Scattering of water waves by thin vertical plate submerged below ice-cover surface*

P. MAITI[1], P. RAKSHIT[2], S. BANERJEA[1]

(1. Department of Mathematics, Jadavpur University, Kolkata 700032, India;

2. Deshapran Birendranath Institution For Boys, 198-B S.P. Mukherjee Road, Kolkata 700026, India)

Abstract The present paper is concerned with scattering of water waves from a vertical plate, modeled as an elastic plate, submerged in deep water covered with a thin uniform sheet of ice. The problem is formulated in terms of a hypersingular integral equation by a suitable application of Green's integral theorem in terms of difference of potential functions across the barrier. This integral equation is solved by a collocation method using a finite series involving Chebyshev polynomials. Reflection and transmission coefficients are obtained numerically and presented graphically for various values of the wave number and ice-cover parameter.

Key words scattering problem, ice-cover surface, hypersingular integral equation, Green's integral theorem, reflection and transmission coefficients

Chinese Library Classification O352.2

2010 Mathematics Subject Classification 76B15

1 Introduction

Linear water wave interaction with thin floating plate has been a subject of interest since early twentieth century because this is perhaps the simplest model of breakwater. Dean[1] studied the problem of water wave scattering by a thin vertical plate completely submerged and extending infinitely downwards in deep water under the assumption of the linearized theory. Later, Ursell[2] used the singular integral equation formulation to obtain the closed form solution of the problem of scattering of water waves by a thin vertical barrier partially immersed in deep water. Evans[3] used a complex variable theory to study the problem of scattering of water waves by a submerged vertical plate present in deep water. The works mentioned above are among the limited number of problems which admit of closed form solution.

During the last decade, immense theoretical advances have been made on the topic of ocean wave and sea ice interaction. As the boundary between the ocean and atmosphere in the polar region, sea ice plays a critical role as a leading indicator in global climatic change. Sea ice acts as both sunscreen and blanket to the ocean surface, which reflects the solar rays. This prevents the solar radiation from heating the water beneath and also prevents the ocean heat from escaping

∗ Received Oct. 28, 2010 / Revised Mar. 22, 2011

Project supported by the Department of Science and Technology of New Delhi (No.SR/SY/MS:521/08)

Corresponding author S. BANERJEA, Ph. D., E-mail: sbanerjee@math.jdvu.ac.in

to air above. This phenomena has important bearing on conserving the marine life. Thus, sea ice acts as Earth's polar refrigerator, cooling it and protecting it from absorbing too much heat from sunlight. Continuous unbroken sheet of ice extending over a large distance that occurs in the polar region, often encounters with waves propagating at free surface. The amplitude of the waves travelling beneath the ice is important to be studied as it causes the ice-cover to bend. The bending of ice-cover is attributed to its elastic property where the continuous thin sheet of ice is modelled as a thin elastic plate. A significant research is carried out to study the problem related to ocean wave interaction with sea ice (see Fox and Squire[4], Squire[5], Chung and Fox[6], Linton and Chung[7], Chakrabarti[8], Gayen et al.[9]). Mathematically, the boundary value problem (BVP) related to ice-cover involves fifth order derivative of the potential function in the ice cover condition whereas the governing partial differential equation is of second order.

The literature concerning a study of ocean wave interaction with ice-cover in the presence of a body submerged beneath the ice-cover floating in a deep water is rather limited.

Das and Mandal[10] investigated the problem of ocean wave and sea ice interaction in the presence of a long horizontal cylinder. Maiti and Mandal[11] considered the problem when the barrier is in the form of a thin vertical plate submerged at an angle with the vertical beneath the ice-cover extending infinitely downwards. They used Green's integral theorem to reduce the BVP to a hypersingular integral equation in terms of the difference of potential across the barrier. Following Parsons and Martin[12] the hypersingular integral equation was then solved by collocation method after approximating the unknown function satisfying the integral equation by a finite series of Chebyshev polynomials.

In the present paper, we study the problem of wave diffraction by a thin vertical plate submerged beneath ice-cover in deep water. Here, we use Green's integral theorem to reduce the boundary value problem to a hypersingular integral equation in terms of the unknown difference of potentials across the barrier. A collocation method is used to solve the hypersingular integral equation after approximating the unknown function satisfying the integral equation by a finite series of Chebyshev polynomials. This reduces the equation to a system of linear equations which can be solved by standard methods. Using the solution to the hypersingular integral equation, the reflection and transmission coefficients are evaluated numerically and depicted graphically against the wave number.

