Indian Economic Review, Vol. XXXV, No. 1, 2000, pp. 21-40

Subsidy Policies and R&D Activities : A Theoretical Analysis*

MANUELA PAL
Bethune College, 181Bidhan Sarani, Calcutta-700006, India

AND

MANASH RANJAN GUPTA
Economic Research Unit, Indian Statistical Institute, Calcutta-700035, India

ABSTRACT

We develop a theoretical model of strategic competition between the home firm and the foreign firm in the home market of a less developed country. Both the firms receive subsidy on the R&D activities from their governments. We analyse the properties of optimal subsidy policies of the governments of the two countries in the two cases : (i) each of the two governments maximizes her own welfare, (ii) both of them maximize the joint welfare.

JEL Classification : H2.

1. INTRODUCTION

Due to the trade liberalization policies of the governments in the developing countries, the firms in those countries, are facing stiff competition from the firms of the developed countries even in their own home markets. According to the GATT accord, the governments of these countries are now being restricted to subsidize exports. Therefore, the governments of the developing countries now are more inclined to subsidize research and development (R&D) activities of the firms to boost their competitiveness. In the less developed countries like India, R&D activities by the firms is a growing phenomenon in more recent times. It has been found[1] that real R&D expenditures in the private sector firms in India have grown at about 7% in the1990's. In the industry groups like telecom, agricultural machinery, chemicals, dye-stuff, drugs and pharmaceuticals, textiles, soaps and cosmetics, glass and cements etc, the R&D expenditure in the 1990's have grown at a rate higher than that in the previous decade. Government of India has been giving tax concessions on R&D expenditure for a long time.

---

* We are grateful to the referee of this journal for his comments on an earlier version of this paper with a different title. Remaining errors are ours only.

1. Evidences are available in Basant(2000)

In this paper, the objective is to do a theoretical analysis of the problem of strategic competition between the home firm and the foreign firm in the home market of a less developed economy. Spencer and Brander(1983) analyse the problem in the context of two firms from two developed countries competing in the export market. We consider a model of two countries, each of them having a single firm. They compete in the home market of one of the two countries (the home country).This home firm is trying to increase the market share in the home market and the foreign firm (country) attempts to increase her export share. The home firm is engaged in R&D activity to reduce the cost of production, the R&D being undertaken before the associated output is produced. The government in the home country commits itself to R&D subsidy before decisions on R&D level is made by the firm. So the government becomes the first player in the multistage game and can influence the equilibrium outcome of the game played by the two firms. The foreign country government can also retaliate by subsidizing the foreign firm's R&D activity in the similar manner. Therefore the two governments can engage themselves in a subsidy game.

The model of the strategic competition is based on a three-stage game. In the first stage, the governments of the two countries play a Nash - Cournot game, choosing the R&D subsidy rate. In the second stage , the firms of the two countries, knowing the subsidy levels choose the R&D levels; and then, in the third stage, the firms finally choose the output levels.They play Cournot game with each other in both these stages. Governments choose the optimum R&D subsidy rates by maximizing their own domestic welfare. For the foreign government, the welfare is defined as the profit of the firm less the total subsidy. However, for the home country in addition to the profit of the firm, domestic consumer surplus is also included in the welfare function, since the output is sold in the home market.In Spencer-Brander(1983),both the countries compete in the export market and so the welfare function of each of the two countries excludes consumer surplus. The firms determine the R&D levels and the output levels by maximizing their profit functions, assuming that the R&D levels and the output levels of the rival firm (in the respective stages) remain constant.

There are some interesting results obtained from the working of this model. When the governments of the two countries play a Nash-Cournot game to choose the R&D subsidy rates, the optimal policy is to subsidize the R&D activity. The optimal R&D subsidy rate of the home country is greater than that of the foreign country even under similar cost conditions of the two firms. However, when the governments of the two countries cooperate and maximize their joint welfare to determine their optimal subsidy rate, then the optimal subsidy rates are shown to be positive. The optimal subsidy rates of the two countries are equal (different) under similar (different) cost conditions in this cooperative solution.

In section 2 of the paper, we develop the model and analyse the determination of the optimal output and R&D levels of the firms. In section 3, we analyse the properties of optimal R&D subsidy policy in the case of non-cooperative game between the two governments. In section 3.1, we have shown how the results are related to different cost conditions. In section 3.2, we analyse whether the optimal R&D subsidy rates are strategic substitutes or strategic complements. In section 3.3, we analyse the case when the two countries are maximizing joint welfare, and then compare the optimal subsidy rates under different cost conditions.

2. THE MODEL AND ITS WORKING

We start by analysing the third stage of the game, i.e., the choice of output stage in the firm's rivalry. The home and the foreign firm produce output, q^h and q^f, at the marginal costs, c^h and c^f, respectively. The R&D levels of the home and foreign firms are x^h and x^f respectively where v^h and v^f are the unit costs of R&D. s^h and s^f denote the R&D subsidy rates of the home country and the foreign country respectively.

Profit functions of the home firm and the foreign firm are

$$\pi^h = R^h (q^h , q^f) - c^h (x^h) q^h - v^h x^h + s^h x^h \tag{1}$$

$$\pi^f = R^f (q^h, q^f) - c^f (x^f) q^f - v^f x^f + s^f x^f \tag{2}$$

We assume that the outputs are substitutes and also that the increase of output by one firm decreases the marginal revenues of the other. This implies

$$R^i_j < 0 \quad \text{and } R^i_{ij} < 0 \qquad \text{(Obviously, } R^i_i > 0,\ R^i_{ii} < 0); \quad i = h , f$$

The increase in the R&D expenditure of the ith firm lowers its marginal cost of production at a decreasing rate. Hence,

$$c^i_x < 0,\ c^i_{xx} > 0.$$

The firms are players of Cournot game in the output market and the equilibrium output levels are found from their respective first order conditions of profit maximization. These conditions are

$$(\delta\pi^h/\delta q^h) = R^h_h \ (q^h, q^f) \ - c^h (x^h) \tag{3}$$

$$(\delta\pi^f/\delta q^f) = R^f_f \ (q^h, q^f) \ - c^f (x^f) \tag{4}$$

We assume that the second order conditions to be satisfied, i.e., $\pi^h_{hh} < 0$ and $\delta^f_{ff} < 0$. We also assume that,

$$A = \pi^h_{hh}\, \pi^f_{ff} - \pi^h_{hf}\, \pi^f_{fh} > 0$$

which implies that the own effect of a change in output on the marginal profit dominates the cross effect. This also ensures the stability of equilibrium in the output space.

