ON A CONSTRUCTION OF GROUP DIVISIBLE PARTIALLY BALANCED INCOMPLETE BLOCK DESIGNS

By SANPEI KAGEYAMA

Indian Statistical Institute

and

Hiroshima University, Japan

and

R. N. MOHAN

D. A. R. College, Nuzvid, India

SUMMARY. A method of constructing group divisible partially balanced incomplete block (GDPBIB) designs is discussed.

1. INTRODUCTION

Many methods of construction of GDPBIB designs are available in literature (cf. Chen, 1980, Kageyama and Mohan, 1984, Kageyama and Tanaka, 1981, Raghavarao 1971, Trivedi and Sharma, 1975). Various types of Kronecker product designs are used to construct block designs.

In this note, a method of constructing GDPBIB designs is described. A balanced incomplete block (BIB) design is used as a basic design. The notation and definitions used here are coincident with those generally used.

2. CONSTRUCTION

Let N_i be the $v_i \times b_i$ incidence matrix of a BIB design with parameters v_i, b_i, r_i, k_i and λ_i, where $i = 1, 2$. Then the matrix $N_1 \otimes J + N_1^* \otimes N_2$, where J is a $v_2 \times b_2$ matrix with every element unity, and N_1^* is the complement of a design N_1, yields a GDPBIB design with parameters

$$v' = v_1 v_2, \quad b' = b_1 b_2, \quad r' = r_1 b_2 + (b_1 - r_1) r_2,$$

$$k' = k_1 v_2 + (v_1 - k_1) k_2, \quad \lambda_1' = r_1 b_2 + (b_1 - r_1) \lambda_2$$

$$\lambda_2' = \lambda_1 b_2 + (b_1 - \lambda_1) r_2.$$

provided that the parameters of the original design N_1 satisfy the condition $b_1 = 2r_1 - \lambda_1$, and in this case a group divisible association scheme of $m = v_1$ groups of $n = v_2$ treatments each is given as $(1, 2, ..., v_2), (v_2+1, v_2+2, ..., 2v_2),$

..., $((v_1-1)v_2+1, ..., v_1v_2)$; in the case $b_2 = 2r_2-\lambda_2$, we again get a GDPBIB design with λ_2' and λ_2^* interchanged and the group divisible association scheme of $m = v_2$ groups of $n = v_1$ treatments each is given as

$$(j, v_2+j, 2v_2+j, ..., (v_1-1)v_2+j), \quad j = 1, 2, ..., v_2.$$

The proof is trivial and omitted here.

Similarly, it can be shown that matrices $N_1 \otimes N_2 + N_1^* \otimes J$ and $N_1 \otimes I + N_1^* \otimes J$ yield GDPBIB designs with appropriate parameters under some parametric conditions. Other incidence matrices of Kronecker product type GDPBIB designs can variously be considered. But if one wants to construct GDPBIB designs by the Kronecker product, then the process may in many cases be essentially same as outlined here.

REFERENCES

CHEN, Z. L. (1980): Combinatorial recursion methods of constructing GD designs. *Acta Math. Appl. Sinica*, **3**, 373-381.

KAGEYAMA, S. and MOHAN, R. N. (1984): Three methods of constructing PBIB designs. *Commun. Statist. Theory & Methods*, **13**(25), 3185-3189.

KAGEYAMA, S. and TANAKA, T. (1981): Some families of group divisible designs. *J. Statist. Planning Inference*, **5**, 231-241.

RAGHAVARAO, D. (1971): *Constructions and Combinatorial Problems in Design of Experiments*, John Wiley & Sons, Inc., New York.

TRIVEDI, H. T. and SHARMA, V. K. (1975): On the construction of group divisible incomplete block designs. *Canad. J. Statist.*, D3, 285-288.

Paper received : November, 1984.