2 Formulation of problem

We consider two-dimensional irrotational motion of an incompressible, inviscid, and homogeneous fluid covered with ice. We choose a rectangular Cartesian coordinate system, in which the y-axis is directed vertically downwards into the fluid region $y \geqslant 0$ and the x-axis is along the rest position of the lower part of ice-cover. Ice cover is modeled as a thin elastic plate of thickness $h_{\rm i}$ and density $\rho_{\rm i}$. A thin rigid plate at $x = 0$, $a < y < b$ is submerged beneath the ice cover. A train of time harmonic waves represented by velocity potential $\mathrm{Re}(\phi^{inc}(x, y)\mathrm{e}^{-\mathrm{i}\sigma t})$ is incident on the barrier from negative infinity, where

$$\phi^{inc}(x, y) = \mathrm{e}^{-\lambda K y + \mathrm{i}\lambda K x}, \tag{1}$$

σ is the angular frequency, and $k = \lambda K$ is the unique real positive root of the dispersion relation

$$(Dk^4 + 1 - \delta K)k - K = 0, \tag{2}$$

$K = \frac{\sigma^2}{g}$, g is the acceleration due to gravity, $D = \frac{L}{\rho g}$, $L = \frac{Eh_{\rm i}^3}{12(1-\nu^2)}$ is the flexural rigidity of ice, E is Young's modulus, ν is the Poisson ratio of the elastic material of the ice-cover, ρ is the density of water, $\delta = \frac{\rho_{\rm i}}{\rho}h_{\rm i}$.

The other roots of (2) are $\lambda_1 K$, $\overline{\lambda}_1 K$, $\lambda_2 K$, and $\overline{\lambda}_2 K$ with $\mathrm{Re}\,\lambda_1 > 0$ and $\mathrm{Re}\,\lambda_2 < 0$. Assuming linear theory, the two-dimensional motion is represented by the velocity potential

$\mathrm{Re}\,(\phi(x,y)\mathrm{e}^{-\mathrm{i}\sigma t})$, where ϕ satisfies

$$\nabla^2\phi = 0, \quad y \geqslant 0, \tag{3}$$

$$\Big(D\frac{\partial^4}{\partial x^4} + 1 - \delta K\Big)\phi_y + K\phi = 0 \quad \text{on} \quad y = 0, \tag{4}$$

$$\phi_x = 0 \quad \text{on} \quad x = 0, \quad a < y < b, \tag{5}$$

$r^{\frac{1}{2}}\nabla\phi$ is bounded as

$$r = ((x-a)^2 + (y-b)^2)^{\frac{1}{2}} \to 0. \tag{6}$$

The far field conditions are

$$\phi(x,y) \to \begin{cases} \phi^{inc}(x,y) + R\phi^{inc}(-x,y) & \text{as} \quad x \to -\infty, \\ T\phi^{\mathrm{inc}}(x,y) & \text{as} \quad x \to \infty, \end{cases} \tag{7}$$

where R and T are the reflection and transmission coefficients.

Let us consider the function $\psi(x,y)$ as

$$\psi(x,y) = \phi(x,y) - \phi^{inc}(x,y), \tag{8}$$

where ψ satisfies

$$\begin{aligned} &\nabla^2\psi = 0, \quad y \geqslant 0, \\ &\Big(D\frac{\partial^4}{\partial x^4} + 1 - \delta K\Big)\psi_y + K\psi = 0 \quad \text{on} \quad y = 0, \\ &\psi_x = \frac{\partial}{\partial x}\phi^{\mathrm{inc}}(0,y), \quad a < y < b, \\ &r^{\frac{1}{2}}\nabla\psi \ \text{ is bounded as } \ r \to 0, \\ &\nabla\psi \to 0 \quad \text{as} \quad y \to \infty, \\ &\psi(x,y) \to \begin{cases} R\phi^{\mathrm{inc}}(-x,y) & \text{as} \quad x \to -\infty, \\ (T-1)\phi^{\mathrm{inc}}(x,y) & \text{as} \quad x \to \infty. \end{cases} \end{aligned} \tag{9}$$

