A Graph Theoretic Approach to Single Row Routing Problems

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Abstract

In this paper, we present a new approach to the classical single row routing problem. The approach is based on a graph theoretic representation, in which an instance of the single row routing problem is represented by three graphs, a circle graph, a permutation graph and an interval graph. Three schemes for decomposition of the problem are presented. The decomposition process is applied recursively until either each sub-problem is non-decomposable or it belongs to one of the special classes of single row routing problem. For some special classes we show that routing can be done optimally, while solution in other cases can be approximated using heuristic algorithms. These solutions of sub-problems are then combined to obtain the solution of the given problem.

1 Introduction

The classical single row routing problem (SRRP) is one of the important problems in the layout design of multilayer circuit boards [8]. It has received considerable attention over past ten years [4,5,6,7,10,11,12]. The problem can be defined as follows. Given a set of two-terminal nets defined on a set of evenly spaced terminals on a real line, called the node axis. Without loss of generality it can be assumed that node axis is a horizontal line. The interconnection for the nets are realized by means of non-crossing paths. Each path consists of horizontal and vertical line segments on a single layer, such that no two paths cross each other. Moreover, no path is allowed to intersect a vertical line more than once, i.e. backward moves of nets are not allowed. For an example consider the net list $L_1 = \{N_1, N_2, \ldots, N_8\}$ where $N_1 = \{1,5\}, N_2 = \{2,6\}, N_3 = \{4,14\}, N_4 = \{3,7\}, N_5 = \{8,11\}, N_6 = \{9,15\}, N_7 = \{10,12\}, N_8 = \{13,16\}$. A single row realization of L_1 is shown in Fig. 1.

The area above the node axis is called the upper street while the area below the node axis is called the lower street. The number of the horizontal tracks in the upper street is called the upper street congestion (C_{ur}) and the number of horizontal tracks in the lower street is called the lower street congestion (C_{ls}). For the realization shown in Fig. 2, $C_{us} = 2, C_{ls} = 2$.

The objective function considered most often is to minimize the maximum of upper and lower street congestions, i.e. minimize Q_0, where $Q_0 = \max\{C_{us}, C_{ls}\}$.

Necessary and sufficient conditions for the optimal realization of single row routing problem are developed in [10]. An interval diagram representation of the problem is presented by Kuh *et al.* [5]. Raghavan and Sahni [6] give an algorithm for determination of the routability of a set of nets when the number of tracks available on each street is known in advance. The time complexity of their algorithm is again exponential in maximum number of tracks available on upper or lower streets. Since the general problem is shown to be NP-Complete [7], several heuristic algorithms have also been proposed [7,11]. Problem of finding a layout with minimum bends (doglegs) is also NP-Complete [7].

In this paper, we present a new approach to the classical single row routing problem. The approach is based on a graph theoretic representation, in which an instance of the single row routing problem is represented by three graphs, a circle graph, a permutation graph and an interval graph. We propose three schemes for decomposition of SRRP. The first decomposition method breaks the SRRP into several sub-problems each of which is represented by a connected component in the circle graph. The other two decomposition methods are based on the connectivity properties of the interval and circle graph representations. For some special classes of the single row routing problem we show that routing can be done optimally, while solution in other cases can be approximated using heuristic

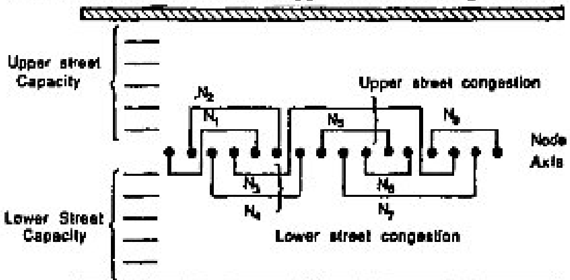

Figure 1: Basic terminology and a single row realization of the net list L_1.

algorithms. These solutions of sub-problems can then be combined to obtain the solution to the given problem.

Furthermore, for some special classes of single row routing problem, using the structure of the graphs representing an instance, we drive exact values for Q_0 in terms of the size of maximum cliques in interval graphs. To our knowledge exact value of Q_0 has not been reported for any class of single row routing problems.

