An Extended Electromagnetic Theory

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Extended formulations of electromagnetic theory *in vacuo* are presented , as being based on a nonzero electric field divergence and a nonzero electric conductivity. These two approaches introduce new features by which some of the so far unanswered questions in electromagnetic field theory may become settled, and the applications of the same theory be enlarged. Examples on this are given by the steady states of leptons and string configurations, and by plane and axisymmetric electromagnetic wave phenomena including photon physics, also with astrophysical applications.

Keywords: photon mass; extended electromagnetic theory

1. Introduction

During more than a century classical electromagnetic theory and its formulation in terms of Maxwell's equations have been successfully applied to numerous problems in physics. Nevertheless there are areas whithin which these equations do not seem to provide a fully adequate description of physical reality. Thus there are several questions the answers of which require further research and new modes of approach.

1. Models in terms of conventional theory can not explain the execessively small observed radial dimension of the electron.
2. There is so far no definite theoretical indication whether or not the neutrino should have a nonzero rest mass.
3. In attempts to develop conventional electrodynamical models of the individual photon, there is a difficulty in finding solutions which both converge at the photon centre and vanish at infinity. This was already realized by Thomson[1].
4. Light appears to be made of waves and simultaneously of particles. In conventional theory the photon is one hand conceived to be a massless particle, still having an angular momentum, and is on the other hand regarded as a wave having the frequency ν and the energy $h\nu$, whereas its angular momentum is independent of the frequency.
5. The Fresnel laws of reflection and refraction of light in non-dissipative media have been known for some 180 years. However, these laws will not apply to total reflection at the boundary between a dissipative medium and a vacuum region[2].

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With the purpose of tackling the problems just mentioned, as well as that of widening the areas of application, several new approaches have been elaborated[3−9] part of which propose modified forms of Maxwell's equations in vacuo.Among these the present review describes two approaches the detailed deductions of which are given elsewhere[5−9]. One is based on the hypothesis of a nonzero electric field divergence in vacuo[5−7], and the other on the assumption of a small nonzero electrical conductivity in vacuo[8,9].

2. Basis of Present Approaches

2.1 Nonzero Electric Field Divergence in Vacuo

Here Maxwell's equations have been modified on the basis of two hypotheses:

1. The electric field divergence can differ from zero in vacuo.
2. The field equations should remain Lorentz invariant.

On this basis the extended form of Maxwell's equations can be written as

$$\Box^2(A, i\phi/c) = -\mu_0(\vec{j}, ic\bar{\rho}) = -\mu_0\bar{\rho}(\vec{C}, ic) = -\mu_0\vec{J} \tag{1}$$

Here, $\bar{\rho}$ denotes charge density and $\vec{j}$ is the corresponding "space charge current density" in vacuo where $\vec{C}^2 = c^2$. This extended form has a connection with the Dirac theory of the electron[10.11].It can be compared to the theories by Dirac and by de Broglie, Vigier and Evans. In the Dirac theory with the relativistic wave function ψ the four- current has the form

$$\vec{J} = ec(\bar{\psi}\alpha_i\psi, i\bar{\psi}\psi) \tag{2}$$

with a given charge e of the electron. In the theory by de Broglie and Vigier[12] and by Evans and Vigier[3] on the individual photon with a given rest mass m_0, the four-current becomes instead

$$\vec{J} = (\frac{2\pi m_0 c^2}{h})(1/\mu_0)(\vec{A}, i\phi/c) \tag{3}$$

For steady electromagnetic equlibria the present form(1) yields an expression for the integrated charge of a particle-like state, and for axisymmetric wave packets it results in an expression for the photon rest mass.

2.2 Nonzero Electric Conductivity in Vacuo

Maxwell's equations in vacuo have been modified by assigning a small but nonzero conductivity coefficient ($\sigma \neq 0$). This gives rise to a displacement current as already observed by Bartlet *et al.*[13]. If we assign a nonzero conductivity coefficient to the Maxwell vacuum instead of space-charge, then the photon looses its energy when it propagates through such a vacuum. But to make this framework fully relativistic, it is needed to introduce a space-charge in vacuum. From the modified Maxwell equations the following situations arise:

1. Plane waves are progressively damped with the factor $\exp(-kz)$, where $k = \omega\beta$.
2. The phase velocity of propagation of the wave is $\frac{1}{\alpha}$ and varies with the frequency.

Here,

$$\alpha^2 = \frac{\chi_e \chi_m}{2c^2}\left[\left\{1 + (\frac{\sigma}{\epsilon \chi_e \omega})^2\right\}^{1/2} + 1\right]$$

and

$$\beta^2 = \frac{\chi_e \chi_m}{2c^2}\left[\left\{1 + (\frac{\sigma}{\epsilon \chi_e \omega})^2\right\}^{1/2} - 1\right].$$

where ω is the frequency.

3. New Features

The present approaches give rise to new phenomena represented by steady equilibria and dynamic states of wave propagation.

3.1 Effects due to Nonzero Electric Field Divergence

The current density in equation(1) leads to steady electromagnetic equilibria in vacuo, being determined by

$$c^2 curl^2 \vec{A} = -\vec{C}(\nabla^2)\phi = \vec{C}\frac{\bar{\rho}}{\epsilon_0} \tag{4}$$

The resulting axisymmetric states consist of “particle-shaped” geometry being bounded both in the axial and radial directions, and of “string-shaped” geometry being uniform in the axial direction.

The particle-shaped solutions yield states both with nonzero and zero net charge and magnetic moment. They may contribute to the understanding of such truly elementary particles as the leptons. The string-shaped equilibria reproduce several features of the earlier proposed string model of the hadron color field.

