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Determining Specifications for the Multi-Dimensions of a Mating Component and a Sampling Plan for Monitoring Incoming Quality

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ABSTRACT *The technology/designs that are used for manufacturing in engineering industries in developing countries, such as India, are usually bought from developed countries. Since the quality of raw materials, skills of workmen and the environmental control are usually poorer in developing countries the variability in the components is higher than the design requirements. Carrying out 100% inspection and sorting of incoming lots are not practicable owing to high cost. Again rejection of too many lots leads to the unavailability of components on the shop floor and deteriorates the relationship with suppliers. Under this circumstance it is often required to make a compromise by setting wider specifications (called working specifications), which allows incidents of 'tolerance stack-up' during assembly or production of non-conforming final products. These, in turn, result in loss of productivity and quality-related problems. This case study carried out in an Indian company demonstrates that the 'tolerance stack-up' and quality related problems can be reduced considerably under the existing constraints by defining appropriately (i) the working specifications for the multi-dimensions of a mating component and (ii) the sampling plan for monitoring its incoming quality. This led to a reduction of incidents of 'tolerance stack-up' from 19.2% to 3.9%, and usage of the optimal sequence of ordering of the dimensions for inspection minimized the inspection cost.*

KEY WORDS: Mating component, multi-dimensions, clearance, working specifications, measurement error, sampling plan

Introduction

The final products in engineering industries are produced by assembling a number of discrete parts or components. The two components whose dimensions interact are called mating components. The common feature of engineering industries in developing countries such as India is that whereas the raw materials and the other resources are indigenous the technology/designs for the manufacturing are bought from developed countries. Since

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the quality of raw materials, skills of workmen and the environmental control are usually poorer in the developing countries than the developed countries, the variability in the components is higher than the design requirements. Carrying out 100% inspection and sorting of incoming lots are not practicable owing to high cost. Again, rejection of too many lots leads to unavailability of components on the shop floor and deteriorates the relationship with suppliers. Often, a compromise is made by setting wider specifications, which allows incidents of 'tolerance stack-up' during assembly or production of non-conforming final products. This paper demonstrates through a case study how the 'tolerance stack-up' and quality related problems can be reduced considerably under the existing constraints by determining appropriate (i) working specifications for the multi-dimensions of a mating component and (ii) a sampling plan for monitoring its incoming quality. This paper also describes determination of the optimal sequence of ordering of the dimensions for inspection that minimizes inspection cost. The simple statistical techniques, such as analysis of variance, scatter diagram, F-test, t-test, $\overline{X} - R$ charts and usage of properties of Normal and Binomial probability distributions, have been useful for this purpose.

The case study was carried out in an Indian electrical switch-gear making company. In the company, one of the common products is the contactor, a controlling device whose function is to make or break an electrical circuit as desired. For production of a contactor, one of the important assembly requirements is to push-fit eight NC contact bases into the eight similar grooves made on a moulded plastic case. During fitting of a NC contact base into a groove the dimensions x_1, x_2, x_3 and x_4 (coded) of the NC contact base interact with the dimensions y_1, y_2, y_3 and y_4 (coded) of the groove respectively. According to the design requirements, the dimensions x_3 and x_4 of the NC contact base should have an interference fit with the dimensions y_3 and y_4 of a groove respectively. This implies that clearance $c_i = y_i - x_i$ $(i = 3, 4)$ should be negative. For the pair of dimensions (y_2, x_2) a partial interference fit is allowed and for the pair of dimensions (y_1, x_1) no interference is allowed. The specifications for the clearance, c_i $(i = 1, 2, \ldots, 4)$, for the four pairs of interacting dimensions are shown in Table 1.

