Properties of hadronic interactions at high energies and the nature of the cosmic-muon charge ratio

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We would like to highlight here the very recently emphasized properties of ultrahigh-energy interactions and to study their bearing on the analysis of the nature of the cosmic-muon charge ratio at ultrahigh energies. It is interesting to note that by combining the recently observed properties of all the relevant aspects of the problem and using the updated values of the necessary parameters, we can give a reasonably good description of the data even with the help of a phenomenological model for particle production of Rossi *et al.* and also with the induction of a hand-inserted value for moderate violation of Feynman scaling at ultrahigh energies. Our results on the muon charge ratio and some other observables will be compared with those of the quark-gluon string model of Erlykin, Krutikova, and Shabelskii, which is also a version of empirical theoretical model with scaling violation as a basic feature.

I. INTRODUCTION

The behavior of the sea-level muon charge ratio at ultrahigh energies is still not well understood,[1] as the problem is intimately linked up with our understanding of the mechanism of multiple production of pions and kaons in the atmosphere and some other problems related with the nature of the primaries and the behavior of the primary nucleon spectra.[2] The problem is more acute because of the fact that the scenario of multiple production of secondaries is overwhelmingly dominated by soft (low transverse momentum) particles, for which, to date, there is no clear and comprehensive theoretical understanding[3] based on a clearly consistent and purely dynamical model. This apart, several other aspects of high-energy interactions need to be understood in greater and more precise detail.

On the other hand, side by side, the experimental measurements of the values of the muon charge ratio have been continuing.[4-7] Our aim here is to first assemble the latest information on various aspects related to this problem in all possible ways and, then, to study the overall consequence, if any, of this updating of the data connected with various physical phenomena, on the final results of the muon charge ratio and muon spectrum. We shall investigate here these problems on the basis of a moderate violation of the Feynman scaling as proposed by some authors and the logarithmic nature of rising cross sections. Very recently, Erlykin, Krutikova, and Shabelskii[2] made a study on the behavior of the muon charge ratio and we shall compare our results with those from the quark-gluon string model (QGSM) of Erlykin, Krutikova, and Shabelskii.[2]

II. BASIC WORKING FORMULAS

The key factor in determining the muon charge ratio μ^+/μ^- is the fractional energy moment defined as[8]

$$Z_{pC}=\frac{\pi}{\sigma^{pp}}\int_0^1 x^{\gamma-1}g_{pC}\,dx \;, \tag{1}$$

where pC represents the production of C particles in proton-proton (pp) collisions and γ is the index of a primary nuclear spectrum with the form

$$dN=N_0E^{-(\gamma+1)}dE \tag{2}$$

with $N_0=P_0+n_0$ where P_0 and n_0 are the proton and neutron intensities at the top of the atmosphere, respectively. Here $N_0=1.87\ (\text{cm}^2\,\text{s}\,\text{sr/nucleon})^{-1}$ and $\gamma=1.7$ for the spectrum measured[9] by the Japanese-American Cooperative Emulsion Experiment (JACEE). In expression (1) the term g_{pC} is defined as

$$g_{pC}=\int_0^\infty f_{pC}(x,p_T)\,dp_T^2 \tag{3}$$

and the mean multiplicity is obtained from the relation

$$\langle n_C\rangle_{pp}=\frac{\pi}{\sigma_{\text{inel}}^{pp}}\int_{x_{\min}}^{x_{\max}}\frac{1}{x}g_{pC}\,dx \;, \tag{4}$$

where $x_{\min}$ and $x_{\max}$ give the realistically measurable values of the scaling variable (instead of zero and one, respectively) with $x_{\min}\sim$ a very small positive quantity always greater than zero in magnitude and $x_{\max}\sim$ a quantity very near to unity[8] and with $\sigma_{\text{inel}}^{pp}\simeq 35$ mb (at the CERN ISR range of energy) and the

$$f_{pC}(x,p_T)=\left[E\frac{d^3\sigma}{dp^3}\right]_{S\to\infty}\left[\frac{S_0}{S}\right]^{\epsilon}. \tag{5}$$

The term $(S_0/S)^{\epsilon}$ represents the degree of Feynman scale breaking in high-energy collisions in the fragmentation region where $\epsilon=0.15$.

