SHORT COMMUNICATIONS

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Effect of Thermocapillarity on the Production of a Conducting Thin Film in the Presence of a Transverse Magnetic Field

The gradual development of a thin liquid film on the surface of a rotating disk is studied analytically when the thermocapillary force is in action in the presence of a transverse magnetic field. It is found that the acting thermocapillary force has a profound effect in enhancing the thinning rate of the film even in the presence of large Hartmann number M. A physical explanation of this result is provided. Large amounts of fluid are depleted in a small span of time when the Hartmann number M is small.

1. Introduction

Coating with a very thin and uniform film of photoresist on silicon wafers for integrated circuits, or the production of a layer of very thin magnetic paint on the substrates which are placed in the grooves of a spinning disk, is known in the literature as spin coating. This technique has also found its application in the production of optical and magnetic recording media. A sizeable amount of works [1–7] has so far been published in the literature by assuming the coating liquids to be either Newtonian or non-Newtonian fluids with or without the admixture of different volatile solvents. It is well known from these studies that the rate of film thinning slows down beyond a specific height (depending on the rotational speed) of the film. In general, the final stage of film thickness is proportional to $\tau^{-1/2}$, for $\tau \to \infty$, where τ is the spining time. So, to obtain the desired thinness of the film, one has to operate the spinner for a quite long time. This is so because the removal of liquid continues in the radial direction from the surface of the disk by the action of the centrifugal force. As a result, the film thins progressively and, at the same time, the outward radial velocity decreases continuously due to the increase of viscous resistance with thinning. After a sufficient lapse of time, the radial flow practically ceases and during this period the chief mechanism of mass loss is due to evaporation only. It is well known that the evaporation starts from the surface layer of the film. During the process of evaporation the latent heat is extracted from the film and as a result, a solid skin is formed on the surface layer which puts greater resistance to the remaining liquid for evaporation. Therefore, coating defects may occur if the convective flow does not completely cease before this skin hardens sufficiently. Recently, Dandapat and Roy [8] have tried to accelerate the rate of thinning such that one can obtain the desired thinning before the hardening of the skin. According to them it is always possible to create an external tangential stress on the film surface in the form of a surface tension gradient by imposing either temperature or concentration differences between the centre of the disk and its periphery. They have shown in [8] that by imposing a specified axisymmetric temperature distribution on the disk it is possible to obtain the film thickness proportional to $(\alpha\tau)^{-1}$ for $\tau \to \infty$, where α is a new nondimensional thermocapillary parameter. Middleman [9] and Regh and Higgins [10] have also noticed that the rate of film thinning increases due to the shear induced by air flow over the surface of the film. In connection with the development of a conducting thin film in the presence of a transverse magnetic field, Ray and Dandapat [11] have recently shown that the magnetic field puts greater resistance on film thinning right from the beginning of the rotation. Hence it may be interesting to study the joint effects of thermocapillarity and of a uniform magnetic field on the production of a conducting thin film on a rotating disk.

In other words, the motivation of this study is to answer the question *"does the thermocapillarity force overcome the magnetic resistance?"*

2. Mathematical formulation and its solution

Consider a uniform film of viscous, electrically conducting fluid on a disk whose radius is large compared with the film thickness h_0. A uniform transverse parallel magnetic field B_0 permeates through the disk which can rotate with a uniform angular velocity Ω about an axis normal to the plane of the disk as depicted in Fig. 1. Initially the system is at room temperature T_0. An axisymmetric temperature distribution which can decrease/increase radially outward from the axis of rotation is imposed simultaneously with the start of the uniform angular rotation of the disk. The origin is fixed at the centre of the disk, and the z-axis is the axis of rotation.