Now, solving the first order conditions, we get the equilibrium levels of output as functions of the R&D levels of both the firms. Mathematically,

$$q^h = q^h (x^h , x^f);$$

and, $$q^f = q^f (x^h, x^f)$$

When the R&D level of a firm is increased, the marginal cost of production of the firm is reduced. Hence her own level of output rises and the output of the rival falls[2]. These effects can be given as follows.

$$q^i_i = (\delta q^i / dx^i) = (R^i_{jj}\, c^i_x / A) \tag{5}$$

$$q^j_i = (\delta q^j / \delta x^i) = (R^j_{ji}\, c^i_x / A) \tag{6}$$

Once the output levels are determined in terms of the R&D levels, the optimal R&D levels are to be determined, given the unit cost of R&D and the R&D subsidy levels. The indirect profit functions of the firms can be expressed in terms of R&D levels. Let g^h and g^f represent these indirect profit functions. Then

$$g^h = R^h (q^h (x^h, x^f), q^f (x^h, x^f)) - c^h (x^h)\, q^h (x^h, x^f) - v^h x^h + s^h x^h \tag{7}$$

$$g^f = R^f (q^h (x^h, x^f), q^f (x^h, x^f)) - c^f (x^f)\, q^f (x^h, x^f) - v^f x^f + s^f x^f \tag{8}$$

The firms play a Cournot game in determining the R&D levels. So the following first order conditions are satisfied while maximizing profit.

$$g_h^h = (\delta g^h / \delta x^h) = R_f^h\, q_h^f - c_x^h q^h - v^h + s^h = 0 \tag{9}$$

$$g_f^f = (\delta g^f / \delta x^f) = R_h^f\, q_f^h - c_x^f q^f - v^f + s^f = 0 \tag{10}$$

(using (3) and (4) respectively.)

The second order condition is

$$g^i_{ii} = R^i_j\, q^j_{ii} + q^j_i\, (R^i_{ji}\, q^i_i + R^i_{jj}\, q^j_i) - c^i_x\, q^i_i - c^i_{xx}\, q_i < 0 \quad \text{for } i = h, f$$

We also assume that

$$B = g^h_{hh}\, g^f_{ff} - g^h_{hf}\, g^f_{fh} > 0$$

which is the stability condition for equilibrium in the R&D space. This condition implies that the own effect of a change in the level of R&D on the marginal profit dominates the corresponding cross effect.

Here, $g^i_{ij} = R^i_j\, q^i_{ij} + q^j_i\, (R^i_{ji}\, q^i_j + R^i_{jj}\, q^j_j) - c^i_x\, q^i_j$ for $i = h, f$.

These would normally be negative. An increase in the rival firm's R&D level reduces the marginal effect of own R&D on own profit.

2. Reaction curve of the firm in the output space shifts upward and that of the rival firm does not shift.

These assumptions ensure that the reaction functions of the two firms in the R&D space have negative slopes. This follows from total derivation of (7) and (8) with respect to x^h and x^f , which yield

$$(dx^h/dx^f)_h = -(g^h_{hf} / g^h_{hh}) \tag{11}$$

$$(dx^f/dx^h)_f = -(g^f_{fh} / g^f_{ff}). \tag{12}$$

These are negative.

So, x^h and x^f , i.e., the R&D levels are strategic substitutes.

The optimal values of x^h and x^f can be derived from equations (7) and (8) as functions of s^h and s^f and of the unit costs of R&D - v^h and v^f. But we ignore the effects of changes in unit costs of R&D as we assume them to be fixed. Therefore, we have the following functions.

$$x^h = x^h (s^h , s^f);$$

and $$x^f = x^f (s^h, s^f)$$

When R&D subsidy rate is increased by the home-government, it increases own R&D level in equilibrium[3] and reduces the R&D level of the rival foreign firm. Similarly, an increase in the foreign R&D subsidy rate increases the foreign R&D level and reduces the home R&D level[4]. Mathematically, these comparative static results are as follows.

$$x^i_{s^i} = (dx^i/ds^i) = -(g^i_{jj} /B) > 0 \tag{13}$$

$$x^j_{s^i} = (dx^j/ds^i) = (g^j_{ji} /B) > 0 \tag{14}$$

for $i = h, f$.

3. R&D SUBSIDY POLICY

In this section, we consider the first stage of the game, namely the determination of optimal R&D subsidy rates by the governments of the home country and of the foreign country.

We consider the social welfare functions of the two countries, which are denoted by W^h and W^f. The social welfare function of the foreign country is given by the profit of the firm net of the total R&D subsidy. So,

---

3. Reaction curve of the home firm shifts upward in the R&D space and that of the foreign firm does not shift.

4. Reaction curve of the foreign firm shifts upward in the R&D space and that of the home firm does not shift.

$$W^f = g^f - s^f x^f \tag{15}$$

But the social welfare of the home country depends on the consumer surplus in addition to producer's surplus because the product is sold in the home market. Therefore, the home social welfare is sum of the profit of the firm and consumer surplus minus the R&D subsidy level. So,

$$W^h = g^h + \phi\,(q^h + q^f) - s^h x^h \tag{16}$$

where $\phi\,(q^h + q^f)$ denotes the consumer surplus. This depends positively on the total quantity sold in the home market, $(q^h + q^f)$. Hence, $\phi'(.) > 0$.

The governments in each of the two countries maximize their respective welfare functions to determine optimal subsidy rates when they are not cooperating. But if the governments decide to cooperate, then they will maximize joint welfare to choose the optimal R&D subsidy rates. We consider both the cases here. Spencer and Brander (1983) also considered both the cases.In the case of non-cooperative equilibrium, subsidy rates are determined by solving the following two first order conditions:

$$(dW^h/ds^h) = 0 \quad \text{and} \quad (dW^f/ds^f) = 0 .$$

But when the two governments cooperate and maximize joint welfare, $W = W^h + W^f$, optimal R&D subsidy rates are decided from the first order conditions $(dW/ds^h) = 0$ and $(dW/ds^f) = 0$.

In the following subsection, we consider non-cooperative equilibrium in R&D subsidies.