As long as the clearance between the ith pair of dimensions, c_i $(i = 1, 2, \ldots, 4)$, is within the range $[c_i^L, c_i^U]$ the fit of the two dimensions is considered proper. Otherwise the fit of the two dimensions is considered improper, i.e. a misfit. It may be noted that since the plastic is not a rigid material, both 'tolerance stack-up' and misfit may occur due to incompatibility between two interacting dimensions. If the incompatibility between the dimensions is large there will be 'tolerance stack-up' in the sense that the NC contact base cannot be pushed. However, if the incompatibility between the dimensions is small the NC contact base will be pushed (maybe with some difficulty) but the two dimensions will be a misfit with respect to specification for clearance. An NC

Table 1. Specifications for clearances

Interacting dimensions	Clearance	
	Minimum (c_i^L)	Maximum (c_i^U)
(y_1, x_1)	0	0.40
(y_2, x_2)	−0.15	0.30
(y_3, x_3)	−0.30	−0.10
(y_4, x_4)	−0.35	−0.08

contact base is considered a misfit if the fit between any of the four pairs of the interacting dimensions is improper.

The Problem

The technology/designs for the manufacturing were supplied to the company by its Japanese collaborator. At present, whereas the plastic cases are produced within the company, the NC contact bases are supplied by a domestic manufacturer. It had been experienced that the actual ranges of variation in the dimensions of incoming NC contact bases were considerably wider than their design specifications and thus it was not practicable to consider the design specifications as product acceptance criteria because it would lead to rejection of a large number of incoming lots, resulting in the unavailability of the component in the assembly shop. The confusion regarding the acceptability criteria of the NC contact base was aggravated due to the fact that the variability in the other mating component (i.e. the plastic case), which is produced within the company, was also larger than the design requirements.

The current practice for monitoring incoming lot quality is as follows: 15–20 items from a lot are inspected, and if all the measured values are in close proximity to the design specifications the incoming lots of the NC contact base are accepted. However, what is meant by 'close proximity' is not well defined and consequently the product acceptance criterion varies with the inspectors.

The result of a snap study carried out in the assembly shop revealed that the occurrences of 'tolerance stack-up' during assembly, with the current variability in the plastic case and incoming quality of the NC contact base, was as high as 19.2%. The percentage of misfit NC contact bases could not be known since it was not possible to know how many times incompatible NC contact bases had been pushed forcefully. But the technical personnel of the company believed that it was quite high and, as a result, the percentage of customer complaints regarding the contactor was relatively high.

Objectives

The objectives of the study were set as follows.

(1) To determine the natural variations (tolerances) of the dimensions of the grooves in plastic cases under the current production system in the company and to examine the scope of reducing the natural variations (tolerances).
(2) To determine the working specifications for the dimensions of NC contact bases such that proportion of its proper fit into the grooves are maximized under the current natural variations of the dimensions of the grooves.
(3) To develop a sampling plan so that acceptance of lots of NC contact bases of poor quality is safeguarded adequately, and
(4) To define the sequence of ordering of the dimensions for inspection such that the total expected cost of inspection is minimized.

Data Collection

In the company, two dies are used for producing the plastic case. In a die there are eight similar blocks, which results in eight similar grooves in a case after the moulding operation. Thus, the two possible sources of systematic variation in a dimension of the grooves are (a) the difference between the two dies and (b) the difference between the blocks within a die. Therefore, it was planned to include moulded cases produced out of die 1 as well as die 2, as samples, and measure all four dimensions in all the grooves of the sample cases. Measuring the dimensions of a groove in a case requires cutting the case in such a way that various cross-sections are exposed. Since measuring the dimensions in the case is a destructive test and it requires considerable time, only one moulded case in a day was collected randomly for measurement purposes. A sample of 16 moulded cases was collected over 16 consecutive days. In the first eight days the cases were produced using die 1 and in the next eight days the cases were produced using die 2. Thus, a total of 128 observations were available on each of the four dimensions of the grooves. The data were recorded keeping the identification of the dies and the blocks within the dies.

Two lots, which arrived from the supplier within a ten-day span were selected randomly. These two lots are supposed to be produced in two different production runs at the supplier's end. From each selected lot a sample of size 30 was drawn randomly. All four dimensions of the sampled NC contact bases were measured. In order to ensure that measurement error is minimized, all the measurements in plastic cases and NC contact bases were made by an inspector using the same digital vernier.