Let $\mathrm{G}(x,y;\xi,\eta)$ be the source potential which describes the motion in water covered with ice due to presence of a line source at (ξ,η). The expression for $\mathrm{G}(x,y;\xi,\eta)$ is given by Chowdhury[13] as follows:

$$\begin{aligned} G(x,y;\xi,\eta) &= -2\int_0^\infty -\frac{\mathrm{e}^{-ky}((Dk^4+1-\delta K)k\cosh(k\eta) - K\sinh(k\eta))}{k((Dk^4+1-\delta K)k - K)}\cos(k(x-\xi))\mathrm{d}k \\ &= \ln\frac{r}{r'} - 2\int_0^\infty -\frac{(Dk^4+1-\delta K)\mathrm{e}^{-k(y+\eta)}}{(Dk^4+1-\delta K)k - K}\cos(k(x-\xi))\mathrm{d}k, \end{aligned} \tag{10}$$

where $r,\ r'=((x-\xi)^2 + (y\mp\eta)^2)^{\frac{1}{2}}$, or alternately,

$$\begin{aligned} G(x,y;\xi,\eta) = &-2\int_0^\infty \frac{L(k,y)L(k,\eta)}{k(k^2(1-\delta K + Dk^4) + K^2)}\mathrm{e}^{-k|x-\xi|}\mathrm{d}k \\ &- 2\pi\mathrm{i}\frac{1}{\lambda(5DK^4\lambda^4 - \delta K + 1)}\mathrm{e}^{-K\lambda(y+\eta)+\mathrm{i}\lambda K|x-\xi|} \\ &- 2\pi\mathrm{i}\frac{1}{\lambda_1(5DK^4\lambda_1^4 - \delta K + 1)}\mathrm{e}^{-K\lambda_1(y+\eta)+\mathrm{i}\lambda_1 K|x-\xi|} \\ &+ 2\pi\mathrm{i}\frac{1}{\overline{\lambda}_1(5DK^4\overline{\lambda}_1^4 - \delta K + 1)}\mathrm{e}^{-K\overline{\lambda}_1(y+\eta)-\mathrm{i}\overline{\lambda}_1 K|x-\xi|}, \end{aligned}$$

where $L(k, y) = k(Dk^4 - \delta K + 1)\cos(ky) - K\sin(ky)$.

We now apply Green's integral theorem to the harmonic function $G(x, y; \xi, \eta)$ and $\psi(x, y)$ in the region bounded by the lines $y = 0, -X \leqslant x \leqslant X; x = \pm X, 0 \leqslant y \leqslant Y; y = Y, -X \leqslant x \leqslant X; x = 0\pm, a \leqslant y \leqslant b$ and a circle of radius ε with centre at (ξ, η) and finally we make X, $Y \to \infty$, and $\varepsilon \to 0$ to get

$$2\pi\psi(\xi, \eta) = -\int_a^b f(y)\frac{\partial}{\partial x}G(0, y; \xi, \eta)\mathrm{d}y, \tag{11}$$

where

$$\begin{aligned} f(y) &= \psi(+0, y) - \psi(-0, y) \\ &= \phi(+0, y) - \phi(-0, y), \quad y \in (a, b). \end{aligned} \tag{12}$$

Noting (8), (11) reduces to

$$\phi(\xi, \eta) = \phi^{\mathrm{inc}}(\xi, \eta) - \frac{1}{2\pi}\int_a^b f(y)\frac{\partial}{\partial x}G(0, y; \xi, \eta)\mathrm{d}y, \tag{13}$$

where $\phi^{inc}(x, y)$ is given by (1).

Now, by the conditions (5) and (1), we have

$$\phi_\xi(0, \eta) = 0, \quad \eta \in (a, b),$$

and

$$\phi_\xi^{inc}(0, \eta) = -\mathrm{i}\lambda K\mathrm{e}^{-\lambda K\eta}, \quad \eta \in (a, b). \tag{14}$$

Using these results in (13), we obtain

$$⨎_a^b f(y)\frac{\partial^2}{\partial x\partial\xi}G(0, y; 0, \eta)\mathrm{d}y = 2\pi K\lambda\mathrm{i}\mathrm{e}^{-\lambda K\eta}, \quad \eta \in (a, b), \tag{15}$$

where $⨎_a^b$ denotes hypersingular integral. Now, from (10), we have

$$G_{x\xi}(0, y; 0, \eta) = -\frac{1}{(y-\eta)^2} - \frac{1}{(y+\eta)^2} - 2K\int_0^\infty -\frac{k\mathrm{e}^{-k(y+\eta)}}{\Delta(k)}\mathrm{d}k, \tag{16}$$