The basic equations also predict $nontranssverse$ waves in vacuo as determined by the equation

$$(\frac{\partial^2}{\partial t}^2 - c^2\vec{\nabla}^2)E + (c^2\nabla + \vec{C}\frac{\partial}{\partial t})(div\vec{E}) = 0 \tag{5}$$

for the electric field. There are three limiting cases :

1. When $div\vec{E} = 0$ and $curl\vec{E} \neq 0$, the result is a conventional $transverse$ electromagnetic wave, henceforth denoted as an “EM wave”.
2. When $div\vec{E} \neq 0$ and $curl\vec{E} = 0$, a purely $longitudinal$ electric space-charge wave arises, here denoted as an “S-wave”.
3. When both $div\vec{E} \neq 0$ and $curl\vec{E} \neq 0$, a hybrid $nontransverse$ electromagnetic space-charge wave appears, heree denoted as an “EMS wave”.

3.2 Effects due to Nonzero Electric Conductivity

If we try to formulate the extended Einstein-Broglie-Proca (EBP) theory i.e. Maxwell’s vacuum with $\sigma \neq 0$ in a fully relativistic manner as well as to make the theory gauge invariant, it is necessary to introduce the concept of a space-charge in vacuo as

$$j = (\vec{j}, j_0) \tag{6}$$

where $j_0 \neq 0, j_0 = ic\bar{\rho}$. Then we have both

$$div\vec{E} \neq 0; \text{and } curl\vec{E} \neq 0 \tag{7}$$

This is nothing but the "EMS" wave of the first approach.

4. Plane Waves

Due to their relative simplicity, plane waves provide a convenient demonstration of the new dynamic states which originate from the present approaches.

4.1 Waves due to a Nonzero Electric Field Divergence

The waves which arise in the case of a nonzero electric field divergence are for any field component assumed to have the form $\exp[i(-\omega t + \vec{k}.\vec{r})]$ where ω and $\vec{k} = (k_x, k_y, k_z)$ are the frequency and wave number, with $\vec{r} = (x, y, z)$. When $\vec{k} \times \vec{E} = 0$ the magnetic field vanishes, $\vec{C} \times \vec{E} = 0$, and $\vec{k} \times \vec{C} = 0$. Then all the field vectors $\vec{k}, \vec{E}$ and $\vec{C}$ become parallel and the dispersion relation is

$$\omega = \pm kc \tag{8}$$

These are the characteristic features of the S wave of the previous section. When $\vec{k}x\vec{E} \neq 0$ there is a magnetic field. Then we also have $\vec{k} \times \vec{C} \neq 0$, and $\vec{E}\dot{\vec{C}} = 0$. The dispersion relation becomes

$$\omega = \vec{k}\dot{\vec{C}}\text{(9)}$$

which characterizes the EMS wave. For this wave the conventional Poynting theorem applies, but not for the S wave. Here it should be noticed that many authors^{2,3} use the name "longitudinal waves" for all modes having at least one field component in the direction of propagation. This would then apply as a common name to both the S wave and the EMS wave. An incident conventional EM wave, which propagates through a dissipative medium and becomes subject to total reflection at a plane vacuum interface, leads to a problem which cannot be solved by means of conventional theory^2. A solution can, however, be provided through EMS waves.

4.2 Waves due to a Nonzero Electric Conductivity

In EBP theory the Maxwell equation can be written as

$$\Box^2 E = k_0^2 E + \sigma\mu_0\chi_m \frac{\partial E}{\partial t} \tag{10}$$

instead of $\Box^2 E = k_0^2 E$ for a Maxwell vacuum. On laboratory scale we can neglect the second term of the right hand side and get back the conventional equations. In EBP theory the real physical (spin 1) pilot waves are considered to be associated with the photons that follow the average line in their motion.

5. Features of Present Axisymmetric Photon Model

A photon model can be based on the axisymmetric solutions of equation(5) as follows.

1. The model can be pictured as having a "bound" ("self-confined") part of radiation which is associated with a rest mass m_0, and which is superimposed on a remaining

"free" part of radiation represented by the mass difference $m - m_0$. In the laboratory frame the bound radiation can be regarded to circulate around the axis of symmetry.. In this way the model unifies the wave and particle concepts of an individual photon.

2. The limit of zero rest mass leads to a divergent EM mode which is physically unacceptable. The rest mass can be allowed to be very small but nonzero. When it is less than 10^{-9} of the electron mass, there should not be any detectable departure from the zero rest mass case in a Michelson-Morley experiment.
3. The total electric charge is zero, and it is not necessary to rely on the idea that the photon is its own antiparticle.
4. The intrinsic magnetic field of the present wave packet model is reconcilable but not identical with the photon model by Evans and Vigier[3,14].
5. The phase and group velocities are equal and constant, thereby being slightly smaller than c which can be considered as an asymptotic velocity limit at infinite photon energy. There is no dispersion in wave number space. The total mass and the rest mass are both proportional to the frequency ν. All parts of the field energy of the wave packet are thus included i the same way in the total energy $h\nu$.
6. The present wave packet model is supported by so far performed experiments. In studies on interference phenomena of individual photons[15], dot-shaped marks are observed at a screen. These marks seem to be consistent with the limited radial extension of an impinging photon. This result is also supported by microwave transmission experiments of in presence of an aperture[16].

6. Possible Astrophysical Implications

The isotropy and homogeneity of our universe are the two basic components of the cosmological principle.

1. The hypothesis of nonzero photon rest mass leads to the violation of isotropy of the light properties.
2. Several attempts have been made to relate the nonzero rest mass of the photon with the vacuum dissipation processes which leads to a new interpretation of the redshift of spectral lines at cosmological scale.
3. The recent astrophysical events at high redshift can be used to place severe limits on the variation of the speed of light[17], the photon mass and the energy scale of quantum gravity.

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