right] dy. \tag{3.9}$$

Similarly, to obtain A_1 we apply Green's theorem to $\varphi_1(x, y)$ and $\Psi_0(-x, y)$ in the same region and it is found that

$$A_1 = B_1. \tag{3.10}$$

To simplify (3.9) further, we need a convenient expression for $\varphi_0(\pm 0, y)$. To obtain this let us define

$$\Psi(y) = \varphi_0(+0, y) - \varphi_0(-0, y).$$

Then from (3.8) we find that

$$\Psi(y) = \pm 2\varphi_0(\pm 0, y) \tag{3.11}$$

so that

$$\Psi(y) = 0 \quad \text{for } y \geqq a.$$

But from (3.3) we find that

$$K\Psi(y) + \frac{d\Psi}{dy} = -2\int_0^\infty (k^2 + K^2)\chi(k) \sin ky \, dk.$$

The integral on the right side can be calculated and it is given by (cf. [1, p. 99])

$$L(y) \quad \text{for } 0 < y < a, \qquad 0 \quad \text{for } y \geqq a,$$

where

$$L(y) = \sigma a \theta_0 \left[(q + ip) \frac{y}{(a^2 - y^2)^{1/2}} - \frac{iK}{2a} y (a^2 - y^2)^{1/2} + \frac{2i}{a\pi}(1 - Kb) y \ln \frac{y}{y + (a^2 - y^2)^{1/2}} \right]. \tag{3.12}$$

Thus utilizing (3.11) and (3.8), we find

$$\varphi_0(\pm 0, y) = \begin{cases} \pm e^{-Ky} \int_y^a e^{Kt} L(t) \, dt & \text{for } 0 < y < a, \\ 0 & \text{for } y \geqq a. \end{cases}$$

Hence from (3.9) and (3.10) we finally obtain

$$A_1 = B_1 = 2iK \int_0^a L(y) g(y) \, dy, \tag{3.13}$$

where

$$g(y) = e^{Ky} \int_0^y \{c'(t) - Kc(t)\} e^{-2Kt} \, dt, \qquad 0 < y < a, \tag{3.14}$$

which is obviously related to the shape of the curved plate. Thus (3.13) is the general result for the first-order correction to the amplitude of the wave motion at a large distance from the curved plate.

We now assume a general Fourier expansion for $G(\theta) \equiv g(a \sin \theta)/a$ $(0 \leqq \theta \leqq \pi/2)$ to deduce A_1. We assume that

$$G(\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos 4n\theta + b_n \sin 4n\theta), \tag{3.15}$$

where a_n, b_n's can be obtained once the shape of the plate is known. Thus we obtain from (3.13)

$$A_1 = B_1 = 2\sigma a^2\theta_0 K\left[\frac{a_0\alpha_0}{2} + \sum_{n=1}^{\infty}(a_n\alpha_n + b_n\beta_n)\right], \tag{3.16}$$

where

$$\alpha_0 = -(p - iq) + \frac{Ka}{3} + \frac{Kb-1}{\pi},$$

$$\alpha_n = \left\{p - iq + \frac{Ka}{2}\frac{16n^2+3}{9-16n^2} + \frac{16n(Kb-1)}{\pi}\right\}\frac{1}{16n^2-1}, \tag{3.17}$$

$$\beta_n = \left\{4(p-iq)n + 2Kan\frac{5-8n^2}{16n^2-9} + \frac{1-Kb}{\pi(2n-1)}\right\}\frac{1}{16n^2-1} + \frac{Kb-1}{2\pi(1-4n^2)}S_n,$$

where

$$S_n = \int_0^{\pi/2}\frac{\sin(4n+2)\theta}{\sin\theta}\,d\theta$$

so that

$$S_n = \frac{4}{1-16n^2} + S_{n-1}, \qquad S_0 = 2,$$

from which all S_n's can be calculated.

α_n's and β_n's are numerical constants independent of the shape of the curved plate and can be evaluated once Ka and b/a are known. Some representative values of these coefficients α_n $(n = 0, 1, 2, 3)$ and β_n $(n = 1, 2, 3)$ are given in Table 1. We note that for any fixed values of the parameters Ka and b/a, α_n's and β_n's decrease as n increases.