Analysis and Results

Analysis of Variance

The observed values on each of the dimensions of the grooves were subjected to an analysis of variance (ANOVA) in order to test the statistical significance of difference between blocks within dies. The following two-stage nested fixed-effect model was adopted for the analysis:

$$y_{ijk} = \mu + \tau_i + \beta_{j(i)} + \varepsilon_{(ij)k}$$

$$(i = 1, 2; j = 1, 2, \ldots, 8; \text{ and } k = 1, 2, \ldots, 8)$$

where,

μ = overall mean;
τ_i = effect of ith die
$\beta_{j(i)}$ = effect of jth block within ith die
$\varepsilon_{(ij)k}$ = random error component
y_{ijk} = kth observation on a dimention of the groove produced due to jth block within ith die

It may be noted that the effects of die 1 and die 2 are confounded with the effects of the first eight days and next eight days respectively. The ANOVA of the dimensions y_1 and y_2 are given in Table 2 and y_3 and y_4 are given in Table 3.

Since all the F values were statistically insignificant at the 5% level, it was concluded that variation in no dimension was inflated due to differences between blocks within dies, and possibly the variation in no dimension was inflated due to the difference between the dies.

Table 2. ANOVA of dimensions y_1 and y_2

		y_1			y_2		
Source	DF	SS	MS	F	SS	MS	F
Die	1	0.00211	0.00211	2.77 ^{NS}	0.00035	0.00035	0.29 ^{NS}
Block (Die)	14	0.00836	0.00060	0.79 ^{NS}	0.01521	0.00109	0.92 ^{NS}
Error	112	0.08523	0.00076		0.01328	0.00119	
Total	127	0.09570			0.14844		

SS = sum of squares; MS = mean sum of squares; ^{NS} = not significant at 5% level.

Test of Pair-wise Independence of the Dimensions

Scatter diagrams for the pairs of dimensions (y_1,y_2), (y_1,y_3), (y_1,y_4), (y_2,y_3), (y_2,y_4) and (y_3,y_4) were made using the observed measurements on the grooves produced from a particular block. These diagrams did not exhibit any relationship between the pair of dimensions (see Appendix 1). Scatter diagrams for the pairs of dimensions (x_1,x_2), (x_1,x_3), (x_1,x_4), (x_2,x_3), (x_2,x_4) and (x_3,x_4) also did not exhibit a relationship between the pairs of dimensions. This confirmed the hypothesis of the technical personnel that the four dimensions in the grooves and NC contact base are pair-wise independent.

Understanding the Stability of the Manufacturing Process of the Plastic Case and NC Contact Base

Deming (1986) discussed the importance of process stability. For a stable process only, the distribution of a quality characteristic (e.g. the dimension) is predictable. The stability of a process is judged through control chart plots of the suitable statistics computed from sample of subgroup size 'n'. Here, for each dimension $y_i (i = 1, 2, \ldots, 4)$ the eight measurements obtained from eight grooves within a case were considered as a subgroup, i.e. $n = 8$. Since the range method loses efficiency as an estimator of variance as n increases, $\overline{X} - S$ control charts are preferred to $\overline{X} - R$ when n is large. Montgomery (1985) has recommended not using $\overline{X} - S$ charts unless $n > 10$. For each dimension $y_i (i = 1, 2, \ldots, 4)$, the $\overline{X} - R$ control charts' limits were estimated and the control chart plots were obtained. For all the dimensions the subgroup range and averages were within the control limits and for no dimension did the R as well as $\overline{X}$ charts exhibit any systematic pattern of variation (see Appendix 2). Therefore it was concluded that the

Table 3. ANOVA of dimensions y_3 and y_4

		y_3			y_4		
Source	DF	SS	MS	F	SS	MS	F
Die	1	0.00005	0.00005	0.06 ^{NS}	0.00012	0.00012	0.11 ^{NS}
Block (Die)	14	0.01826	0.00130	1.55 ^{NS}	0.00711	0.00051	0.46 ^{NS}
Error	112	0.09447	0.00084		0.12363	0.00110	
Total	127	0.11278			0.13086		

plastic moulding process was stable. Since two dies were used during the data collection period it was further concluded that there was no difference between the two dies.