where

$$\Delta(k) = k(Dk^4 - \delta K + 1) - K. \tag{17}$$

Using (16), the hypersingular integral equation (15) transforms to

$$\begin{aligned} &⨎_a^b\Big(\frac{1}{(y-\eta)^2} + \frac{1}{(y+\eta)^2} + \frac{2K^2\pi\mathrm{i}\lambda\mathrm{e}^{-\lambda K(y+\eta)}}{5D(\lambda K)^4 + 1 - \delta K} + 2K\int_0^{\lambda K}\frac{\mathrm{e}^{-k(y+\eta)}}{k(Dk^4 - \delta K + 1) - K}\mathrm{d}k \\ &+ 2K(\lambda K)^2\int_0^1\frac{\mathrm{e}^{-\frac{\lambda K}{x}(y+\eta)}x^2}{\lambda K(D\lambda^4K^4 + x^4 - \delta Kx^4) - Kx^5}\mathrm{d}x\Big)f(y)\mathrm{d}y \\ =& -2\pi\lambda K\mathrm{i}\mathrm{e}^{-\lambda K\eta}, \quad a < \eta < b. \end{aligned} \tag{18}$$

To solve the above equation, we substitute

$$\begin{cases} y = \Big(\dfrac{b+a}{2} + \dfrac{b-a}{2}\Big)t, \\ \eta = \Big(\dfrac{b+a}{2} + \dfrac{b-a}{2}\Big)u \end{cases} \tag{19}$$

into (18) to get

$$\oint_{-1}^{1}\Big(\frac{1}{(t-u)^2}+L(u,t)\Big)F(t)\mathrm{d}t=h(u),\quad -1<u<1,\tag{20}$$

where

$$\begin{aligned}L(u,t)=&\frac{(b-a)^2}{4(b+a)^2+4(b^2-a^2)(t+u)+(b-a)^2(t+u)^2}\\&+\frac{(b-a)^2}{2}\frac{K^2\pi\lambda\mathrm{i}\mathrm{e}^{-\mu\lambda K}}{5D(\lambda K)^4+1-\delta K}+\frac{(b-a)^2}{2}\int_0^{\lambda K}\frac{2Kk\mathrm{e}^{-k\mu}}{k(Dk^4+1-\delta K)-K}\mathrm{d}k\\&+(\lambda K)^4\frac{(b-a)^2}{4}2K\int_0^1\frac{\mathrm{e}^{-\frac{\lambda K}{x}\mu}x^2}{\lambda K(D\lambda^4K^4+x^4-\delta Kx^4)-Kx^5}\mathrm{d}x,\end{aligned}\tag{21}$$

$$\mu=b+a+\frac{b-a}{2}(t+u),\tag{22}$$

$$F(t)=f\Big(\frac{b+a}{2}+\frac{b-a}{2}t\Big),\quad -1<t<1,\tag{23}$$

$$F(\pm1)=0,\tag{24}$$

and

$$h(u)=-2\pi\lambda K\mathrm{i}\frac{b-a}{2}\mathrm{e}^{-\lambda K(\frac{b+a}{2}+\frac{b-a}{2}u)},\quad -1<t<1.\tag{25}$$

Following the methodology used by Parsons and Martin[12], we assume

$$F(t)=(1-t^2)^{\frac{1}{2}}g(t),\tag{26}$$

so that

$$F(\pm1)=0.$$

Here, the bounded function $g(t)$ is approximated as

$$g(t)=\sum_{n=0}^{N}a_nU_n(t),\quad -1<t<1,\tag{27}$$

where $U_n(t)$ is Chebyshev polynomial of second kind given by

$$U_n(\cos\theta)=\frac{\sin((n+1)\theta)}{\sin\theta},\tag{28}$$

and a_n are unknowns to be determined.

Substituting (26) and (27) into (20), we have

$$\sum_{n=0}^{N}a_nA_n(u)=h(u),\quad -1<u<1,\tag{29}$$

where

$$A_n(u)=-\pi(n+1)U_n(u)+\int_{-1}^{1}(1-t^2)^{\frac{1}{2}}U_n(t)L(u,t)\mathrm{d}t.\tag{30}$$

Now, the collocation points $u = u_j$ are defined as (see Parsons and Martin[12])

$$u_j = \cos\Big(\frac{(j+1)\pi}{N+2}\Big), \quad j = 0, 1, \cdots, N,$$

and also

$$u_j = \cos\Big(\frac{(2j+1)\pi}{2N+2}\Big), \quad j = 0, 1, \cdots, N.$$

This reduces (30) to the linear system

$$\sum_{n=0}^{N} a_n A_n(u_j) = h(u_j), \quad j = 0, 1, \cdots, N. \tag{31}$$

Now, using standard method, the linear system of equations can be solved for a_n. Knowing a_n, one can obtain $g(t)$ and hence $F(t)$ from (26). Knowing $F(t)$, $f(t)$ can be obtained from (23).