3.1. *Optimal R&D Subsidy rates in Non-Cooperative Equilibrium*

The government of the home country subsidizes the R&D activity of the home firm; and the optimal subsidy rate is determined from the first order condition of welfare maximization, given by

$$dW^h/ds^h = 0$$

or

$$g_h^h x^h_{s^h} + g_f^h x^f_{s^h} + \phi'(.)\,(q^h_h x^h_{s^h} + q_f^h x^f_{s^h}) + \phi'(.)\,(q^f_h x^h_{s^h} + q_f^f x^f_{s^h}) + g^h_{s^h} - s^h x^h_{s^h} - x^h = 0 \tag{17}$$

Now, from equation (9) we know that $g^h_h = 0$.

Also, from equation (7) we get $g^h_{s^h} = x^h$. So, the equation (17) reduces to

$$g_f^h x^f_{s^h} + \phi'(.)\,(q^h_h x^h_{s^h} + q_f^h x^f_{s^h}) + \phi'(.)\,(q^f_h x^h_{s^h} + q_f^f x^f_{s^h}) - s^h x^h_{s^h} = 0 \tag{17a}$$

Again from (13) and (14) and using the results (11) and (12), we get

fr $x^f_{sh} = (dx^f / dx^h)_f \, x^h_{sh}.$

Therefore, equation (17a) can be written as

$$g^h_f \, (dx^f / dx^h)_f + \phi'(.) \, (q^h_h + q^h_f \, (dx^f / dx^h)_f) + \phi'(.) \, (q^f_h + q^f_f \, (dx^f / dx^h)_f) - s^h = 0$$

So, the optimal rate of R&D subsidy of the home country is given by

$$s^h = g^h_f \, (dx^f / dx^h)_f + \phi'(.) \, (q^h_h + q^h_f \, (dx^f / dx^h)_f) + \phi'(.) \, (q^f_h + q^f_f \, (dx^f / dx^h)_f) \quad (18)$$

Now, $g^h_f = R^h_f q^f_f < 0$ (using equation (3)). Here $(dx^f/dx^h)_f < 0$; and hence the first term of the rigft hand side of equation (18) is positive. Hence, g^h_f represents the marginal profit of the home firm with respect to the reduction in the R&D level of the foreign firm. This marginal effect takes place through the decrease in the output level of the foreign firm. So the first term on the right side of the optimal subsidy expression given by (18) shows the above mentioned marginal profit brought by an increase in the R&D level of the home firm.

The combined second and third term in the right hand side of the equation (18) indicates the marginal effect on consumer surplus with respect to change in the R&D level of the home firm. Second (third) term shows that part of the effect caused by the change in the level of production of the home (foreign) firm. The level of output of the home (foreign) firm rises (falls) and so the marginal effect on consumer surplus occurring through the change in the volume of sale of the home (foreign) firm is positive (negative). So the second (third) term is positive (negative).Therefore, the consumer surplus may either increase or decrease, depending on whether the increase in home output level is greater or less than the reduction of foreign output level.

If the consumer surplus is increased, then the optimal R&D subsidy rate of the home country will be positive and greater than the optimal R&D subsidy rate found in Spencer-Brander (1983). However, if the consumer surplus is diminished, then the optimal R&D subsidy rate is positive (negative), when the increase in profit is greater (smaller) than the reduction in consumer surplus; and is always less than that in Spencer-Brander(1983) model. In Spencer-Brander (1983) model, the firms sell their products in a third market. Therefore, consumer surplus was not included in the social welfare function of any country and hence the effect of R&D subsidy on consumer surplus was absent in their analysis.

Now, under the assumption of a linear demand function given by $p = \alpha - \beta q$ and $q = q^h + q^f$, R&D activity of either country raises total output q. But the effect of a change in the R&D subsidy of home country on the total output is ambiguous. In this case, the change in total output with respect to home R&D subsidy is

$$(q_h^h + q^h_f \, (dx^f / dx^h)_f + q^f_h + q^f_f \, (dx^f / dx^h)_f) = -\,(c^h_x / 3\beta) + (c^f_x / 3\beta) \, (g^f_{fh} / g^f_{ff}) \quad (18a)$$

Now, if we assume that

$|x^h_{s^h}| > |x^f_{s^h}|$

that is, the absolute value of the effect of a change in home R&D subsidy on its own R&D activity is more than the absolute value of the change in home R&D subsidy on the foreign R&D level, it implies (from the results (13) and (14)) that

$|g^f_{ff}| > |g^f_{fh}|$

So, total output will increase when $|c^h_x| \geq |c^f_x|$ and hence consumer surplus will also increase in this case. Total output will decrease only if $|c^h_x| < |c^f_x|$ and only if the second term of the right side of (18a) dominates the first term. Consumer surplus will also decrease in this case. Otherwise, total output and consumer surplus will increase.

The optimal R&D subsidy rate of the foreign country is similarly determined from the first order condition of her own welfare maximization. So, from $dW^f/ds^f = 0$, s^f is determined. Now, $dW^f/ds^f = 0$ implies

$$g^f_h x^h_{s^f} + g^f_f x^f_{s^f} + g^f_{s^f} - s^f x^f_{s^f} - x^f = 0 \tag{19}$$

Now, $g^f_f = 0$ from (8). Also, from (8) we get $g^f_{s^f} = x^h$; and

substituting in (19), we get,

$$s^f = g^f_h (x^h_{s^f} / x^f_{s^f}) = g^f_h (dx^h / dx^f)_h \tag{20}$$

(using the results (13) and (14) and (11))

Now, $g^f_h = R^f_h q^h_h < 0$ (using the result in equation (4)).This is the marginal effect on profit of the foreign firm with respect to the increase in the home firm's R&D level. So the term on the right side of equation (20) indicates the marginal effect on the profit of the foregn firm with respect to the R&D level of the foreign firm which takes place through a reduction in home R&D level. Therefore, the optimal rate of R&D subsidy in the foreign country is positive and this rate is same as the optimal R&D subsidy rate found in Spencer-Brander (1983) model.

For the optimal R&D subsidy rate to be positive in the case of the foreign country, we need not make any assumption about the relationship between marginal cost of production of the two firms. Also, this rate is not affected by the nature of the relationship between the change in marginal cost of two firms-c^h_x and c^f_x .

The major result can be summarized in the form of the following proposition.