Using the different $(E\,d^3\sigma/dp^3)_{pp\to CX}$ for $\pi^{\pm}$ and $K^{\pm}$ we can obtain the values of Z_{pC} for different particles. The letter C stands for the observed particle and X denotes the rest unobserved. For πp (or πA) collisions the procedure for determining $Z_{\pi C}$ is exactly the same with $\sigma_{\text{inel}}^{\pi p}=25$ mb at the ISR range of energies and with $(E\,d^3\sigma/dp^3)_{\pi p\to CX}$. The charge ratio for cosmic-ray muons is given by

$$\frac{\mu^+}{\mu^-}=\frac{P_+ + kK_+}{P_- + kK_-}, \quad (6)$$

where

$$P_\pm=\frac{\ln(\Lambda_\pi/\Lambda_N)}{1/l_\pi}Z_{p\pi}\pm\frac{\Delta_0}{N_0}\frac{\ln(\Lambda'_\pi/\Lambda'_N)}{1/l'_\pi}Z_{p\pi}^- \quad (7)$$

with

$$\Lambda_n=\frac{\lambda_N}{1-Z_{pp}-Z_{pn}}, \quad \Lambda'_N=\frac{\lambda_N}{1-Z_{pp}+Z_{pn}}, \quad (8)$$

$$\Lambda_\pi=\frac{\lambda_\pi}{1-Z_{\pi^+\pi^+}-Z_{\pi^-\pi^-}}, \quad \Lambda'_\pi=\frac{\lambda_\pi}{1-Z_{\pi^+\pi^+}+Z_{\pi^+\pi^-}}, \quad (9)$$

$$\frac{1}{l_\pi}=\frac{1}{\Lambda_N}-\frac{1}{\Lambda_\pi}, \quad \frac{1}{l'_\pi}=\frac{1}{\Lambda'_N}-\frac{1}{\Lambda'_\pi}, \quad (10)$$

and

$$Z_{p\pi}^\pm=Z_{p\pi^+}\pm Z_{p\pi^-} \quad (11)$$

and similar expressions for $K^\pm$. The term $\Delta_0/N_0=(p_0-n_0)/(p_0+n_0)=0.80$ measures the proton excess. The values of all the parameters used in this section are taken from Ref. 9(b).

III. SOME FOUNDATIONAL PROPOSITIONS

Our present work is based on the acceptance of the following facts and/or assumptions.

(i) We accept the conclusion arrived at by Minorikawa and Mitsui[8(c)] that the pions from the fragmentation region $(0.1\le x\le 1.0)$ mainly dominate the muon charge ratio for which we make use of the breaking of the Feynman scaling by a factor $(S/S_0)^\epsilon$ with $\epsilon=0.15$ and $S_0=100$ GeV^2 supported by the UA5 Collaboration.[10]

(ii) (a) We assume basically the normal mass composition of the primary nuclei as given [11,12] in Table I and also the fact that the primary mass composition is taken to be a constant independent of energy.[13] (b) But it must be admitted that the composition of the primary cosmic rays is an unsettled question for which we consider it fit to study also the effect of the changing composition of the primaries on the nature of the muon charge ratio in one of the subsequent sections (Sec. VIII) of the present work.

TABLE I. Composition of cosmic rays from high-energy data.

Mass number (A)	1	4	14	26	51
Fraction of particles in percentage	41	9	15	17	18

(iii) We accept the conclusion made by Sreekantan, Tonwar, and Viswanath[14] that a hadron-hadron collision is not exactly equivalent to a hadron-nucleus reaction at least in the forward fragmentation region. However, in order to simplify the calculations we accept here that nucleons inside the projectile nucleus behave in nucleus-nucleus collisions like an assembly of free nucleons. The physics of the European Muon Collaboration (EMC) effect which essentially reflects a departure from this is not taken into account here. This is in sharp contrast to the work of Minorikawa and Mitsui.[8(c)]

(iv) The phenomenological model for the production of hadron secondaries by Rossi *et al.*[15] has basically been applied here for calculations which, however, do not take into account the rise of the K/π ratio with energy.[16,17] It has been assumed to have a constant value ~ 0.15.