R and T can be obtained from (11) by making $\xi \to \pm\infty$, and noting the behaviour of $\psi(\xi, \eta)$ from (9) as $\xi \to \pm\infty$. This gives

$$R = -\int_a^b \frac{K\mathrm{e}^{-K\lambda y}}{(5DK^4\lambda^4 - \delta K + 1)} f(y)\mathrm{d}y, \tag{32}$$

and

$$T = 1 + \int_a^b \frac{K\mathrm{e}^{-K\lambda y}}{(5DK^4\lambda^4 - \delta K + 1)} f(y)\mathrm{d}y. \tag{33}$$

Putting $y = \frac{b+a}{2} + \frac{b-a}{2}t$ and after some manipulation, we have

$$R = -\frac{(b-a)K}{2}\sum_{n=0}^{N} a_n \int_{-1}^{1} \frac{(1-t^2)^{\frac{1}{2}} U_n(t)}{(5DK^4\lambda^4 - \delta K + 1)} \mathrm{e}^{-K\lambda(\frac{b+a}{2} + \frac{b-a}{2}t)}\mathrm{d}t, \tag{34}$$

and

$$T = 1 + \frac{(b-a)K}{2}\sum_{n=0}^{N} a_n \int_{-1}^{1} \frac{(1-t^2)^{\frac{1}{2}} U_n(t)}{(5DK^4\lambda^4 - \delta K + 1)} \mathrm{e}^{-K\lambda(\frac{b+a}{2} + \frac{b-a}{2}t)}\mathrm{d}t. \tag{35}$$

Now, R and T can be evaluated numerically after finding the values of a_n.

3 Numerical results

Numerical computations of $|R|$ and $|T|$ are carried out for different values of non-dimensional parameters $D' = \frac{D}{a^4}$ and $\delta' = \frac{\delta}{a^4}$ and the wave number Ka. For numerical computation of the reflection coefficient $|R|$, we choose $N = 15$ in the expansion (31). The different integrals are evaluated by using Mathematica.

In Table 1, $|R_{\text{exact}}|$ is tabulated from the exact result given by Evans[3] for water with free surface (i.e., D'=0 and δ'=0) against the wave number Ka for $b/a = 4$. Also, $|R|$ is computed from (34) for various values of Ka for very small values of D', δ' (D$'$ $= 0.0001$, $\delta' = 0.0001$) and presented in Table 1.

Table 1 Comparison of $|R_{\text{exact}}|$ and $|R|$ with different Ka

Ka	$\lvert R_{\text{exact}}\rvert$ $D'=0$ $\delta'=0$	$\lvert R\rvert$ $D'=0.000\,1$ $\delta'=0.000\,1$
0.01	0.000 709 8	0.000 714 2
0.05	0.014 959 9	0.015 165 2
0.10	0.047 756 2	0.046 083 0
0.15	0.084 700 6	0.084 845 9
0.20	0.117 640 0	0.117 580 0
0.25	0.143 025 0	0.143 069 0
0.30	0.160 293 0	0.160 345 0
0.35	0.170 347 0	0.170 379 0
0.40	0.174 585 0	0.174 611 0
0.45	0.174 403 0	0.174 422 0
0.50	0.171 004 0	0.171 018 0
1.00	0.091 450 3	0.091 412 6
1.50	0.038 470 5	0.038 401 1
2.00	0.015 189 1	0.015 127 5

It is observed from Table 1 that $|R_{\text{exact}}|$ coincides with $|R|$ upto 3–4 decimal places. This demonstrates the effectiveness of the numerical scheme based on numerical solution of the hypersingular integral equation.

In Figs. 1 and 2, $|R|$ and $|T|$ are plotted, respectively, against the wave number Ka for $\frac{b}{a}=4$, $D'=0.000\,1$, $\delta'=0.000\,1$ and $D'=0.5$, $\delta'=0.01$. In Fig. 1, it is observed that $|R|$ increases at first and then decreases as Ka increases to unity. Similarly in Fig. 2, $|T|$ is observed to decrease as Ka increases and then increases as Ka increases to unity. This is plausible since large values of wave number correspond to short waves, which are confined near the ice-cover surface, are mostly transmitted over the barrier. In both Figs. 1 and 2, it is observed that for $0<Ka<0.15$, the values of $|R|$ almost coincide for $D'=0.000\,1$, $\delta'=0.000\,1$ and $D'=0.5$, $\delta'=0.01$. Similar behaviour is exhibited by $|T|$ in Fig. 2 for $0<Ka<0.15$. This phenomena occurs since the small values of wave number correspond to the waves with long wave length and the long waves travel towards the bottom of the ocean and thereby do not fell the effect of ice-cover. As Ka increases from 0.15, $|R|$ increases and reaches a peak and then decreases with increase in Ka. It is observed that the peak value in R decreases as D' increases from 0.000 1 to 1.5 and δ' from 0.000 1 to 0.01. This shows that presence of ice-cover diminishes the reflection. Similarly, from Fig. 2, it is observed that presence of ice-cover increases the transmission.