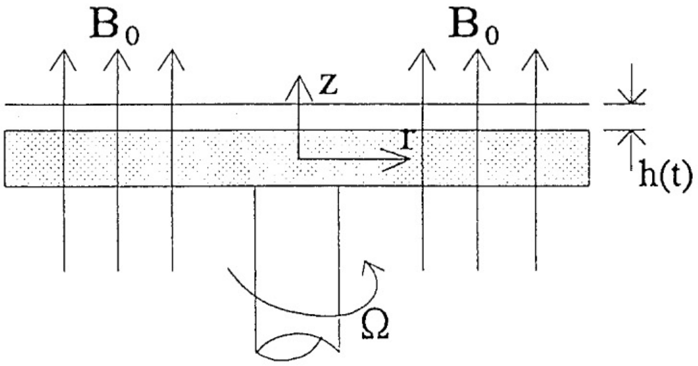

Fig. 1. Schematic drawing of the geometry

For axisymmetric motion, the governing equations in cylindrical coordinates (r, θ, z) become

$$u_r + (u/r) + w_z = 0\,, \tag{1}$$

$$u_t + uu_r - (v^2/r) + wu_z = -(1/\varrho)\, p_r + \nu[u_{rr} + (u/r)_r + u_{zz}] - (\sigma B_0^2/\varrho)\, u\,, \tag{2}$$

$$v_t + uv_r + (uv/r) + wv_z = \nu[v_{rr} + (v/r)_r + v_{zz}] - (\sigma B_0^2/\varrho)\, v\,, \tag{3}$$

$$w_t + uw_r + ww_z = -(1/\varrho)\, p_z + \nu[w_{rr} + (w_r/r) + w_{zz}]\,, \tag{4}$$

where u, v, w, p, ϱ, ν, and σ denote, respectively the velocity components along the radial, circumferential and axial directions, pressure, density, kinematic viscosity, and electrical conductivity of the fluid. Further, a subscript denotes the derivative with respect to the indicated variable. Due to the thinness of the film (Takashima [12] and Dandapat and Kumar [13]) the buoyancy effects can be neglected in the present problem.

The energy equation becomes

$$T_t + uT_r + wT_z = K[T_{rr} + T_r/r + T_{zz}]\,, \tag{5}$$

where K is the thermal diffusivity.

The initial conditions are

$$u(r,\, z,\, 0) = v(r,\, z,\, 0) = w(r,\, z,\, 0) = 0\,; \tag{6a}$$

$$T(r,\, z,\, 0) = T_0\,, \qquad h(0) = h_0\,, \qquad h_t(0) = 0\,. \tag{6b}$$

The boundary conditions are

(i) *at the disk* $z = 0$

$$u(r,\, 0,\, t) = 0\,, \qquad v(r,\, 0,\, t) = \Omega r\,, \qquad w(r,\, 0,\, t) = 0\,, \tag{7a}$$

and the imposed temperature distribution

$$T(r,\, 0,\, t) = T_0 - \lambda(r^2/2)\, T_1\,, \tag{7b}$$

where T_0 and T_1 are positive constants. λ may take either -1 or $+1$ depending on heating or cooling the disk from below. Here heating (cooling) refers to a situation where the temperature of the rotating substrate increases (decreases) with the distance from the axis of rotation;

(ii) *at the free surface* $z = h(t)$

$$-p + 2\mu w_z = 0\,, \tag{8a}$$

$$\mu(u_z + w_r) = -(\bar{\sigma}_T)\, T_r\,, \tag{8b}$$

$$\mu v_z = -(\bar{\sigma}_T)\, T_z\,, \tag{8c}$$

where μ is the dynamic viscosity and $\bar{\sigma}$ denotes surface tension. It changes with temperature according to

$$\bar{\sigma} = \bar{\sigma}_0 - \gamma(T - T_0)\,.$$

Here $\bar{\sigma}_0$ is the surface tension at temperature T_0 and $\gamma\ (= -\bar{\sigma}_T)$ is positive for most common liquids [8, 12].

Assuming a *planar interface*, equations (8a)–(8c) are derived from the free surface conditions, i.e., the jump in the normal stress across the interface is balanced by surface tension times curvature, and the shear stress equals the thermal stress along the interface.

The thermal boundary condition at $z = h(t)$ is given by the Newton's law of cooling

$$T_z + L(T - T_g) = 0\,, \tag{8d}$$

where L and T_g denote the heat transfer coefficient at the free surface and temperature in the gas phase respectively. The kinematic condition at the free surface becomes

$$h_t = w(r,\, h,\, t)\,. \tag{9}$$

Following Dandapat and Ray [6] we can seek the similarity solutions in the form

$$u = rf(z,\,t)\,, \qquad v = rg(z,\,t)\,, \qquad w = w(z,\,t)\,, \tag{10a}$$

$$p = -(r^2/2)\,A'(z,\,t) + B'(z,\,t)\,, \tag{10b}$$

$$T = T_0 - \lambda(r^2/2)\,\bar{\theta}(z,\,t) - \lambda\bar{\varphi}(z,\,t)\,. \tag{10c}$$

It should be noted here that the similarity solution (10c) is compatible with the boundary condition (7b) and as r is assumed to be large but finite so T never tend to $\mp\infty$.