(v) It will not be out of place to reiterate the fact that in the primary-proton energy range from 10^2 GeV/nucleon to 5×10^6 GeV/nucleon, the differential energy spectrum is assumed to have the shape

$$1.87E_p^{-2.7}\ (\text{cm}^2\,\text{s}\,\text{sr}\,\text{GeV/nucleon})^{-1}$$

which was formulated by the JACEE group and extensively used by Mitsui, Minorikawa, and Komori.[16]

(vi) We do not take into account the effects arising out of the energy dependence of the absorption mean free paths as was done by Volkova, Zatsepin, and Kuzmichev.[18]

IV. BEHAVIOR OF THE TOTAL CROSS SECTION

The rise of the total cross section for the production of hadrons with energy was reported[19] long ago from cosmic-ray experiments. Since then this has more or less been an established fact. Only the rate of rise is still in question. Contrary to the QCD prediction of a rapid rise[20] of the cross section, the experiments at the highest CERN SPS $p\bar{p}$ collider energy at $S^{1/2}=900$ GeV confirm that a $\ln^2 S$ rise fits[21] excellently well with the data. The separate cosmic-ray experiments also support[22] the same

TABLE II. Comparison between the various rates of rise of cross sections.

$\sqrt{S}$ (TeV)	$\ln^2(S/S_0)$ model [Block and Cahn (Ref. 26)]	QCD-inspired model [Margolis *et al.* (Ref. 27)]
0.54 SPS $p\bar{p}$	63.1 ± 0.72	69.5 ± 0.9
1.80 (Fermilab Tevatron)	80.8 ± 1.34	92.6 ± 1.54
40.00 (Superconducting Super Collider)	138.2 ± 3.50	167.7 ± 3.02

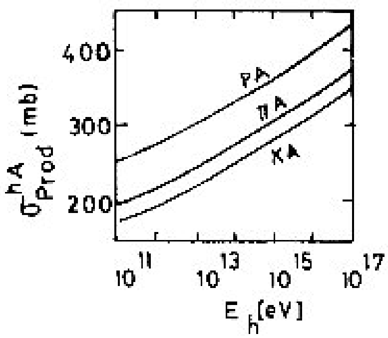

FIG. 1. Present calculations of total cross sections σ^{hA} for high-energy collisions of proton-air, pion-air, and kaon-air.

rate. And the very recent theoretical analysis also corroborates the same contention[23,24] where the rising cross section is expressed in the form of $\sigma_{\rm rising}=\ln^2(S/S_{\rm cut})$ where $S_{\rm cut}\sim 100$ GeV^2. From the study of unitarity bounds on diffraction the hadronic cross sections at ultrahigh energies are given by the form $\sim C+d\ln^2 S$ (Ref. 25), where C and d have specific values. For ultrahigh-energy proton-proton collisions, $C\sim 38$ mb and $d\sim 0.4$ mb and for proton-air collisions the values of C and d are ~ 280 mb, respectively, at very high energies.

The disagreement between the calculations of the $\ln^2(S/S_0)$ model and those of the QCD-inspired models is shown in Table II. The $\ln^2(S/S_0)$ model in the table is taken from the work of Block and Cahn[26] and the QCD-inspired one is taken from Margolis *et al.*[27] In our calculations for muon spectrum and muon charge ratio we shall use the following expressions for the rising cross section of Prokoshkin.[28] The rising nature of cross sections shown in Fig. 1 for proton-proton and pion-proton collisions is also obtained on the basis of the following expressions, of course, with the A dependence having been taken into account according to the subsequent section.