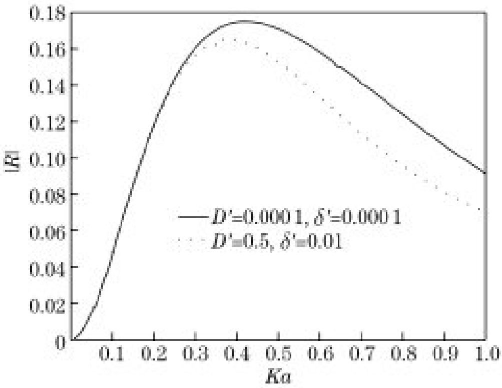

Fig. 1 $|R|$ against Ka for $b/a=4$

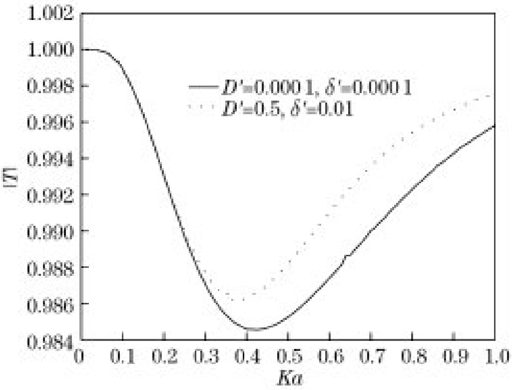

Fig. 2 $|T|$ against Ka for $b/a=4$

In Figs. 3 and 5, $|R|$ is plotted against Ka for $\frac{b}{a}=4$, $\delta'=0.01$ and for $D'=1.0$ and 1.5, respectively. Similarly in Figs. 4 and 6, $|T|$ is depicted against Ka for $\frac{b}{a}=4$, $\delta'=0.01$ and

$D' = 1.0$ and 1.5, respectively. It is observed from Figs. 3–6 that behaviour of $|R|$ and $|T|$ are complementary to each other. In fact, for all values of D', δ', and Ka in Figs. 1–5 the energy identity $|R|^2 + |T|^2 = 1$ is verified.

In Figs. 3 and 5 for a fixed length of barrier values of $|R|$ increases at first and then decreases as Ka increases. However, unlike in Fig. 1, here the values of $|R|$ exhibit sharp oscillatory for $0.15 < Ka < 0.3$ and $D' = 1.0$ and 1.5. Similar oscillatory behaviour is observed for $|T|$ in Figs. 4 and 6 for these values of D' and δ'. This peculiar behaviour in $|R|$ and $|T|$ may be attributed to the elasticity of the ice-cover. As the flexural rigidity of ice-cover increases, the waves interaction with ice-cover and submerged barrier produce sharp fluctuation in the values of $|R|$ and $|T|$.