The following dimensionless variables

$$\begin{gathered} \tau = t/t_c\,, \quad \xi = z/h_0\,, \quad H = h/h_0\,, \quad F = h_0 f/U_0\,, \quad G = g/\Omega\,, \quad W = w/U_0\,, \\ \theta = (h_0^2/\Delta T)\,\bar{\theta}\,, \quad \varphi = \bar{\varphi}/\Delta T\,, \quad A = A'/\Omega^2\,, \quad B = B'/(h_0\Omega)^2 \end{gathered} \tag{11}$$

are used in those equations which are obtained after using (10) in (1–9) and equating the different order of r. The set of dimensionless equations thus obtained are:

$$\begin{aligned} &2F + W_\xi = 0\,, \\ &\mathrm{Re}(F_\tau + F^2 + WF_\xi) = F_{\xi\xi} + G^2 - M^2F\,, \\ &\mathrm{Re}(G_\tau - GW_\xi + WG_\xi) = G_{\xi\xi} - M^2G\,, \\ &\mathrm{Re}(W_\tau + (1/2)\,W_\xi^2) - W_{\xi\xi} + B_\xi = 0\,, \\ &A_z = 0\,, \\ &\Pr\mathrm{Re}(\theta_\tau + W\theta_\xi - W_\xi\theta) = \theta_{\xi\xi}\,, \\ &\Pr\mathrm{Re}(\varphi_\tau + W\varphi_\xi) = \varphi_{\xi\xi} + 2\theta\,. \end{aligned} \tag{12}$$

As explained earlier that, during the course of rotation a situation will arise when viscous shear and the centrifugal force across the film are of comparable magnitude. The characteristic time scale $t_c\ (\equiv \nu/h_0^2\Omega^2)$ used in equation (11) represents that time when a balance is reached between the above stated forces. At this stage radial characteristic velocity U_0 becomes very small as a result $\mathrm{Re}\ (\equiv U_0h_0/\nu)$ the Reynolds number is also very small. The dimensional parameters $M\ (\equiv B_0h_0(\sigma/\varrho\nu)^{1/2})$ and $\Pr\ (\equiv \nu/K)$ are defined as Hartmann number and the Prandtl number respectively. We have assumed $\Delta T = |\lambda h_0^2 T_1|$.

Corresponding boundary and initial conditions become

$$\left.\begin{aligned} &F(0,\,\tau) = W(0,\,\tau) = 0\,, \qquad G(0,\,\tau) = 1 \\ &\theta(0,\,\tau) = 1\,, \qquad \varphi(0,\,\tau) = 0 \end{aligned}\right\} \quad \text{at} \quad \xi = 0\,. \tag{13}$$

$$\left.\begin{aligned} &F_\xi(H,\,\tau) = \alpha\lambda\theta(H,\,\tau)\,, \qquad G_\xi(H,\,\tau) = 0 \\ &\theta_\xi(H,\,\tau) = \varphi_\xi(H,\,\tau) = 0 \\ &H_\tau(\tau) = W(H,\,\tau) \\ &A = 0 \end{aligned}\right\} \quad \text{at} \quad \xi = H\,. \tag{14}$$

$$\left.\begin{aligned} &F(\xi,\,0) = G(\xi,\,0) = W(\xi,\,0) = 0 \\ &\theta(\xi,\,0) = \varphi(\xi,\,0) = 0 \\ &H(0) = 1\,, \qquad H_\tau(0) = 0 \end{aligned}\right\} \quad \text{at} \quad \tau = 0\,. \tag{15}$$

where $\alpha\,(\equiv \gamma\,\Delta T/\varrho_0 h_0^2 \Omega^2)$ is a parameter due to thermocapillarity. It is clear from equations (12e) and (14f) that $A = 0$. Now, $B(z, t)$ can be evaluated by integrating the equation (12d) with respect to ξ from ξ to $\xi = H(\tau)$ and finally pressure can be obtained from the equation (10b).