The total cross section is represented by the sum of the Regge expressions and the other growing component as given by

$$\sigma_{\rm tot}(S)=\sigma^R_{\rm tot}(S)+\sigma^{\rm gr}_{\rm tot}(S)\ , \quad (12)$$

where

$$\sigma^R_{\rm tot}(S)=\sigma^R_\infty(1+a/\sqrt{S})\ . \quad (13)$$

The values used for σ^R_∞ and a are shown in Table III and the growing part $\sigma^{\rm gr}_{\rm tot}$ is taken of the form

$$\sigma^{\rm gr}_{\rm tot}(S)=\alpha\ln(S/S_0)-\beta\ln^2 S/S_0\ , \quad (14)$$

where $\alpha=0.46$ mb, $\beta=0.27\pm0.10$ mb, and $\sigma^{\rm gr}_{\rm tot}(S<S_0)=0$. We have plotted in Fig. 1 the values of

TABLE III. Parameters used in expression (12).

Collisions between the particles	Value of σ^∞ (mb)	Value of a (GeV)
pp	37.1	0.32
π^-p	20.8	0.88
π^-p	20.8	1.29

TABLE IV. Values of S_0 used.

Collisions between the particles	Values of S_0 used (GeV^2)
pp	80
πp	50

$(\pi^+p+\pi^-p)/2$ for pion-proton collisions. The rising part of the cross sections denoted by $\sigma^{\rm gr}_{\rm tot}$ has the following notable features.

(i) The values of $\sigma^{\rm gr}_{\rm tot}$ are very nearly the same for particles and antiparticles.

(ii) The values of $S=S_0$ at which $\sigma^{\rm gr}_{\rm tot}$ begins to grow are larger for nucleon-nucleon collision than for meson-nucleon collision which is a reflection of the structure of particles (see Table IV).

(iii) The rising part of the cross section as a function of the dimensionless variable (S/S_0) is universal in nature. It is clear that the $\ln^{1.8}(E_p/100\ {\rm GeV})$ form of rise of cross section proposed by Gaisser *et al.*[29] has not been taken into consideration here.

V. EFFECT OF NUCLEAR COLLISIONS

The A dependence of the hadron-nucleus collision is inserted here in the manner of Minorikawa and Mitsui:[8(c)]

$$E\frac{d^3\sigma}{dP^3}(hA\to cX)=E\frac{d^3\sigma_h}{dP^3}(hp\to cX)\times\eta^c_h(y,p_T)\exp[\alpha^c_h(y,p_T)\ln A]\ . \quad (15)$$

From the same we obtain an acceptable relation $(\sigma_{hp}/\sigma^0_h)\eta^c_h=1.25-1.5$ which helps us to arrive at $\eta^c_h(y,p_T)$, the parameter measuring the charge mixing effect in hadronic collisions in the production and detection of the C particle and the α^c_h factor involves a y-dependent polynomial to include the kinematic changes in the nuclear collision. The other set of collisions of paramount importance are pion-proton (hydrogen) or pion-nucleus collisions. We would like to take the effect of these collisions into consideration on the basis of the contention by Shabelskii[30] that the inclusive spectra of all charged particles are approximately the same in proton-proton and pion-proton collisions at least up to 10^6 GeV^2 and the contention that violation of Feynman scaling is weaker in πp collisions than in pp scattering at high energies.

VI. NATURE OF INCLUSIVE CROSS SECTIONS USED AND THE RESULTS

The nature of our choice of the empirical fits to the inclusive cross sections proposed by Rossi *et al.*[15] relies on a very recent observation by Alner *et al.*[10] that the Feynman scaling at the highest SPS $p\bar p$ collider energy, $S^{1/2}=900$ GeV is quite valid in the central region and that there is a breaking of the Feynman scaling only in the fragmentation region by a marginal amount of 10–20 percent. This can be taken care of with the help of the prescription by Wdowczyk and Wolfendale[31] which forms one of the basic foundations of this work. The theoretical prediction of a larger violation of Feynman scaling by Cheung and Mackeown[32] has not been taken into account.