PROPOSITION 1: When the firms play a non-cooperative game in choosing optimal R&D subsidies, and when the marginal effect of home firm's R&D activity on consumer surplus is positive, the R&D subsidy rates of both the home and foreign country are positive and the subsidy rate of the home country is greater than that of the foreign country.

3.2. *Best response functions of the governments in R&D subsidy space*

We have found the optimal R&D subsidy rates of the two countries, by maximizing their respective welfare function. Now the welfare functions as well as the first order conditions of welfare maximization depend on both the R&D subsidy rates. Therefore, from the equations (17) and (19), the reaction functions of the two governments in the R&D subsidy space can be derived.

Now, the slope of the reaction function of the home firm in the R&D subsidy is given by

$$ds^h / ds^f = - (W^h_{s^h s^f} / W^h_{s^h s^h})$$

where,

$$\begin{aligned} W^h_{s^h s^f} = {} & (g^h_{ff} x^f_{s^f} x^f_{s^h} + g^h_{fh} x^h_{s^f} x^f_{s^h} + g^h_f x^f_{s^h s^f}) + \phi'(.)\,(q^h_{hh} x^h_{s^f} x^h_{s^h} + q^h_{hf} x^f_{s^f} x^h_{s^h} \\ & + q^h_h x^h_{s^h s^f}) + \phi'(.)\,(q^h_{fh} x^h_{s^f} x^f_{s^h} + q^h_{ff} x^f_{s^f} x^f_{s^h} + q^h_f x^f_{s^h s^f}) + \phi'(.)\,(q^f_{hh} x^h_{s^f} x^h_{s^h} \\ & + q^f_{hf} x^f_{s^f} x^h_{s^h} + q^f_h x^h_{s^h s^f}) + \phi'(.)\,(q^f_{fh} x^h_{s^f} x^f_{s^h} + q^f_{ff} x^f_{s^f} x^f_{s^h} + q^f_f x^f_{s^h s^h}) - s^h x^h_{s^h s^f} \end{aligned}$$

and

$$\begin{aligned} W^h_{s^h s^h} = {} & (g^h_{ff} (x^f_{s^h})^2 + g^h_{fh} x^h_{s^h} x^f_{s^h} + g^h_f x^f_{s^h s^h}) + \phi'(.)\,(q^h_{hh} (x^h_{s^h})^2 + q^h_{hf} x^f_{s^h} x^h_{s^h} + q^h_h x^h_{s^h s^h}) \\ & + \phi'(.)\,(q^h_{fh} x^h_{s^h} x^f_{s^h} + q^h_{ff} (x^f_{s^h})^2 + q^h_f x^f_{s^h s^h}) + \phi'(.)\,(q^f_{hh} (x^h_{s^h})^2 + q^f_{hf} x^f_{s^h} x^h_{s^h} \\ & + q^f_h x^h_{s^h s^h}) + \phi'(.)\,(q^f_{fh} x^h_{s^h} x^f_{s^h} + q^f_{ff} (x^f_{s^h})^2 + q^f_f x^f_{s^h s^h}) - (s^h x^h_{s^h s^h} + x^h_{s^h}) \end{aligned}$$

$W^h_{s^h}$ is the marginal contribution of the home government's subsidy on the welfare of the home country. So, $W^h_{s^h s^f}$ ($W^h_{s^h s^h}$) is the rate of change in the above mentioned marginal welfare of the home country with respect to the rate of R&D subsidy of the foreign (home) government.

We now try to explain each term of the expresion $W^h_{s^h s^f}$ and detmine its sign.

Now, $g^h_f x^f_{s^h}$ is the marginal contribution of home government's subsidy on the profit of the home firm occurring through the change in the foreign R&D level. The bracketed first term

$$(g^h_{ff} x^f_{s^f} x^f_{s^h} + g^h_{fh} x^h_{s^f} x^f_{s^h} + g^h_f x^f_{s^h s^f})$$

is the rate of change of the marginal contribution of the home government's subsidy on the profit of the home firm with respect to the R&D subsidy of the foreign government.

We assume that $x^f_{s^h s^f} > 0$. This implies that the rate at which foreign R & D level is decreased as a result of an increase in home country's R&D subsidy, is reduced when the foreign R&D subsidy rate is increased. This makes the bracketed first term negative.

$\phi'(.)\ (q^h x^h_{s^h} + q^h_f x^f_{s^h})$ is the marginal contribution of the home government's subsidy on the consumer surplus through a change in the home firm's output. The combined bracketed second term and the third term,

$$\phi'(.)\ (q^h_{hh} x^h_{s^f} x^h_{s^h} + q^h_{hf} x^f_{s^f} x^h_{s^h} + q^h_h x^h_{s^h s^f}) \text{ and } \phi'(.)\ (q^h_{fh} x^h_{s^f} x^f_{s^h} + q^h_{ff} x^f_{s^f} x^f_{s^h} + q^h_f x^f_{s^h s^f})$$

indicate the rate of change in this marginal contribution with respect to the foreign R&D subsidy rate, s^f. The second term indicates the rate of change coming through the increase in own R&D level and the third term indicates the rate of change taking place through the fall in foreign R & D level. Here, we assume that $x^h_{s^h s^f} > 0$.

This implies that the home R&D level increases at a decreasing rate when foreign R&D subsidy is increased. However, the signs of both the second term and third term are ambiguous. So, the rate of change in marginal contribution on consumer surplus with respect to an increase in s^f may have any sign.

Now, $\phi'(.)\ (q^f_h x^h_{s^h} + q^f_f x^f_{s^h})$ is the marginal contribution of home governments R & D subsidy on consumer surplus through a change in foreign firm's output level. Therefore, the combined bracketed fourth term and the fifth term

$$\phi'(.)\ (q^f_{hh} x^h_{s^f} x^f_{s^h}) + q^f_{hf} x^f_{s^f} x^h_{s^h} + q^f_h x^h_{s^h s^f}) + \phi'(.)\ (q^f_{fh} x^h_{s^f} x^f_{s^h} + q^f_{ff} x^f_{s^f} x^f_{s^h} + q^f_f x^f_{s^h s^h})$$

shows the rate of change in this above mentioned marginal contribution with respect to the foreign R&D subsidy rate. The fourth term shows the part of the rate of change which comes indirectly through a change in home country's R&D level. The fifth term explains the other part of the rate of change taking place through the change in foreign R&D level. These two terms are also ambiguous in sign.