2 Preliminaries

In this section we present a graph formulation of the single row routing problem. Most of the graph terminology used in this paper follows from Golumbic, Harary [2,3].

Let R be a set of evenly spaced terminals on the node axis. A *net* N is defined to be a subset of nodes in R i.e. $N \in R, |N| \geq 2$. N is called a simple net if $|N| = 2$, otherwise it is called a multi-net. In this paper we only deal with simple nets and in the remainder of the paper a net would always mean a simple net. Let $L = \{N_1, N_2, \ldots, N_n\}$ be a set of nets defined on R. Each net N_i can be uniquely specified by two distinct terminals l_i and r_i called the left touch point and the right touch point, respectively, of N_i. Abstractly, a net can be considered as an interval bounded by left and right touch points. Thus for a given set of nets, an interval diagram depicting each net as an interval can be easily constructed. For example, Figure 2(a) shows an example of interval diagram corresponding to a set of nets. Given an interval diagram corresponding to a set of nets, two graphs models representing the single row routing problem can be defined as follows.

Define an *overlap graph* $\vec{G}_O = (V, \vec{E}_O)$,

$$V = \{v_i \mid v_i \text{ represents interval } I_i \text{ corresponding to } N_i\}$$

$$\vec{E}_O = \{(v_i, v_j) \mid l_i < l_j < r_i < r_j\}$$

Similarly, define a *containment graph* $\vec{G}_C = (V, \vec{E}_C)$, where the vertex set V is the same as defined above and $\vec{E}_C$ a set of directed edges defined below:

$$\vec{E}_C = \{(v_i, v_j) \mid l_i < l_j, r_i > r_j\}$$

Since we will be dealing with both directed and undirected version of these graphs therefore the arrow head on top will be omitted to indicate the undirected graph. For example by G_C we mean the graph $\vec{G}_C$ ignoring the directions on the edges i.e; $G_C = (V, E_C)$.

We also define an interval graph $G_I = (V, E_I)$ where the vertex set V is the same as above, and two vertices are joined by an edge if and only if their corresponding intervals have a non-empty intersection. Fig. 2 shows an example of the overlap graph, the containment graph and the interval graph for a set of nets.

It is well known that the class of overlap graphs is equivalent to the class of *circle graphs* and the class of containment graphs is equivalent to the class of *permutation graphs* [2]. Both interval graphs and permutation are well known classes of perfect graphs and has been studied extensively [2]. Polynomial time algorithms are known for recognition, maximum clique, maximum independent set problems among others.

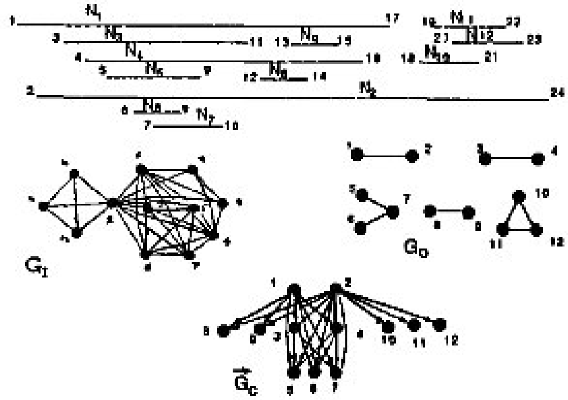

Figure 2: Interval diagram and Graph representation for a set of nets.

Let D_0 denote the minimum number of doglegs required in optimal realization of a set of nets L e.g., for the illustration in Fig. 1 $D_0 = 3$. It is important to note that layouts with minimum doglegs may not have minimum Q_0. Therefore we consider D_0 only for for those layouts with minimum Q_0.

The cut number of a terminal is number of nets intersecting a vertical line through this terminal excluding the net for which this terminal is a touch point. The *cut number* q_i of a net N_i is defined as the maximum of the cut numbers of its left and right touch points. Furthermore, let q_{min} and q_{max} be respectively the minimum and maximum cut numbers among cut numbers of all nets. It has been shown in [5] that for each feasible realization $Q_0 \geq \max\{q_{min}, \lceil q_{max}/2 \rceil\}$. However, our model immediately leads to a tighter lower bound. It follows from the definitions of G_O, G_C and G_I that for every clique $C \in G_O$ there exists a clique $C' \in G_I$ such that $V_{C'} \supseteq V_C$. If C_O, C_C and C_I denote the maximum cliques in G_O, G_C and G_I respectively then it is obvious that $|C_I| \geq \max(|C_O|, |C_C|)$, and it follows that $q_{max} = |C_I| - 1$. Therefore a lower bound tighter than that given in [5] can be obtained as follows.