Before writing down the expressions for inclusive cross sections in terms of the rapidity variable let us first define it in the Lorentz frame:

$$y^{\text{c.m.}}=\tfrac{1}{2}\ln(E+p_L)/(E-p_L) \tag{16}$$

with the conversion factor

$$y^{\text{lab}}=y^{\text{c.m.}}-\ln[\bar{\alpha}(1+\bar{\beta})] , \tag{17}$$

where $\bar{\alpha}$ is the Lorentz factor of the c.m. system and $[\ln\bar{\alpha}(1+\bar{\beta})]$ is the rapidity of the incident particle in the c.m. system. This is the additive property of rapidity. In terms of (S,y,p_T),

$$E\frac{d^3\sigma}{dp^3}=f(S,y,p_T) . \tag{18}$$

At very high energies, by using the property of factorization, the above expression might be written on the basis of the violation of the Feynman scaling in the manner Wdowczyk and Wolfendale as

$$f(y,p_T)=g\left[y\left[\frac{S}{S_0}\right]^{\epsilon}\right]h\left[p_T\left[\frac{S}{S_0}\right]^{\epsilon}\right]$$

$$=g'(y)h'(p_T) , \tag{19}$$

where the term $(S/S_0)^{\epsilon}$ represents the moderate violation of scaling with $\epsilon=0.15$ and $S_0=100\ \text{GeV}^2$. The nature of the dependence of the inclusive cross sections on transverse momenta is taken from Rossi *et al.*[15] as was done by Ganguly and Sreekantan:[33]

$$h'(p_T)=\exp(-b'p_T) \tag{20}$$

with

$$b'_{\pi^+}=6.7(\text{GeV}/c)^{-1} ,$$

$$b'_{\pi^-}=6.5(\text{GeV}/c)^{-1} , \tag{21}$$

$$b'_{k^+}=b'_{k^-}=4.5(\text{GeV}/c)^{-1} .$$

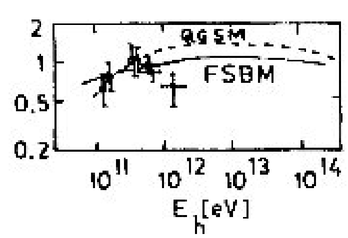

FIG. 2. Plot of calculated ratios of the fluxes of pions and protons at mountain altitudes as a function of the energy. The two models are compared.

TABLE V. Values of the parameters of expression (22).

Particles produced	A'_1	A'_2	A'_3	α'
π^+	250	75	0.40	4.0
π^-	186	200	4.5	5.25
k^+	72	4.0	0.01	1.75
k^-	70	4.5	0.01	1.70

The nature of the rapidity dependence is also taken in a manner similar to Ganguly and Sreekantan:[33]

$$g'\left[y\left[\frac{S}{S_0}\right]^{\epsilon}\right]=A_1\exp\left[-A'_2\left\{Z'+y\left[\left[\frac{S}{S_0}\right]^{\epsilon}\right]^{\alpha'}\right\}\right]+A'_3 , \tag{22}$$

where $Z'-Z$ is a constant $=2$ for charged pions and $Z'=0$ for all kaons. The constants A'_1, A'_2, A'_3, and α' are given in Table V.

Figure 2 depicts the ratio of the fluxes of pions and protons wherein the pions are the secondary product and the proton spectrum is taken to be that of primary one. The flux of pions is obtained from the solution of the standard transport equation which is used here in an implicit manner.

VII. USE OF SOME OPTIMUM PARAMETERS

As prescribed by Ng and Wolfendale[34] long ago, we will now put into use the empirical and optimal values for the favored production of positive particles, especially the positive pions and kaons and the ratio of kaon to pion. The following are the suggested values in the energy-independent and p_T-independent form despite the fact that currently they might be of doubtful validity:

$$\pi^+/\pi^-\simeq 1.64,\quad k^+/k^-\simeq 1.4,\quad k/\pi\simeq 0.15 . \tag{23}$$