The term $(s^h x^h_{s^h} + x^h)$ is the marginal effect of subsidization of R&D activity by the home government on its total subsidy payment. The sixth term $s^h x^h_{s^h s^f}$ is the rate of change in this marginal effect with respect to the foreign R&D subsidy rate. This is assumed to be negative.

Therefore, $W^h_{s^h s^f}$ can be either positive or negative.

Now, $W^h_{s^h s^h}$ is assumed to be negative. Otherwise the second order condition of welfare maximization will not be satisfied.[5]

5. However, if we examine the different terms in this expression closely, we find that many of them are ambiguous in sign.

Therefore, (ds^h / ds^f) which is the slope of the best response function of the home government will be negative (positive) when $W^h{}_{s^h s^f}$ is negative (positive).

Next, we find the slope of the reaction function of the foreign government in the R&D subsidy space. It is given as

$$(ds^f / ds^h) = - (W^f{}_{s^f s^h} / W^f{}_{s^f s^f})$$

where

$$W^f{}_{s^f s^h} = (g^f{}_{hh}\, x^h{}_{s^h}\, x^h{}_{s^f} + g^f{}_{hf} x^f{}_{s^h}\, x^h{}_{s^f} + g^f{}_h\, x^h{}_{s^f s^h}) - s^f x^f{}_{s^f s^h} \quad (21)$$

and

$$W^f{}_{s^f s^f} = (g^f{}_{hh}\, (x^h{}_{s^f})^2 + g^f{}_{hf} x^f{}_{s^h}\, x^h{}_{s^f} + g^f{}_h\, x^h{}_{s^f s^f}) - (x^f{}_{s^f} + s^f x^f{}_{s^f s^f}) \quad (22)$$

Now, $W^f{}_{s^f}$ marginal contribution of the R&D subsidy of foreign government on the welfare of the foreign country. Therefore, $W^f{}_{s^f s^h}$ $(W^f{}_{s^f s^f})$ is the rate of change in the marginal welfare of the foreign country with respect to the rate of R&D subsidy of the home (foreign) government.

Now, $g^f{}_h\, x^h{}_{s^f}$ is the marginal contribution of the foreign government's R & D subsidy on the profit of its own firm through a change in the home R&D level. The bracketed term

$$(g^f{}_{hh}\, x^h{}_{s^h}\, x^h{}_{s^f} + g^f{}_{hf} x^f{}_{s^h}\, x^h{}_{s^f} + g^f{}_h\, x^h{}_{s^f s^h}) \text{ in the equation (21)}$$

is the rate of change in this marginal contribution with respect to the rate of R&D subsidy of the home government. Here, we assume that $x^h{}_{s^f s^h} > 0$. This means that the marginal decrease in the R&D level of the home firm with respect to foreign subsidy is reduced when the home R&D subsidy is increased. The sign of this bracketed is ambiguous.

The second term $s^f x^f{}_{s^f s^h}$ in equation (21) is the rate of change in the marginal effect of subsidization of R&D activity on the total subsidy payment of the foreign country when the rate of home R & D subsidy is changed. The term $x^f{}_{s^f s^h} < 0$, because we assume that the foreign R&D level increases at a decreasing rate when the R&D subsidy of the home country is increased.

Therefore, if the bracketed first term of the right hand side of equation (21) is negative and dominates the second term, then

$W^f{}_{s^f s^h} < 0$. Otherwise, it is positive

Again, the expression $W^f{}_{s^f s^f} < 0$ by assumption, because it is the second order condition of welfare maximization[6].

---

6. But once the terms in the expression are closely examined, all of them are not of the same sign.

So, (ds^f / ds^h) , which is the slope of the best response function of the foreign government in the R&D subsidy space, is negative (positive) when

$W^f_{s^f s^h}$ is negative (positive)

The R&D subsidy policies are strategic substitutes (complements) when the slopes of the best response functions of the two governments in the R&D subsidy space are negative (positive).Here, we can't rule out the possibility that the R&D subsidy policies can be strategic complements. However, for the non-cooperative equilibrium in R&D subsidy rates to be stable, the necessary condition is that the absolute value of the slope of the best response function of the home government should be more than that of the foreign government, i.e.,

$|(ds^f/ds^h)_h| > |(ds^f/ds^h)_f|$, irrespective of whether the R&D subsidies are strategic substitutes or complements. Mathematically, the necessary condition for stability can be expressed as follows:

$$\frac{W^h_{s^h s^h}}{W^h_{s^h s^f}} > \frac{W^f_{s^f s^h}}{W^f_{s^f s^f}}$$

or, $(W^h_{s^h s^h} W^f_{s^f s^f} - W^h_{s^h s^f} W^f_{s^f s^h}) > 0$

3.3. *R&D subsidies of the two countries: Cooperative equilibrium*

In this section we assume that the governments of the two countries maximize their joint welfare to determine the optimal subsidy rates. The joint welfare is given by

$$W = W^h + W^f,$$

or, $$W = g^f - s^f x^f + g^h + \phi (q^f + q^h) - s^h x^h.$$

To determine the optimal R&D subsidy rates s^f and s^h , we solve simultaneously the two first order conditions

$$dW/ds^f = 0 \text{ and } dW/ds^h = 0 ;$$

and solving them get the following expressions:

$$s^f = g_f^h + \phi'(.)\,(q_f^f) + \phi'(.)\,(q_f^h) \tag{23}$$

$$s^h = g_h^f + \phi'(.)\,(q_h^f) + \phi'(.)\,(q_h^h) \tag{24}$$

[7]

7. See Appendix 3 for detailed derivation.

So, the optimal R&D subsidy rates given to the firms depend on the marginal effect of her own R&D level on the profit of the rival firm and on the consumer surplus.

Now, $g^j_i < 0$, i.e., the marginal effect of the R&D level of a firm on the profit of the rival firm is negative. However, the marginal effect of the R&D on the consumer surplus is ambiguous, because the increase in R&D of a firm increases her own output, but reduces the output level of the rival firm.

So, the rate of optimal R&D subsidy will be negative if the decrease in profit is more than the increase in consumer surplus[8] or when the decrease in profit is accompanied by a fall in consumer surplus. The optimal subsidy can be positive only when the consumer surplus is increased and this increase more than compensates the fall in profit.