We now consider particular explicit forms for the shape function $c(y)$. Let us take

$$c(y) = a\left(1 - \frac{y}{a}\right)^m, \tag{3.18}$$

where m is a positive integer. Then $G(\theta)$ can be evaluated explicitly from (3.14) and is given by

$$G(\theta) = \frac{1}{2}\{e^{-Ka\sin\theta}(1-\sin\theta)^m - e^{Ka\sin\theta}\} + \sum_{j=1}^{m}\frac{m(m-1)\cdots(m-j+1)}{(-2)^{j+1}(Ka)^j}\{e^{-Ka\sin\theta}(1-\sin\theta)^{m-j} - e^{Ka\sin\theta}\}. \tag{3.19}$$

TABLE 1
$b/a = 0.1$.

Ka	α_0	α_1	α_2	α_3	β_1	β_2	β_3
0.1	−.8414829 −.0053821i	−.3078702 +.0003588i	−.1521472 +.000085i	−.1022436 +.0000376i	.2670097 +.001435i	.0933961 +.0006834i	.0568813 +.0004516i
0.5	−.7461219 −.1709089i	−.3270991 +.0113939i	−.1487424 +.0027128i	−.0991381 +.0011952i	.2988610 +.0455756i	.1115262 .0217027i	.0691492 +.0143419i
1.0	.0782676 +.4469231i	−.3981478 −.029794i	−.1556798 −.007094i	−.1001877 −.0031253i	.1506249 −.1191794i	.0446412 −.0567521i	.0253076 −.0375039i
1.5	−.0184488 +.2210263i	−.4077895 −.014735i	−.1479965 −.0035083i	−0.947959 −.0015456i	.2480171 −.0589403i	.094722 −.0280668i	.0587614 −.0185476i

The Fourier coefficients a_n $(n=0, 1, 2, \cdots)$ and b_n $(n=1, 2, \cdots)$ in the expansion (3.15) of $G(\theta)$ in $(0, \pi/2)$ cannot be found explicitly. However, they can be calculated numerically once Ka and m are known.

4. Discussion. Taking $\varepsilon=.001$ and $.0005$, we have computed the quantities $|A_0^*|$, $|A^*|$, and $|B^*|$ for various values of Ka between 0.1 and 2.0 and m between one and five, where

$$|A_0^*|=\frac{|A_0|}{g\theta_0 a^{3/2}}, \quad |A^*|=\frac{|A_0+\varepsilon A_1|}{g\theta_0 a^{3/2}}, \quad |B^*|=\frac{|-A_0+\varepsilon A_1|}{g\theta_0 a^{3/2}}.$$

Here $|A_0|$ is the amplitude (actual) of the wave motion at large distances set up by the rolling oscillations of a vertical plate, $|A|=|A_0+\varepsilon A_1|$ and $|B|=|-A_0+\varepsilon A_1|$ are the same (up to first order of ε) for a curved plate at its right and left sides, respectively. A representative set of these values are given in Tables 2(a) and 2(b).

From (3.5) we note that $|A_0|$ first increases and then decreases with Ka. This is also reflected in the numerical results for $|A_0^*|$. Here $|A_0^*|$ first increases and then descreases as Ka increases from 0.1 to 2.0. For any fixed m, a similar behaviour is observed for the values of $|A^*|$ and $|B^*|$. Thus the qualitative behaviour of the amplitudes at infinity of the wave motion set up due to rolling oscillations of an immersed curved plate on its two sides is the same as that for a vertical plate with respect to the wave number Ka.

Again for a fixed Ka (and fixed ε), the value of $|A^*|$ decreases while that of $|B^*|$ increases rather slowly as m increases from 1. However, these variations are observed only beyond the fourth decimal place. This is plausible since the variation of m $(m=1, 2, \cdots, 5)$ causes a very slow change in the free surface slope $(=-1/\varepsilon m)$ of the curved plate as ε is much smaller compared to m. It is also observed that the values of $|A^*|$ or $|B^*|$ differ from $|A_0^*|$ (the vertical plate result) beyond the fourth

TABLE 2(a)
$c(y)=a(1-y/a)^m$, $b/a=0.1$.
$\varepsilon=.001$.