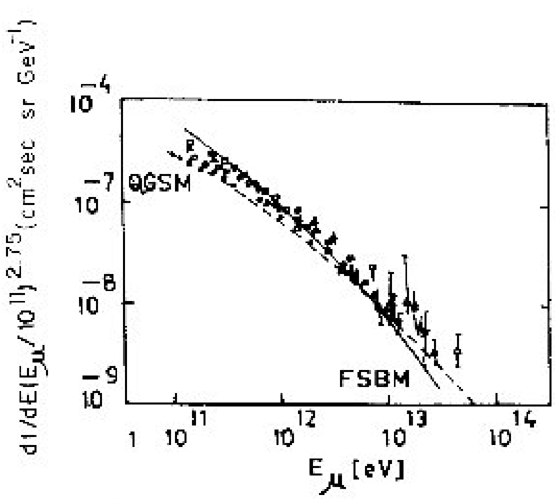

FIG. 3. Energy spectrum of the vertical muons at sea level. Experimental points are from the compilation by Erlykin, Krutikova, and Shabelskii (Ref. 2). The two scale-breaking models are compared: the quark-gluon string-model calculations are taken from Ref. 2 and the other model [Feynman-scaling-breaking model (FSBM)] is the main concern for the present study.

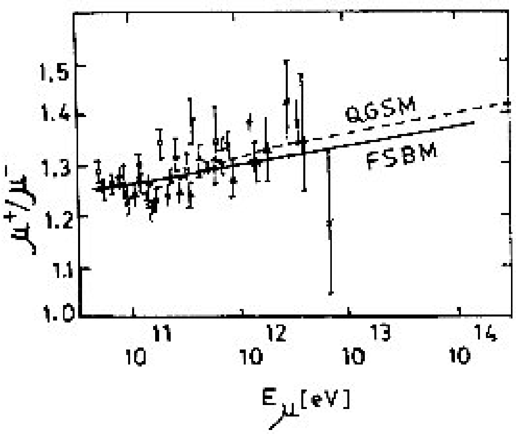

FIG. 4. Plot of the behavior of the sea-level muon charge ratio vs the energy of the muons. Other information is in Fig. 3.

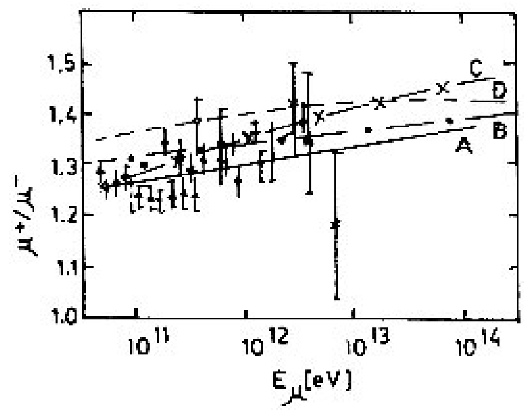

FIG. 5. Variations in the values of the muon charge ratio with changing composition of the primary cosmic radiation. The composition for each curve is taken from Table VI.

The favored production of the positive pion to negative pion at large X (fragmentation region) is explained quite well in the framework of the quark model and the maximum value might be raised to as high as 5 (instead of 2) when neutrons actually come into the picture. But this is not true of the kaon-pion ratio or the ratio of positive kaon to negative kaon production.

The final results of the muon spectrum and muon charge ratio with the present Feynman-scale-breaking model (FSBM) have been presented in Figs. 3 and 4, respectively. They have been compared with the popular quark-gluon string model (QGSM). Both the models present tolerable and agreeable results with experimental measurements.

VIII. EFFECT OF CHANGING COMPOSITION OF PRIMARY COSMIC RADIATION ON MUON CHARGE RATIO

We are interested in studying the behavior of the muon charge ratio at high energies for which we have relied on the composition supported at ultrahigh energies. But even within the ultrahigh-energy region there are some changes[35] in the percentage composition depending on the energy value and the nature of the fit chosen. We have examined here the effect of changes on the muon charge ratio by using the composition depicted in Table VI. The results are shown in Fig. 5 (curves B and C). It is also a fact that reliable data on the composition exist only at the lower-energy domain for which we have tentatively repeated our calculations with such drastic changes[36–38] of composition, as shown by row (D) in Table VI and D in Fig. 5.