Expanding the dependent variables in power of Re in the form

$$\psi(\xi, \tau) = \sum_{j=0}^{\infty} \mathrm{Re}^j\, \psi_j(\xi, \tau) \tag{16a}$$

and

$$H(\tau) = \sum_{j=0}^{\infty} \mathrm{Re}^j\, H_j(\tau)\,. \tag{16b}$$

Using (16) in the system of equations (12)–(15) we can obtain different sets of equations by equating the different powers of Re. Avoiding details, the solutions to the zeroth-order set satisfying the zeroth-order boundary conditions are

$$\begin{aligned}
F_0 &= \frac{\alpha\lambda \sinh M\xi}{M \cosh MH} - \frac{\cosh(M[H-\xi])\,[\cosh(2MH)-3] - \cosh(MH)\,[\cosh(2M[H-\xi])-3]}{6M^2 \cosh^3(MH)}\,,\\
G_0 &= \frac{\cosh[M(H-\xi)]}{\cosh(MH)}\,,\\
W_0 &= \frac{4\lambda\alpha \sinh^3(M\xi/2)}{M^2 \cosh(MH)} - \frac{\sinh[2M(H-\xi)] - \sinh(2MH) + 6M\xi}{6M^3 \cosh^2(MH)}\\
&\quad + \left[\frac{\cosh(2MH)-3}{3M^3 \cosh^3(MH)}\right] [\sinh[M(H-\xi)] - \sinh(MH)]\,,\\
\theta_0 &= 1\,, \qquad \varphi_0 = 2H\xi - \xi^2\,.
\end{aligned} \tag{17}$$

It should be pointed out here that the zero-order solution (17) which we have obtained does not satisfy the initial condition (15). This is due to the fact that our characteristic time t_c is considered to be large. To satisfy the initial condition one should stretch the time and follow the procedure as described in [6]. This detail is avoided here since our object is the final film thickness for $\tau \to \infty$, and the joint effects on it due to thermocapillarity and magnetic force.

The leading order film thickness equation can be obtained from (14e) after using (16) and (17c) as

$$H_{0\tau} = -\frac{4\lambda\alpha \sinh^2(MH_0/2)}{M^2 \cosh(MH_0)} - \frac{1}{3M^3}\,[\tanh^2 MH_0 + 3\,\mathrm{sech}^2\, MH_0 (MH_0 - \tanh MH_0)]\,. \tag{18}$$

It is clear from (18) that $H_{0\tau} < 0$ for all times irrespective of the values of M or α provided $\lambda = +1$, i.e., the disk is cooled from below. To obtain the leading-order film thickness, one has to integrate (18) subject to the initial condition

$$H_0(0) = 1\,. \tag{19}$$

Condition (19) is obtained from the small-time solution of (14e) and (15) (details in (6)).

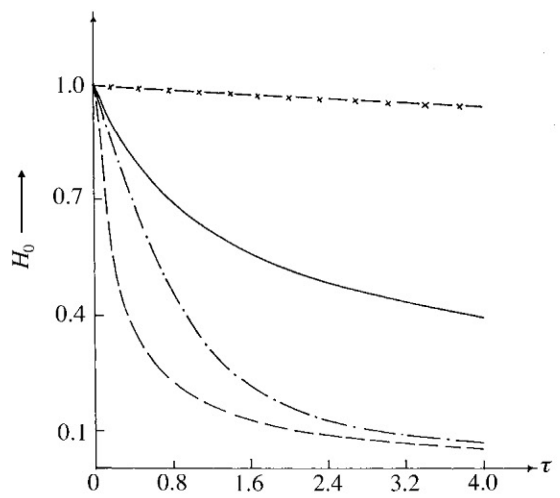

Fig. 2. Variation of film thickness H_0 with time τ for different values of (α and M).
For $\lambda = 1$: ——— (0, 0), – – – (4, 0), –×–× (0, 3), – · – · (4, 3)