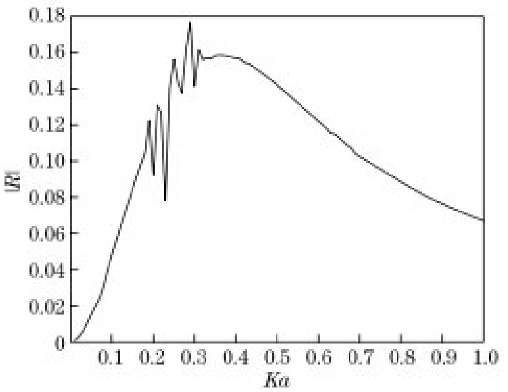

Fig. 3 $|R|$ against Ka for $b/a = 4$, D'=1.0, and δ'=0.01

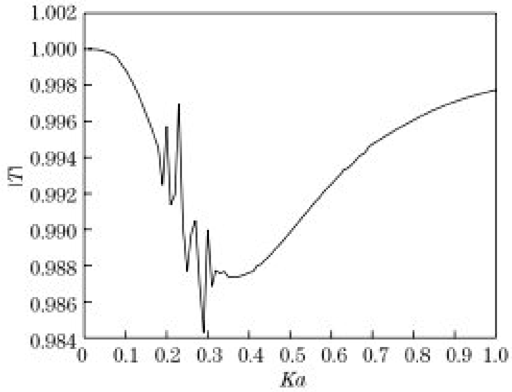

Fig. 4 $|T|$ against Ka for $b/a = 4$, D'=1.0, and δ'=0.01

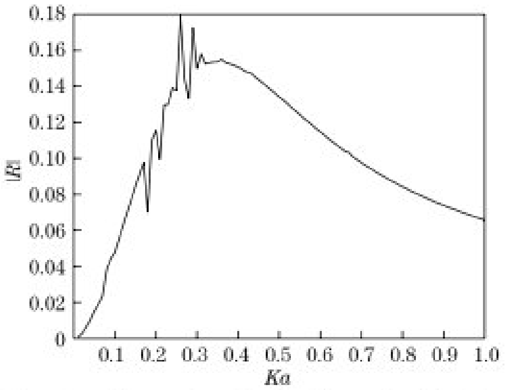

Fig. 5 $|R|$ against Ka for $b/a = 4$, D'=1.5, and δ'=0.01

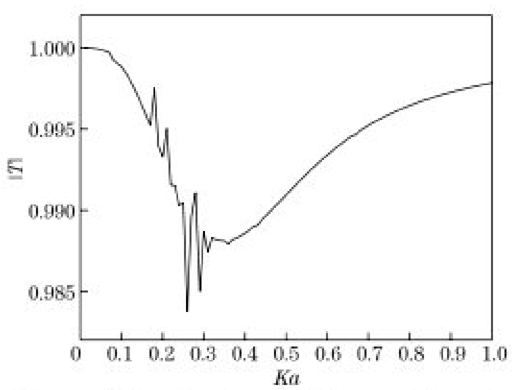

Fig. 6 $|T|$ against Ka for $b/a = 4$, D'=1.5, and δ'=0.01

In Figs. 7 and 8, $|R|$ and $|T|$ are plotted for $\frac{b}{a} = 4, Ka > 1, D' = 0.000\,1, \delta' = 0.000\,1, D' = 0.5, 1.0, 1.5, \delta'$=0.01. It is observed from Figs. 7 and 8 that $|R|$ decreases and $|T|$ increases as Ka increases. However, the decrease in $|R|$ or the increase in $|T|$ is very sharp for $D' = 0.000\,1$, $\delta' = 0.000\,1$ as compared to $D' = 0.5, 1.0, 1.5$, $\delta' = 0.01$. Thus, the increase in flexural rigidity of ice-cover reduces the rate of diminishing of $|R|$ and the rate of increase of $|T|$.

In Fig. 9, $|R|$ is plotted against the wave number Ka for $\frac{b}{a} = 4$, 10, and 15, respectively for the values of ice-cover parameters $D' = 1.5$ and δ'=0.01. It is observed that $|R|$ shows sharp oscillation for Ka=0.1 to 0.3. It is also seen that with the increase of length of the plate, the maximum peak value of $|R|$ increases. This implies that longer plate produces high peaked oscillation of reflection of coefficient waves.