The overall effect of change in composition on the muon charge ratio has been shown in Fig. 5. The point to be emphasized is that the change in proton percentage apparently affects the result most prominently. We get a relatively higher set of values of muon charge ratio with the overwhelming abundance of protons (77.7%) as demonstrated by curve D in Fig. 5. Curve A provides the background for comparison here which is taken to be the standard composition for calculation of the high-energy muon charge ratio. This study, of course, does not quantitatively reveal the influence of changes of the helium component compared to that of the proton component. This limitation could be removed by making a study with the proton contribution kept at a constant value. However, we are basically driven to a conclusion, despite these limitations, that our initial choice of values of primary composition deduced from the high-energy data give a better description of the charge ratio data than what is obtained on the basis of the low-energy composition picture. And this is presumably due only to the cumulative effect of several inputs given in the present work.

IX. DISCUSSION AND CONCLUSIONS

Our results are in satisfactory agreement with the experimental data and in tolerable agreement even with the quark-gluon string model (QGSM). This latter agree-

TABLE VI. Variation in percentage composition of primary cosmic rays with energy and their distribution with mass number (A).

Ranges of primary energy	Percentage distribution for Hydrogen ($A=1$)	Helium ($A=4$)	Others $A>4$
(A) E_c (primary energy) = 10^3 TeV	41	9	50
(B) E_p (primary energy) = 10^6 TeV	51	9	40
(C) E_p (primary energy) = 1 TeV	40	19	41
(D) E_p (primary energy) < 1 TeV	77.9	15.6	6.5

ment is due to several reasons: first, although in an entirely different form, we have reckoned here the effect of the violation of Feynman scaling as was done by Erlykin, Krutikova, and Shabelskii;[2] second, we have considered here the effect of the rising cross sections as was done by Erlykin, Krutikova, and Shabelskii[2] with the only difference in the form of the nature of the rise; third, we have taken into account here the very basic fact that hardon-nucleus collisions are not exactly equivalent to hadron-hadron collisions. The slight differences in the numerical values of the final results stem mainly from the difference in models applied and some other physical foundations used in the present calculations. Erlykin, Krutikova, and Shalbelskii[2] proposed from their theoretical study on muon spectrum and charge ratio a slightly stronger violation of Feynman scaling than what their model predicted. Although the present model for limited violation of Feynman scaling in the fragmentation region does not satisfy this criterion the results seem to give a little better description of the data owing to the other extraneous factors considered here. Thus, physically, violation of the Feynman-scaling hypothesis seems to favor the explanation of the behavior of both the muon spectrum and the muon charge ratio. But the degree of violation of Feynman scaling which is compatible with the totality of the data cannot be ascertained from this study as it is known that these two features could be understood even with the help of a scaling-type model, e.g., the radial scaling hypothesis as shown by Badhwar, Stephens, and Golden.[39]

The charge composition of cosmic rays plays an important role in the determination of the neutron fraction as a function of the energy per nucleon.[40] And the helium nuclei contribute in the maximum to the origin of the neutrons. On the other side, it was argued by Faessler[41] that $\alpha\alpha$ collisions play an important part in understanding the behavior of the muon charge ratio. This is a plain fact—whatever might be the role of the helium nuclei we did not take into account their effect in this calculation. Another limitation of this study is that we have taken here the charge excess parameter (neutron/proton ratio) to be a constant which is actually not the case at cosmic-ray energies. It actually decreases slowly with increasing energy as was shown by Avakyan *et al.*[42] The effect of changing the helium composition in the primary cosmic radiation of the muon charge ratio could also not be ascertained, as stated in the previous section, which is another limitation of the present study. However, the disagreements observed in curves B to D and especially in D of Fig. 5 show that the use of the flux compositions obtained from the data analysis of the distribution of muons in extensive air showers is not the only reason for getting a fairer agreement with A. The other data too have a high degree of relevance in arriving at the better results. We reserve here our comments on the compatibility claimed by Kopeliovich, Nicolaev, and Potashnikova[43] of the nature of rising cross sections between the QCD prediction and the cosmic-ray measurements.

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