These R&D subsidy rates are found to be different from those in Spencer-Brander (1983) model. There $s^i = g^i_j$ ($i = h, f$). So there optimal rate of R&D subsidy is always negative. However, in the present model, the effect of the R&D subsidy on the consumer surplus is to be incorporated in the case of both the countries when the objective is to maximize joint welfare and the products are sold in the home market of one of the two countries. Therefore ($\phi'(.)q^i_i + \phi'(.)\, q^j_i$) appears in the expression of optimal R&D subsidy rate.

Assuming the linear demand function, $p = \alpha - \beta q$, the optimal R&D subsidy rates can be written as

$$s^f = (2q^f c^h_x / 3) + \phi'(.)\,(-\,c^f_x / 3\beta)$$

and

$$s^h = (2q^h c^f_x / 3) + \phi'(.)(-c^h_x / 3\beta)$$

The effect of R&D level on the total output is positive. It is independent of the marginal costs of production c^h and c^f, but depends on the change in marginal cost with respect to the R&D level (i.e., on c^h_x and c^f_x). Therefore, the change in consumer surplus does not depend on c^h and c^f, but depends on c^h_x and c^f_x. But the change in profit, g^j_i depends on the marginal costs as well as on their changes[9].

Now, if the marginal costs of the two countries as well as their changes with respect to the corresponding R&D levels are equal, i.e., if $c^h = c^f$, and $c^h_x = c^f_x$, then the two output levels as well as the marginal effects of changes in the R&D levels of the two firms on the total output are equal, that is, $q^f = q^h$ and $(q^h_f + q^f_f) = (q^h_h + q^f_h)$. In that case, the two optimal R&D subsidy rates - s^h and s^f are equal. However, if the changes in the marginal costs are not equal, then the subsidy rates are different . If, the absolute value of the rate of change of the home firm's marginal cost is greater than that of the foreign firm's marginal

8. In the case the consumer surplus is increased, $|\phi'(.)q^i_j| > |\phi'(.)q^j_i|$.

9. Derivation in detail are given in Appendix 1.

cost, i.e., if $|c^h_x| > |c^f_x|$, then $(q^h_f + q^f_f) < (q^h_h + q^f_h)$. Also, the absolute value of the effect of change of foreign R&D on the profit of the home firm is greater than that of home R&D on the profit of the foreign firm, i.e., $|g^h_f| > |g^f_h|$. Hence, $s^h > s^f$. But when the absolute value of the rate of change of the home marginal cost is less than that of the foreign marginal cost, i.e., $|c^h_x| < |c^f_x|$, then $(q^h_f + q^f_f) > (q^h_h + q^f_h)$ and $|g^h_f| < |g^f_h|$. Therefore, $s^f > s^h$.

Next, we consider those cases when the marginal costs are different for the two firms. First, if the marginal cost of home firm c^h is greater than the marginal cost of foreign firm c^f, then the output of home firm q^h will be less than the output of the foreign firm q^f. Also if the absolute value of the change in marginal cost of foreign firm is greater than that of the home firm, i.e., if $|c^f_x| > |c^h_x|$, then the marginal contribution of foreign R&D on total output is greater than the marginal contribution of home R&D on total output, i.e., $(q^h_f + q^f_f) > (q^h_h + q^f_h)$. Therefore, the marginal effect on consumer surplus is higher in the case of foreign R&D. However, the effect on the change in profits of one firm due to the increase in rival firm's R&D level are not directly comparable and the relationship between g^f_h and g^h_f is indeterminate. So, if $|g^h_f| \leq |g^f_h|$, then $s^f > s^h$. But, if $|g^h_f| > |g^f_h|$, then the relationship between s^f and s^h is ambiguous.

Now, if the absolute value of the change in marginal cost of foreign firm is less than that of the home firm, i.e., if $|c^h_x| > |c^f_x|$, then the marginal contribution of foreign R&D on total output is less than the marginal contribution of home R&D on total output, i.e., $(q^h_h + q^f_h) > (q^h_f + q^f_f)$. Also, $|g^h_f| > |g^f_h|$. Hence, $s^h > s^f$

But, if $|c^h_x| = |c^f_x|$, then $(q^h_h + q^f_h) = (q^h_f + q^f_f)$ and therefore the marginal contributions of the R&D levels on consumer surplus are also equal. But, $|g^h_f| > |g^f_h|$. Therefore, $s^h > s^f$.

So, $s^h > s^f$ when $c^h > c^f$, and when $|c^h_x| \geq |c^f_x|$. So, when the two countries are maximizing their joint welfare, the optimal R&D subsidy rate is found to be higher in the country in which the marginal cost is higher, as well as the rate of decrease in the marginal cost with respect to R&D activity is higher.

Similarly, if the marginal cost of production of home country is less than that of the foreign firm, i.e., if $c^h < c^f$, then $q^h > q^f$. Also, if $|c^f_x| > |c^h_x|$, the marginal effect of home R&D on total output is less than the marginal effect of foreign R&D on total output, i.e., $(q^h_f + q^f_f) > (q^h_h + q^f_h)$. Hence the rate of increase in consumer surplus due to unit increase in foreign R&D level is greater than that increase due to unit increase in home R&D level. Also, the absolute value of the change in home profit with respect to increase in foreign R&D is less than the absolute value of the change in foreign profit with respect to fall in home R&D level, i.e., $|g^h_f| < |g^f_h|$. Due to all these effects together, we find that the rate of optimal R&D subsidy given to the foreign firm is greater than that given to the home firm, i.e., $s^f > s^h$.

But when $|c^f_x| = |c^h_x|$, then $(q^h_f + q^f_f) = (q^h_h + q^f_h)$. But, $|g^h_f| < |g^f_h|$. Therefore, in this case also, $s^f > s^h$.

In the last case, with $c^h < c^f$, and $|c^h_x| > |c^f_x|$, $q^h > q^f$ and also $(q^h_f + q^f_f) < (q^h_h + q^f_h)$. So, $\phi'(.)(q_h) > \phi'(.)(q_f)$. Also, $|g^f_h| - |g^h_f|$ can take any sign.So, applying the same reasons as in the first case, we can conclude that $s^h > s^f$ if $|g^h_f| > |g^f_h|$ but if $|g^h_f| \leq |g^f_h|$, then the relationship between s^h and s^f is indeterminate.