3. Result and discussion

The numerical solution to the equation (18–19) is obtained by using Gill's modified method [14]. Figure 2 depicts the variation of film height with time for various values of α and M when the disk is allowed to cool. It is clear from the figure that α has a strong effect in producing a thin film even in the presence of a magnetic field. The thinning effect of α may be explained as follows: The thermocapillary parameter α is a measure of the variation of surface tension with temperature. Since the disk is allowed to cool axisymmetrically, the surface tension is low at the centre of the disk and hence the thermocapillary force acts as a tangential stress on the surface of the film along the favourable flow direction. This leads α to enhance the film thinning when the disk is cooled from below. The role of α will be adverse if the disk is heated axisymmetrically from below. This can be seen from equation (18) for $\lambda = -1$ in that the first term on the right hand side is now positive which implies a slow rate of thinning. In this case, the temperature of the disk will be higher at the periphery than at the centre and α will act along the adverse flow direction. It is further clear from the figure that the magnetic parameter M slows down the rate of thinning. This is so because, as M increases, the strength of the lines of force will also increase. This, in turn, will put more resistance to the flow for thinning. The trend may be envisaged from the asymptotic solution of H_0 for large time ($\tau \to \infty$) as

$$H_0 \sim (2/M)\tanh^{-1}(M/2\lambda\alpha\tau)\,. \tag{20}$$

Equation (20) shows that H_0 depends on both α and M even for extremely thin films. For an ultra thin film, one may expect that the effects of intermolecular forces, like van der Waals force of attraction etc., will come into play, and that also film rupture may occur. These questions will be delt in a future communication.

For different conditions one can obtain the applicable asymptotic forms of H_0 for large τ as follows:

Case I In the absence of α we get

$$H_0 \sim (1/M)\tanh^{-1}[(3M^2/4\tau)^{1/2}]\,. \tag{21}$$

Case II When $M \to 0$

$$H_0 \sim (\lambda\alpha\tau)^{-1}\,. \tag{22}$$

Case III For $\alpha = 0$ and $M \to 0$, we have

$$H_0 \sim \tau^{-1/2}\,. \tag{23}$$

This shows that the solution (20) is a general asymptotic form since other forms can be obtained as a particular case.

From Figure 3, it is clear that the maximum amount of fluid flows out of the disk in a small span of time as τ increases, and ultimately attains an asymptotic value. Here $\bar{Q}(\tau)$ is the rate of liquid depleted in time τ at a fixed radial distance $r = r_0$ (say).

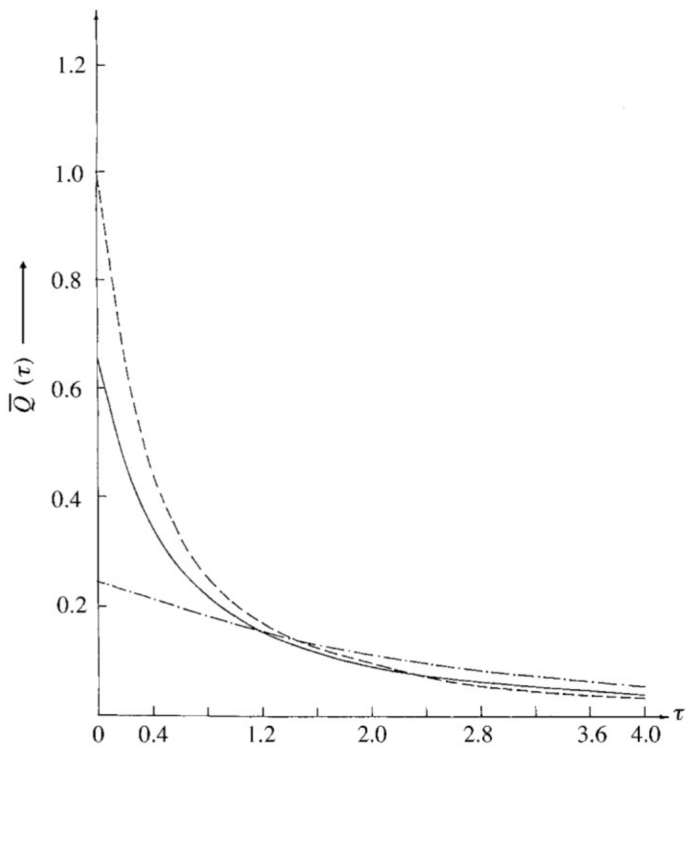

Fig. 3. Variation of $\bar{Q}(\tau)$ with time τ for different values of (α and M).
——— $(0,0)$, – – – $(5,0)$, – · – · $(0,1)$