Lemma 1 *For each feasible realization*

$$Q_0 \geq \max\{q_{min}, \lceil |C_I|/2 \rceil\}$$

3 Summary of Results

3.1 Problem Decomposition

In this section, we present three schemes for decomposition of SRRP. We investigate conditions under which these schemes are applicable.

Decomposition based on the connected components of G_O This decomposition is based on the connected components of the circle and the interval graphs. Consider the interval graph G_I of a given instance of a single row routing problem. It is easy to see that each connected component can be routed independently of others. Therefore, in the rest of the paper we consider only those instances of the problem for which graph G_I has only one connected component.

Now consider the overlap graph G_O of the given single row routing problem. G_O may have more than one connected components although net lists represented by these components are physically overlapping. Let $H_1, H_2, \ldots, H_r$ be the connected components then we define a directed graph $\vec{T} = (V^h, E^h)$. where $V^h = \{v_1^h, v_2^h, \ldots, v_r^h\}$ such that v_i^h corresponds to the connected component H_i. The edge set $\vec{E}^h$ is defined as follows

$$\vec{E}^h = \{(v_i^h, v_j^h) | \exists u \in H_i, \exists v \in H_j, (u,v) \in \vec{E}_C\}$$

In other words, a directed edge is drawn from v_i^t to v_j^t if and only if the *composite interval* corresponding to H_j is completely contained in the composite interval of H_i. By composite interval we mean the interval defined by $(\min_k l_k, \max_k r_k)$ for all $k, N_k \in H_i$. It is easy to see that $\vec{T}$ is a permutation graph. Let $\vec{T}_t = (V^h, \vec{E}_t^h)$ be the transitively reduced graph corresponding to $\vec{T}$. Fig. 3 shows $\vec{T}$ and $\vec{T}_t$ for the example in figure 2. We have the following theorem.

Theorem 1 *$\vec{T}_t$ is a rooted directed tree.*

For a given optimal routing of an instance define $Q_1 = \min\{C_{us}, C_{ls}\}$. An optimal routing is called super-optimal if it also minimizes Q_1.

Theorem 1 forms the basis of an efficient algorithm for super-optimal routing of G_O if super-optimal routing is known for each of its connected components.

Theorem 2 *If super-optimal routing is known for $H_i, 1 \leq i \leq r, H_i \in G_O$ then Algorithm A produces super-optimal routing for G_O .*

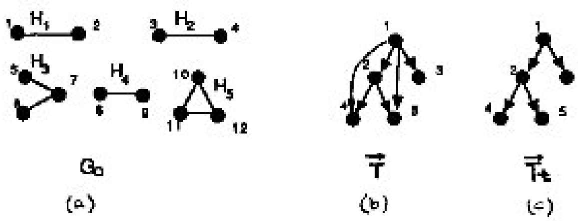

Figure 3: Examples of $\vec{T}, \vec{T}_t$ for the example in Fig. 2.

Decomposition based on Clique intersections This scheme is based on decomposition of a net list with respect to clique intersection in the corresponding G_I. Let $C_1, C_2, \ldots, C_n$ be a linear ordering of cliques in G_I. The clique intersection, I_i of two cliques C_i and C_{i+1} is defined as follows

$$I_i = C_i \cap C_{i+1},\ 1 \leq i < n$$

By decomposing a net list we mean that certain nets can be 'physically' cut by creating dummy terminals equal to twice the number of nets that are cut. The two sub net lists can now be routed independently and two routings can be put together later and 'rewired' so that dummy terminals are eliminated.