Major results analysed in this section are summarized in the form of the following proposition.

PROPOSITION 2. When the firms play a cooperative game and maximize the joint welfare, then the optimal R&D subsidy rates may be of any sign. But whether they are equal or different, depend on certain conditions : (*i*) When $c^h = c^f$, and $|c^h_x| = |c^f_x|$, then $s^h = s^f$; (*ii*) When $c^h = c^f$, and $|c^h_x| > |c^f_x|$, or, when $c^h > c^f$, and $|c^h_x| \geq |c^f_x|$, then $s^h > s^f$; (*iii*) When $c^h = c^f$, and $|c^f_x| > |c^h_x|$, or, when $c^f > c^h$ and $|c^f_x| \geq |c^h_x|$, then $s^f > s^h$.

4. CONCLUSION

In the present paper, we have considered the R&D subsidy policies by the governments of the two countries when the subsidy receiving firms of the two countries are competing in the home market of one of the two countries. The optimal policy of a government is to subsidize the R&D of its firm in order to increase its market share. However, the rate of optimal subsidy in the home country is greater than that in the exporting country, because the consumer surplus of the home country is considered in the welfare function. These subsidy policies in the non-cooperative equilibrium can be either strategic substitutes or strategic complements depending on various conditions imposed on the marginal cost functions of the two firms.

We have also considered the joint welfare maximization by the two governments to determine the optimal R&D subsidy rate, which, in general, will be positive. Here, if the cost conditions are same for the two firms, then the optimal subsidy rates will be equal. But the two subsidy rates will be different if the marginal costs and rate of change in the marginal costs of the two firms are different.

REFERENCES

Basant, R. , 2000, Corporate response to economic reforms, *Economic and Political Weekly* XXXV, 813-822.

Brander, J.A. ,1995, " Strategic trade policy", G. Grossman and K. Rogoff (eds) Handbook of International Economics Volume III, North Holland.

Brander, J. and Spencer, B., 1981, Tariffs and the extraction of foreign monopoly rent under potential entry, *Canadian Journal of Economics* 14, 371-389.

Brander, J. and Spencer, B., 1983, Strategic commitment with R&D : The symmetric case, *Bell Journal of Economics* 14, 225-235.

Brander, J. A. and Spencer, B. J. , 1985, Export subsidies and international market share rivalry, *Journal of International Economics* 18, 83-100.

Miyagiwa, K. and Ohno,Y. ,1997, Strategic R&D policy and appropriability, *Journal of International Economics* 42,125-148.

Munniagaria, M.E. and Singh, N. ,1997, Spillovers and R&D policy, *International Economic Review* 38, 405-430.

Neary, J.P., 1994, Cost asymmetries in international subsidy games: Should governments help winners or losers?, *Journal of International Economics* 37,
197-218.

Spencer, B. and Brander, J. , 1983, International R&D rivalry and industrial strategy, *Review of Economics Studies* L, 707-722.

APPENDIX 1

Under the assumptions of a linear demand function $p = \alpha - \beta\,(q^h + q^f)$, the profit functions of the home and foreign firms are given by

$$\pi^h = (\alpha - \beta\,(q^h + q^f))\,q^h - c^h\,(x^h)\,q^h - v^h\,x^h + s^h x^h\,; \quad \text{and}$$

$$\pi^f = (\alpha - \beta\,(q^h + q^f))\,q^f - c^f\,(x^f)\,q^f - v^f\,x^f + s^f x^f.$$

The first order conditions of profit maximization are given by

$$(\delta\pi^h/\delta q^h) = \alpha - 2\beta q^h - \beta q^f - c^h\,(x^h) = 0; \quad \text{and}$$

$$(\delta\pi^f/\delta q^f) = \alpha - \beta q^h - 2\beta q^f - c^f\,(x^f) = 0.$$

Solving the above two equations, we get the equilibrium output levels produced by the two firms, as

$$q^f = (\alpha - 2c^f + c^h)/3\beta \quad \text{and}$$

$$q^h = (\alpha - 2c^h + c^f)/3\beta$$

Total output produced and sold in the home market is, therefore,

$$q = (2\alpha - c^h - c^f)/3\beta$$

The effect of home and foreign R&D levels on the output levels are given by

$q^i_i = (-2c^i_x)/\ 3\beta > 0$

$q^i_j = (c^j_x)/\ 3\beta < 0$ and

$q_i = (-c^i_x)/\ 3\beta > 0$; for $i, j = f, h$.

Also, the rate of change in the marginal output levels with respect to the R&D subsidy levels can be found as

$q^i_{ii} = (-2c^i_{xx})/\ 3\beta < 0;$

$q^i_{ij} = 0;$

$q^i_{jj} = (c^j_{xx})/\ 3\beta > 0;$

$q^i_{ji} = 0,$ for $i, j = f, h$.

Now, substituting the optimal values of the output levels in the profit functions, we can find the derived profit functions, denoted by g^h and g^f, which are functions of the R&D levels. The first order conditions of profit maximization with respect to the R&D levels are given by

$(\delta g^h/\delta x^h) = g^h_h = -\ \beta\ q^h q^f_h - c^h_x q^h - v^h + s^h = 0$ and

$(\delta g^f/\delta x^f) = g^f_f = -\ \beta q^f q^h_f - c^f_x q^f - v^f + s^f = 0$

The cross marginal profits are

$g^f_h = -\ \beta q^f q^h_h = (2q^f c^h_x)/3 < 0$; and

$g^h_f = -\ \beta q^h q^f_f = (2q^h c^f_x)/3 < 0.$

Also,

$g^i_{jj} = (-\ \beta q^i q^j_{jj} - \beta q^i_j q^j_j) > 0$; and

$g^i_{ji} = (-\ \beta q^i q^j_{ji} - \beta q^i_i q^j_j) = (-\ \beta q^i_i q^j_j\ ) < 0$ (because $q^j_{ji} = 0$) for $i, j = f, h$.

