OBJECT EXTRACTION FROM IMAGE USING HIGHER ORDER ENTROPY

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ABSTRACT

Entropy of order q (depending on the information contained in a sequence of gray levels of length q) and conditional entropy of an image are defined. Using these definitions, two algorithms alongwith their superiority for image thresholding (object-background classification) are formulated and implemented with the help of its co-occurence matrix.

INTRODUCTION

Shannon's definition of entropy [1] has been used by several authors [2-3] to image processing problems. Pun [2] used Shannon's concept to define the entropy of an image assuming that an image is entirely represented by its gray level histogram only. He used this concept to derive an expression for an upper bound of the a-posteriori entropy and finally used it to segment an image into object and background. Kapur et al. [3] have found some flaws in Pun's derivation and also used a similar concept to partition the image into object and background. They, instead of considering one probability distribution for the entire histogram, used two separate probability distributions; one for the object and the other for the background. The total entropy of the image is then maximised to arrive at the threshold for segmentation.

It is to be mentioned here that these entropy based methods were developed without highlighting the adequateness of the concept of Shannon's entropy in the case of an image. For example, the dependency of pixel intensities in an image and hence its spatial distribution are not taken into account in defining its entropy. As a result different images with identical histograms will always result in the same enttopic value and same threshold. This is, of course, not intutively acceptable. Moreover, in the algorithm of Pun [2] the maximisation of an upperbound of the a-posteriori entropy, to avoid the trivial result with the a-posteriori entropy is not justified.

The present work attempts to formulate two other definitions of entropy, namely entropy of order q, q=1,2,..., and the conditional entropy of an image. These new concepts are then introduced to develop two algorithms for object background image classification (segmentation) problem. Effectiveness of these algorithms is demonstrated for two images and their superiority in performance over those of Pun [2] and Kapur et al. [3] is also established.

Global and Local Entropy

We know that in an image pixel intensities are not independent of each other. This dependency of pixel intensities can be incorporated by considering sequences of pixels to estimate the entropy. In order to arrive at the expression of entropy of an image the following theorem due to Shannon [1] can be stated.

Theorem : Let $p(s_i)$ be the probability of a sequence s_i of graylevels of length q. Let us define

$$H^{(q)} = \frac{1}{q}\sum_i p(s_i) \log_2 p(s_i) \qquad (1)$$

where the summation is taken over all gray level sequences of length q. Then $H^{(q)}$ is a monotonic decreasing function of (q) and

$$\lim_{q \to \infty} H^{(q)} = H, \text{ the entropy of the image.}$$

For different values of q we get various orders of entropy.

Case 1 : q = 1, i.e., sequence of length one. If q=1 we get

$$H^{(1)} = -\sum_{i=0}^{L-1} p_i \log_2 p_i \qquad (2)$$

where p_i probability of occurrence of the gray level i. $H^{(1)}$ is the global entropy used by Pun [2] and Kapur et al. [3].

Case 2 : q = 2, i.e., sequences of length two. Hence,

$$H^{(2)} = -\frac{1}{2}\sum_i p(s_i) \log_2 p(s_i)$$

where s_i is a sequence of gray level of length two.

$$= -\frac{1}{2}\sum_i \sum_j p_{ij} \log_2 p_{ij} \qquad (3)$$

where p_{ij} is the probability of co-occurrence of the gray levels i and j. Therefore, $H^{(2)}$ can be obtained from the co-occurrence matrix.

Obviously $H^{(2)}$ takes into account the spatial distribution of gray levels. Expressions for higher order entropies (q > 2) can also be deduced in a similar manner. $H^{(i)}$, $i \geq 2$ may be called 'local entropy' of order i of an image.

Conditional Entropy

Suppose the object X consists of the gray levels $\{x_i\}$ and the background Y contains the gray levels $\{y_j\}$. The conditional entropy of the object X given the background Y i.e., the average amount of information that may be obtained from X given that one has viewed the background Y, can be defined as

$$H(X/Y) = -\sum_{x_i \in X} \sum_{y_j \in Y} p(x_i/y_j)\log_2 p(x_i/y_j) \quad (4)$$

Similarly, the conditional entropy of the background Y given the object X is defined as

$$H(Y/X) = -\sum_{y_j \in Y} \sum_{x_i \in X} p(y_j/x_i)\log_2 p(y_j/x_i) \quad (5)$$

The pixel y_j, in general, can be an mth order neighbour of the pixel x_i. Since the estimation of such a probability is very difficult, we impose another constraint, while estimating them, that x_i and y_j must be adjacent pixels.