	m	1		2		3		4	
Ka	$\|A_0^*\|=\|B_0^*\|$	$\|A^*\|$	$\|B^*\|$	$\|A^*\|$	$\|B^*\|$	$\|A^*\|$	$\|B^*\|$	$\|A^*\|$	$\|B^*\|$
0.2	.0501307	.0501263	.0501354	.0501260	.0501358	.0501259	.0501360	.0501257	.0501361
0.6	.2831708	.2829558	.2833875	.2829388	.2834049	.282982	.2834156	.2829200	.2834240
1.0	.3564812	.3558018	.3571626	.3557473	.3572176	.3557087	.3572565	.3556779	.3572876
1.4	.3175393	.3166035	.3184769	.3165367	.3185441	.3164818	.3185994	.3164349	.3186465
1.8	.2756030	.2750337	.2761739	.2750272	.2761807	.2750030	.2762053	.2749736	.2762349

TABLE 2(b)
$c(y)=a(1-y/a)^m$, $b/a=0.1$.
$\varepsilon=.0005$.

	m	1		2		3		4	
Ka	$\|A_0^*\|=\|B_0^*\|$	$\|A^*\|$	$\|B^*\|$	$\|A^*\|$	$\|B^*\|$	$\|A^*\|$	$\|B^*\|$	$\|A^*\|$	$\|B^*\|$
0.2	.0501307	.0501284	.0501330	.0501283	.0501332	.0501282	.0501333	.0501281	.0501334
0.6	.2831708	.2830631	.2832789	.2830546	.2832876	.2830493	.2832929	.2830451	.2832972
1.0	.3564812	.3561413	.3568217	.3561140	.3568491	.3560947	.3568685	.3560792	.3568841
1.4	.3175393	.3170712	.3180079	.3170378	.3180414	.3170103	.3180690	.3169867	.3180926
1.8	.2756030	.2753181	.2758882	.2753148	.2758916	.2753026	.2759038	.2752879	.2759186

decimal place for small values of Ka (here for $Ka < 0.3$) while this difference occurs in the third decimal place as Ka increases (here for $Ka > 0.3$). This shows that the influence of ε and m on the amplitudes at large distances from the plate is quite significant when the wavenumber is not small.

5. Conclusion. First-order corrections to the wave amplitudes at large distances, of the motion set up by the rolling oscillations of an immersed nearly vertical plate, are obtained here by using a simplified perturbational approach. For some explicit forms of the shape function of the plate, the wave amplitudes at large distances from the plate are calculated up to first order analytically. Their magnitudes are computed numerically and it is observed that the influence of the shape of the plate on these values is to some extent significant.

Appendix A. Derivation of (2.3). To deduce (2.3) we note that as the barrier is hinged at $(\varepsilon c(b), b)$ and performs rolling oscillations with small amplitude θ_0 and frequency σ, its angular velocity is

$$\mathbf{w} = w\mathbf{k},$$

where

$$w = \operatorname{Re}(-i\sigma\theta_0 e^{-i\sigma t}) \tag{A1}$$

and $\mathbf{k}$ is the unit vector along the z-direction. Let

$$\mathbf{r} = \mathbf{i}(x - \varepsilon c(b)) + \mathbf{j}(y - b) \tag{A2}$$

so that r is the distance of any point on the plate from the point $(\varepsilon c(b), b)$, where $\mathbf{i}$, $\mathbf{j}$ are the unit vectors along the x, y directions, respectively. The velocity $\mathbf{v}$ at any point on the plate is then given by

$$\mathbf{v} = \operatorname{Re}[i\sigma\theta_0 e^{-i\sigma t}\{\mathbf{i}(y-b) + \mathbf{j}\varepsilon(c(y) - c(b))\}]. \tag{A3}$$

As the plate communicates its normal velocity to the adjacent fluid particles,

$$v_n = \operatorname{Re}\left\{\frac{\partial \varphi}{\partial n} e^{-i\sigma t}\right\}$$

on the curved plate. Now

$$v_n = \mathbf{v} \cdot \mathbf{n},$$

where $\mathbf{n}$ is the unit vector normal to the curved plate and to the first order of ε, is given by

$$\mathbf{n} = \mathbf{i} - \mathbf{j}\varepsilon c'(y) + O(\varepsilon^2). \tag{A4}$$