APPENDIX 2

Totally differentiating equation (17) , we get,

$$[(g^h_{ff}(x^f_{s^h})^2 + g^h_{fh}\, x^h_{s^h}\, x^f_{s^h} + g^h_f\, x^f_{s^h s^h}) + \phi'(.)\,(q^h_{hh}(x^h_{s^h})^2 + q^h_{hf}\, x^f_{s^h}\, x^h_{s^h} + q^h_h\, x^h_{s^h s^h})$$
$$+ \phi'(.)\,(q^h_{fh}\, x^h_{s^h}\, x^f_{s^h} + q^h_{ff}(x^f_{s^h})^2 + q^h_f\, x^f_{s^h s^h}) + \phi'(.)\,(q^f_{hh}(x^h_{s^h})^2 + q^f_{hf}\, x^f_{s^h}\, x^h_{s^h} + q^f_h\, x^h_{s^h s^h})$$
$$+ \phi'(.)\,(q^f_{fh}\, x^h_{s^h}\, x^f_{s^h} + q^f_{ff}(x^f_{s^h})^2 + q^f_f\, x^f_{s^h s^h}) - (s^h\, x^h_{s^h s^h} + x^h_{s^h})]\, ds^h +$$

$$[(g^h_{ff}\, x^f_{s^f}\, x^f_{s^h} + g^h_{fh}\, x^h_{s^f}\, x^f_{s^h} + g^h_f\, x^f_{s^h s^f}) + \phi'(.)\,(q^h_{hh}\, x^h_{s^f}\, x^h_{s^h} + q^h_{hf}\, x^f_{s^f}\, x^h_{s^h} + q^h_h\, x^h_{s^h s^f})$$
$$+ \phi'(.)\,(q^h_{fh}\, x^h_{s^f}\, x^f_{s^h} + q^h_{ff}\, x^f_{s^f}\, x^f_{s^h} + q^h_f\, x^f_{s^h s^f}) + \phi'(.)\,(q^f_{hh}\, x^h_{s^f}\, x^h_{s^h} + q^f_{hf}\, x^f_{s^f}\, x^h_{s^h} + q^f_h\, x^h_{s^h s^f})$$
$$+ \phi'(.)\,(q^f_{fh}\, x^h_{s^f}\, x^f_{s^h} + q^f_{ff}\, x^f_{s^f}\, x^f_{s^h} + q^f_f\, x^f_{s^h s^h}) - s^h\, x^h_{s^h s^f}]\, ds^f = 0$$

or, $\quad W^h_{s^h s^h}\, ds^h + W^h_{s^h s^f}\, ds^f = 0$

Again, totally differentiating equation (19), we get,

$$[(g^h_{hh}\, x^h_{s^h}\, x^h_{s^f} + g^f_{hf}\, x^f_{s^h}\, x^h_{s^f} + g^f_h\, x^h_{s^f s^h}) - s^f\, x^f_{s^f s^h}]\, ds^h$$
$$+ [(g^f_{hh}(x^h_{s^f})^2 + g^f_{hf}\, x^f_{s^f}\, x^h_{s^f} + g^f_h\, x^h_{s^f s^f}) - (x^f_{s^f} + s^f\, x^f_{s^f s^f})]\, ds^f = 0$$

or, $W^f_{s^f s^h}\, ds^h + W^f_{s^f s^f}\, ds^f = 0$

APPENDIX 3

The joint welfare of the two countries are given by

$$W = g^f - s^f x^f + g^h + \phi\,(q^f + q^h) - s^h x^h$$

The first order conditions of welfare maximization are $dW/ds^f = 0$ and $dW/ds^h = 0$. Now,

$$\frac{dW}{ds^f} = g^f_f\, x^f_{s^f} + g^f_h\, x^h_{s^f} + g^f_{s^f} - x^f - s^f x^f_{s^f} + g^h_f\, x^f_{s^f} + g^h_h x^h_{s^f} + g^h_{s^f}$$
$$+ \phi'(.)\,(q^f_f\, x^f_{s^f} + q^f_h\, x^h_{s^f}) + \phi'(.)\,(q^h_f\, x^f_{s^f} + q^h_h\, x^h_{s^f}) - s^h x^h_{s^f} = 0$$

Here, $g^f_f = 0$, $g^h_h = 0$, $g^h_{sf} = 0$, $g^f_{sf} = x^f$ and $x^h_{sf} = (dx^h / dx^f)_h \, x^f_{sf}$

Substituting these values in the above expression, we get,

$$s^f + s^h (dx^h / dx^f)_h = g^f_h (dx^h / dx^f)_h + g^h_f + \phi'(.) (q^f_f + q^f_h (dx^h / dx^f)_h)$$

$$+ \phi'(.) (q^h_f + q_h^h (dx^h / dx^f)_h \quad \text{(A1)}$$

This can be written as (for simplicity)

$$s^f + s^h (dx^h/dx^f)_h = K_1 \quad \text{(A1')}$$

where K_1 represents the right hand side of equation (A1)

Similarly,

$$\frac{dW}{ds^h} = g^f_f x^f_{sh} + g^f_h x^h_{sh} + g^f_{sh} - s^f x^f_{sh} + g^h_f x^f_{sh} + g_h^h x^h_{sh} + g^h_{sh}$$

$$+ \phi'(.) (q^f_f \, x^f_{sh} + q^f_h \, x^h_{sh}) + \phi'(.) (q^h_f x^f_{sh} + q^h_h \, x^h_{sh}) - s^h x^h_{sh} - x^h$$

Again, $g^f_f = 0$, $g^h_h = 0$, $g^f_{sh} = 0$, $g^h_{sh} = x^h$ and $x^f_{sh} = (dx^f / dx^h)_f \, x^h_{sh}$

Substituting, we get,

$$s^f (dx^f / dx^h)_f + s^h = g^f_h + g^h_f (dx^f / dx^h)_f + \phi'(.) (q^f_f (dx^f / dx^h)_f)$$

$$+ \phi'(.) (q^h_f (dx^f / dx^h)_f) \quad \text{(A2)}$$

This is written as

$$s^f (dx^f/dx^h)_f + s^h = K_2 \quad \text{(A2')}$$

where K_2 represents the right hand side of equation (A2).

Now, solving (A1') and (A2') simultaneously, we get

$$s^f = \frac{K_1 - K_2\ (dx^h / dx^f)_h}{1 - (dx^f / dx^h)_f\ (dx^h / dx^f)_h}$$

$$s^h = \frac{K_2 - K_1\ (dx^h / dx^f)_h}{1 - (dx^f / dx^h)_f\ (dx^h / dx^f)_h}$$

Now, substituting the values of K^1 and K^2, we get the expressions of the R&D subsidy rates.