Theorem 3 *If $\exists i, 1 \leq i < n$ such that , G_I has a clique intersection $I_i \leq 2$ and if optimal routing is known for the two components that are formed by decomposing G_I at this clique intersection then G_O can be optimally routed.*

Decomposition based on Connectivity of G_O This scheme is based on the connectivity of the circle graph of a single row routing instance. Given a connected graph $G = (V, E)$, an edge $e \in E$ is called a cut edge if removal of e from G disconnects G. The decomposition scheme proceeds by finding a cut edge e in G_O and decomposes G_O into two graphs G_1 and G_2 defined as follows. Let v_1, v_2 be the two vertices connected by e and G' and G'' are the two components formed by removal of e such that $v_1 \in G'$ and $v_2 \in G''$. Then G_1 and G_2 are defined as the subgraphs induced by $G' + v_2$ and $G'' + v_1$ respectively. We study this decomposition under the restriction that either the composite intervals of $G' - v_1$ and $G'' - v_2$ are disjoint or if composite interval of $G' - v_1$ contains the composite interval of G'' then after removal of v_1, G_O should not have more than two connected components.

Theorem 4 *Assume that a decomposition by cut edge of a given SRRP exists. Then an super-optimal routing of SRRP can be obtained from the super-optimal routing of the two components formed by the decomposition process.*

3.2 Routing of special classes of SRRP

In this section we investigate routing of some special classes of single row routing problem. We show that exact values for Q_0 can be derived if G_O is a path, cycle, clique or a complete bipartite graph. We also present a sufficient and necessary condition for feasibility of routing without doglegs.

A connected acyclic graph with no vertex having degree greater than 2 is called a path. For a large class of routing problems G_O can be a path. Note that for a path of length n, $n > 1$, $|C_O| = 2$, $|C_I| \leq n$. Since for paths $q_{min} \leq \lceil |C_I|/2 \rceil$ therefore $Q_0 \geq \lceil |C_I|/2 \rceil$. A simple routing algorithm for paths gives us the following lemma.

Lemma 2 *If G_O is isomorphic to a path of length n, $n \geq 1$ then $Q_0 = \lceil |C_I|/2 \rceil$.*

By generalization of paths to cycles of length n, $n \geq 3$, we note that the routing specifications for which G_O is a cycle becomes quite restricted as stated in the lemma below. In fact for cycles $|C_I| = 3$.

Lemma 3 *If G_O is a cycle of length n, $n \geq 3$ then $Q_0 = \lceil |C_I|/2 \rceil$ and*

$$D_0 = \begin{cases} 1 & \text{if } n \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

This lemma leads to characterization of SRRP for which realization is possible without doglegs.

Theorem 5 *Given an instance of single row routing, a realization without doglegs exists if and only if its corresponding G_O is a bipartite graph.*

Following Theorem 5, we define routing of a given SRRP without doglegs as *natural bipartition routing*. Moreover, natural bipartition routing when unique if G_O is connected. It may be noted that Theorem 5 presents a simple characterization of natural bipartition routing problems as compared to one presented by Raghavan *et al.* [7]. One must note that natural bipartition routing may not be optimal i.e.,Q_0 may not be minimum. In these cases doglegs are required to obtain the optimal value of Q_0. This Theorem leads to a lower bound on the number of doglegs for a given instance of SRRP.

Corollary 1 *Given an instance of single row routing, then $D_0 \geq |V| - |V_b|$ where $G_b = (V_b, E_b)$ is a maximum bipartite subgraph of G_O corresponding to the instance.*

The exact value of Q_0 and D_0 for the classes of SRRP for which G_O is a complete bipartite graph or a clique can be derived as given below. Both classes can be routed easily.

Lemma 4 *If G_O is a complete bipartite graph $K_{m,n}$, $m \geq n$ then $Q_0 = \lceil |C_I|/2 \rceil$ and*

$$D_0 = \begin{cases} 2n-1 & \text{if } m > n+1 \\ 0 & \text{otherwise} \end{cases}$$

Lemma 5 *If G_O is a complete graph K_n then $Q_0 = \lceil |C_I|/2 \rceil$*

4 Conclusions

In this paper, we have introduced a graph theoretic approach to the classical single row routing problem. An instance of the single row routing problem is represented by three graphs, a circle graph, a permutation graph and an interval graph. We propose three schemes for decomposition of SRRP. Furthermore, for some special classes of single row routing problem, using the structure of the graphs representing an instance, we drive exact values for Q_0 in terms of the size of maximum cliques in interval graphs. The approach also leads to many interesting theoretical insights into the single row routing problem.

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