The conditional entropy of the image is then defined as

$$H^{(c)} = (H(X/Y) + H(Y/X))/2 \quad (6)$$

APPLICATION TO IMAGE SEGMENTATION

Based on the new definitions of entropy of an image, the following two algorithms for object background classification are proposed.

Algorithm - 1

Let t_{ij} be the (i,j)th entry of the co-occurrence matrix. Then the probability of co-occurrence p_{ij} of gray levels i and j is

$$p_{ij} = t_{ij}/(\sum_i \sum_j t_{ij}) \quad (7)$$

obviously $0 \leq p_{ij} \leq 1$.

If s, $0 \leq s \leq L-1$, is a threshold, then s partitions the co-occurrence matrix into four quadrants, namely A, B, C and D [Fig. 1].

Normalising the probabilities within individual quadrant, such that the sum of the probabilities of each quadrant equals to one, we get the following cell probabilities.

$$p^A_{ij} = t_{ij} \Big/ \sum_{i=0}^{s} \sum_{j=0}^{s} t_{ij} \quad (8)$$

$$p^B_{ij} = t_{ij} \Big/ \sum_{i=0}^{s} \sum_{j=s+1}^{L-1} t_{ij} \quad (9)$$

$$p^C_{ij} = t_{ij} \Big/ \sum_{i=s+1}^{L-1} \sum_{j=s+1}^{L-1} t_{ij} \quad (10)$$

and

$$p^D_{ij} = t_{ij} \Big/ \sum_{i=s+1}^{L-1} \sum_{j=0}^{s} t_{ij} \quad (11)$$

Now the second order local entropy of the object and of the background can be defined as

$$H^{(2)}_A(s) = -\sum_{i=0}^{s} \sum_{j=0}^{s} p^A_{ij} \log_2 p^A_{ij} \quad (12)$$

$$H^{(2)}_C(s) = -\sum_{i=s+1}^{L-1} \sum_{j=s+1}^{L-1} p^C_{ij} \log_2 p^C_{ij} \quad (13)$$

The gray level corresponding to the maximum of

$$H^{(2)}_T(s) = H^{(2)}_A(s) + H^{(2)}_C(s) \quad (14)$$

gives the threshold for object-background classification.

Algorithm - 2

Suppose s is an assumed threshold. Then,

$$H(X/Y) = -\sum_{i=0}^{s} \sum_{j=s+1}^{L-1} p^B_{ij} \log_2 p^B_{ij} \quad (15)$$

and

$$H(Y/X) = -\sum_{i=s+1}^{L-1} \sum_{j=0}^{s} p^D_{ij} \log_2 p^D_{ij} \quad (16)$$

The conditional entropy of the image

$$H^{(C)}_T = (H(X/Y) + H(Y/X))/2 \quad (17)$$

is then maximised with respect to s.

IMPLEMENTATION AND RESULTS

The segmentation (object-background classification) algorithms described earlier are implemented on two images. The threshold levels produced by different methods are presented in Table-I.

Fig. 2(a) represents the image of a biplane with two dominant modes in its gray level histogram. The segmented images produced by different methods are shown in Figs. 2(b)-2(e). From the results one can see that except for the conditional entropic method (eqn. 17), the propeller in front of the biplane is lost and some portion of the background got mixed up with the object. Fig. 3(a) represents the input image of Abraham Lincoln. Its histogram has a number of deep valleys. The thresholds produced by different methods are shown in Table-I and the corresponding segmented images are shown in Figs. 3(b)-3(e). In this case too, all the methods except the conditional entropic method (Algorithm-2) have produced comparable result. The best result is produced by the Algorithm-2 which has clearly separated the object from the background. All other methods failed to discriminate between the beard and the background at the South-East corner of the image.

REFERENCES

1 C.E. Shannon, Bell Syst. Tech. Journal, Vol. 27, pp. 379-423, July 1948.

2 T. Pun, Signal Processing, Vol. 2, pp. 223-237, 1980.

3 J.N. Kapur, P.K. Sahoo and A.K.C. Wong, Computer Graphics, Vision and Image Processing, Vol. 29, pp. 273-285, 1985.

TABLE-I

Thresholds for object-background classification

IMAGES	THRESHOLDS			
	ALGO-1	ALGO-2	ALGO OF PUN	ALGO. KAPUR ET. AL
BIPLANE (Fig.2)	22	12	24	21
LINCOLN (Fig.3)	17	10	16	15