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BOOK REVIEW

Ablowitz, M. J.; Fokas, A. S.: Complex Variables: Introduction and Applications. Cambridge, Cambridge University Press 1997. XII, 647 pp. £ 55.00, US$ 80.00; ISBN 0-521-48058-2 (hardback); £ 19.95, US$ 34.95; ISBN 0-521-48523-1 (paperback) (Cambridge Texts in Applied Mathematics)

Das vorliegende Buch besteht aus zwei Teilen. Im Teil I (ca. 300 Seiten) werden die gängigen Grundlagen der klassischen Funktionentheorie bis zum Residuenkalkül und der Auswertung komplexer Integrale entwickelt. Einige über den Standardstoff hinausgehende Themen sind: Verallgemeinerte Cauchy-Formel für nicht-analytische Funktionen, Satz von Mittag-Leffler, Einführung in die Fourier- und Laplace-Transformation mit Anwendungen auf partielle Differentialgleichungen. Teil I endet mit dem Argumentprinzip und dem Satz von Rouché. Die Darstellung ist direkt, ohne unnötige Finessen und stets von zahlreichen Beispielen begleitet. Wo möglich, werden Ergebnisse physikalisch interpretiert. Im Teil II (ca. 315 Seiten) werden in drei unabhängig voneinander lesbaren Kapiteln von je ca. 100 Seiten spezielle anwendungsorientierte Themen behandelt. Kap. 5 ist den konformen Abbildungen und ihrer Bedeutung für praktische Fragen gewidmet. Polygone und Kreisbogenpolygone treten auf und eine kurze Einführung in die Strömungstheorie. Der konforme Modul eines Vierecks wird eingeführt und physikalisch interpretiert. Die Integralgleichung von Theodorsen wird abgeleitet und ihre numerische Lösung skizziert. Kap. 6 bringt die asymptotische Auswertung von Integralen. Es beginnt mit illustrierenden Beispielen, behandelt sodann das asymptotische Verhalten von Laplace-Integralen, Watsons Lemma und die Methode der stationären Phase. Die Methode des steilsten Abstiegs samt Anwendungen auf partielle Differentialgleichungen werden eingehend geschildert. Schließlich werden in Kap. 7 Riemann-Hilbert-Probleme (RH) eingehend theoretisch sowie in praktischer Anwendung behandelt. Zunächst werden die Plemeljschen Formeln für Integrale vom Cauchy-Typ abgeleitet, sodann ihre Anwendung auf RH-Probleme für geschlossene Kurven und Bögen behandelt. Die Lösung singulärer Integralgleichungen via RH-Problemen wird diskutiert. Wertvoll sind die zahlreichen Anwendungen in der Strömungstheorie, auf die Radon-Transformation und die Lösung von Integralgleichungen vom Abelschen Typ oder mit logarithmischem Kern. Schließlich werden einige Probleme für verallgemeinerte analytische Funktionen behandelt, das sind Funktionen, welche $(*)\ \partial\Phi/\partial\bar{z} = A(z, \bar{z})\,\Phi + B(z, \bar{z})\,\bar{\Phi}$ in einem Gebiet genügen. Die allgemeine Lösung von $(*)$ läßt sich in Integralform geschlossen darstellen. Auch in Teil II ist die Darstellung erfreulich ‚straightforward', theoretisch gut fundiert, aber immer mit Blick auf praktische Anwendungen und mit vielen Beispielen und Übungsaufgaben versehen.

So eignet sich das Buch hervorragend für Studierende der Angewandten Mathematik, aber auch für Freunde der Funktionentheorie, die Anwendungen ihrer Disziplin kennenlernen wollen. Da das Buch von Lawrentjew und Schabat (Methoden der komplexen Funktionentheorie, Berlin 1967), obwohl im Inhalt und in der Zielgruppe vergleichbar, nun schon 30 Jahre alt ist, so ist das Erscheinen des vorliegenden Werkes sehr zu begrüßen, und es ist ihm eine weite Verbreitung zu wünschen.

Gießen D. Gaier