Thus from (A3) and (A4) we find that

$$\begin{aligned} v_n &= \operatorname{Re}[i\sigma\theta_0 e^{-i\sigma t}\{\mathbf{i}(y-b) + \mathbf{j}(c(y) - c(b))\} \cdot \{\mathbf{i} - \mathbf{j}\varepsilon c'(y)\} + O(\varepsilon^2)] \\ &= \operatorname{Re}[i\sigma\theta_0 e^{-\sigma i t}(y-b) + O(\varepsilon^2)]. \end{aligned}$$

Thus

$$\frac{\partial \varphi}{\partial n} = i\sigma\theta_0(y - b) \quad \text{on the plate.}$$

Appendix B. Derivation of $\varphi_0(x, y)$. A representation for $\varphi_0(x, y)$ can be obtained from [7] with obvious modifications (the notation A and B in [7] will be changed to D, E, respectively, here) and is given by

$$\text{(B1)}\quad \begin{aligned} \varphi_0(x, y) &= \sigma a \theta_0 \bigg[(D - iE)\, e^{-Ky+iKx} \\ &\quad + \int_0^\infty \{S(k) + iC(k)\}\, e^{-kx}(k \cos ky - K \sin ky)\, dk \bigg], \qquad x > 0, \\ \varphi_0(x, y) &= -\varphi_0(-x, y), \qquad x < 0. \end{aligned}$$

Then in the notation of (3.3), we have

$$A_0 = \sigma a \theta_0 (D - iE), \qquad \chi(k) = \sigma a \theta_0 \{S(k) + iC(k)\}.$$

Explicit expressions for D, E, and $C(k)$ are given in [7]. However the expression for $S(k)$ is not given there, but can be calculated easily from the relation

$$-\frac{\pi}{2}(k^2 + K^2)kS(k) = \int_0^\infty f(y)(k \cos ky - K \sin ky)\, dy,$$

where

$$f(y) = \begin{cases} 0 & \text{for } 0 < y < a, \\ q \dfrac{d}{dy}\left(e^{-Ky} \displaystyle\int_a^y \frac{u\, e^{Ku}}{(u^2 - a^2)^{1/2}}\, du \right) & \text{for } y > a, \end{cases}$$

q being an unknown constant. Then $S(k)$ is found as

$$\text{(B2)}\qquad S(k) = qa \frac{J_1(ka)}{k^2 + K^2}.$$

Again, an expression for $C(k)$ is given in [7] and this can be further simplified as (cf. [3, p. 683])

$$\text{(B3)}\quad \begin{aligned} &-\frac{\pi}{2} k(k^2 + K^2)C(k) \\ &= \frac{\pi}{4} KaJ_2(ka) + \frac{\pi}{2}(1 - Kb)\{J_1(ka)\mathbf{H}_0(ka) - J_0(ka)\mathbf{H}_1(ka)\} - \frac{\pi}{2} pkaJ_1(ka), \end{aligned}$$

where p is a constant.

For finding the constants D, E, and q, Ursell [7] first obtained the expressions for D and E connecting only p. These can be further simplified as follows (cf. [3, p. 683]):

$$\text{(B4)}\quad \begin{aligned} D &= \frac{Kb - 1}{K^2 a}[1 + Ka\{K_1(Ka)\mathbf{L}_0(Ka) + K_0(Ka)\mathbf{L}_1(Ka)\}] + \frac{a}{2} K_2(Ka) + paK_1(Ka), \\ E &= (1 - Kb)\{I_1(Ka)\mathbf{L}_0(Ka) - I_0(Ka)\mathbf{L}_1(Ka)\} + \frac{Ka}{2} I_2(Ka) - \pi apI_1(Ka) \end{aligned}$$

(an obvious misprint in the expression of E, i.e., B in [7], is corrected here).

Again, [7] also obtained

$$\text{(B5)}\qquad D = qa\pi I_1(Ka), \qquad E = qaK_1(Ka).$$

Explicit expressions for q and p can now be obtained and these are given in (3.5). D and E are then found from (B4). It is now a simple task to find $\chi(k)$ and A_0, and these are given by (3.4) and (3.5) (the last relation), respectively.

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