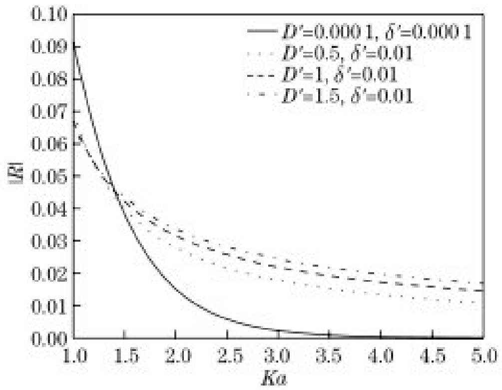

Fig. 7 $|R|$ against Ka for $b/a = 4$

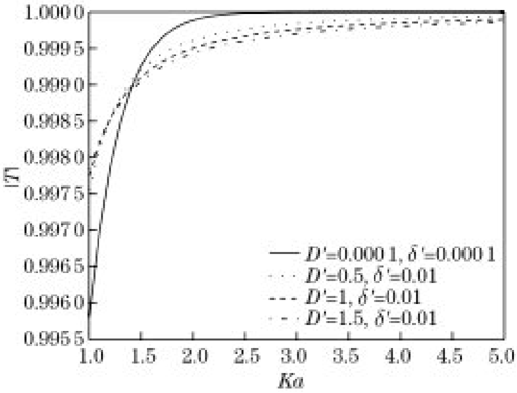

Fig. 8 $|T|$ against Ka for $b/a = 4$

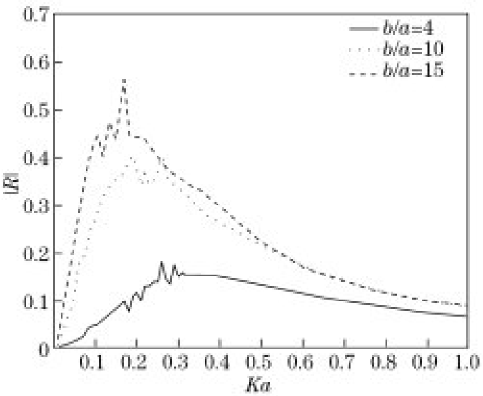

Fig. 9 $|R|$ against Ka for $b/a = 4$, D'=1.5, and δ'=0.01

4 Discussions

Water wave scattering by a thin vertical plate submerged beneath an ice cover in an infinite depth water is investigated by using hypersingular integral equation formulation. The reflection coefficient $|R|$ and transmission coefficient $|T|$ are computed numerically by using the approximate solution to the hypersingular integral equation. Also the effect of the presence of ice cover on $|R|$ and $|T|$ are visualised from the figures. It is observed that the behaviour of $|R|$ is complementary to that of $|T|$, $|R|$ and $|T|$ satisfy the energy identity $|R|^2 + |T|^2 = 1$. Due to the presence of ice cover, $|R|$ or $|T|$ changes the values rapidly for the range of values of $Ka = 0.15$ to 0.3 as the flexural rigidity of ice-cover increases. Also, it is observed that due to increase of the length of the plate produces more reflection of wave energy.

The results in the present paper are based on the linear theory, which may be regarded as benchmark. However, one can consider nonlinear scattering problems in water with ice-cover but that constitutes a separate class of problems. In recent past, Marchenko and Shrira[14] studied two-dimensional nonlinear waves in a liquid covered by ice, which may be utilized to carry out further study in this direction.

Acknowledgements This work is supported by the Council of Scientific and Industrial Research, New Delhi through a research associateship to P. MAITI and partially supported by the Department of Science and Technology through a project No. SR/SY/MS:521/08 through S. BANERJEA.

References

[1] Dean, W. R. On the reflection of surface waves by a submerged plane barrier. *Proc. Camb. Phil. Soc.*, **41**, 231–238 (1945)

[2] Ursell, F. The effect of a fixed vertical barrier on surface waves in deep water. *Proc. Camb. Phil. Soc.*, **43**, 374–382 (1947)

[3] Evans, D. V. Diffraction of surface waves by a submerged vertical plate. *J. Fluid Mech.*, **40**, 433–451 (1970)

[4] Fox, C. and Squire, V. A. On the oblique reflection and transmission of ocean waves at shore fast sea-ice. *Phil. Trans. R. Soc. Lond. A*, **347**, 185–218 (1994)

[5] Squire, V. A. Review of ocean and sea ice revisited. *Cold Regions Science and Technology*, **49**, 110–133 (2007)

[6] Chung, H. and Fox, C. Calculation of wave-ice interaction using the Wiener-Hopf technique. *New Zealand J. Math.*, **31**, 1–18 (2002)

[7] Linton, C. M. and Chung, H. Reflection and transmission at the ocean/sea-ice boundary. *Wave Motion*, **38**, 43–52 (2003)

[8] Chakrabarti, A. On the solution of the problem of scattering of surface water waves by the edge of an ice-cover. *Proc. R. Soc. Lond. A*, **456**, 1087–1099 (2000)

[9] Gayen, R., Mandal, B. N., and Chakrabarti, A. Water wave scattering by an ice-strip. *J. Eng. Math.*, **53**, 21–37 (2005)

[10] Das, D. and Mandal, B. N. Wave scattering by a horizontal circular cylinder in a two-layer fluid with an ice-cover. *Inter. J. Eng. Sci.*, **45**(10), 842–872 (2007)

[11] Maiti, P. and Mandal, B. N. Wave scattering by a thin vertical barrier submerged beneath an ice-cover in deep water. *Applied Ocean Research*, **32**, 367–373 (2010)

[12] Parsons, N. F. and Martin, P. A. Scattering of water waves by submerged plates using hypersingular integral equations. *Applied Ocean Research*, **14**, 313–321 (1992)

[13] Chowdhury, R. *Water Wave Scattering by Obstacles and Surface Discontinuities*, Ph. D. dissertation, Calcutta University (2004)

[14] Marchenko, A. V. and Shrira, V. I. Theory of two-dimensional nonlinear waves in liquid covered by ice. *Fluid Dynamic Research*, **26**(4), 580–587 (1991)