On the other hand, observations on each dimension of the NC contact base x_i ($i = 1, 2, \ldots, 4$) obtained from a lot were plotted on normal probability paper and no outlier was detected. The mean and standard deviation (SD) of each dimension in each lot were estimated and the equality of SDs and means were tested using F-test and t-test respectively. It was found that for no dimension did the means and SDs in the two lots differ statistically. It was therefore assumed that both the lots of the NC contact bases came from a stable process at the supplier's end.

Estimating Parameters of the Distribution of y_i ($i = 1, 2, \ldots, 4$)

It is well known that dimensional measurements follow a Normal distribution, which is defined by two parameters – the mean and the standard deviation (SD). It may be noted that the measured values of the dimensions in the plastic case do not match the measured values of the corresponding dimensions in the die due to contraction of the plastic after cooling, and thus the true means of the dimensions in the plastic case are unknown. Since none of the possible sources of systematic variation was significant and there was no evidence to assume that the moulding process was unstable, the mean and SD of each dimension of the groove were estimated from all 128 observations. It was decided to consider the natural tolerances of the dimensions as their working specifications. The estimated means, SDs and natural tolerances of the dimensions of the grooves are given in Table 4.

Estimating Parameters of the Distribution of x_i ($i = 1, 2, \ldots, 4$)

Since it was reasonable to assume that the lots of NC contact bases came from a stable normal process at the supplier's end, the mean and SD of each dimension in the NC contact base were estimated by pooling the observations from the two lots. These estimates are given in Table 5.

Estimating Current Proportion of Proper Fit of NC Contact Bases into the Grooves

It is well known that if $X \sim N(\mu_x, \sigma_x^2)$ and $Y \sim N(\mu_y, \sigma_y^2)$ and the two variables are independent, then $(Y - X) \sim N(\mu_x - \mu_y, \sigma_x^2 + \sigma_y^2)$. Thus here,

$$c_i = (y_i - x_i) \sim N(\mu_{y_i} - \mu_{x_i}, \sigma_{y_i}^2 + \sigma_{x_i}^2)$$

Table 4. Estimates of mean, SD and natural tolerances of y_i ($i = 1, 2, \ldots, 4$)

Dimension (y_i)	Estimated Mean ($\bar{y}_i$)	Estimated SD ($\hat{\sigma}_{y_i}$)	Natural Tolerances ($\bar{y}_i \pm 3\hat{\sigma}_{y_i}$)
y_1	8.178 mm	0.02745 mm	8.096 mm–8.260 mm
y_2	8.106 mm	0.03418 mm	8.003 mm–8.208 mm
y_3	1.335 mm	0.02980 mm	1.246 mm–1.424 mm
y_4	1.401 mm	0.03210 mm	1.305 mm–1.497 mm

Table 5. Estimates of mean and SD of x_i ($i = 1, 2, \ldots, 4$)

Dimension (x_i)	Mean ($\bar{x}_i$)	SD ($\hat{\sigma}_{x_i}$)
x_1	7.990 mm	0.0460 mm
x_2	8.062 mm	0.0413 mm
x_3	1.591 mm	0.0480 mm
x_4	1.641 mm	0.0508 mm

The proportion of proper fit between the pair of dimensions (y_i, x_i) was estimated as

$$P(c_i^L \le c_i \le , c_i^U) = P[c_i \le c_i^U] - P[c_i \le c_i^L]$$

$$= \Phi\left(\frac{c_i^U - (\mu_{y_i} - \mu_{x_i})}{\sqrt{(\sigma_{y_i}^2 + \sigma_{x_i}^2)}}\right) - \Phi\left(\frac{c_i^L - (\mu_{y_i} - \mu_{x_i})}{\sqrt{(\sigma_{y_i}^2 + \sigma_{x_i}^2)}}\right) \qquad (1)$$

The value of $\Phi(z)$ is available in a standard normal table. Using the estimates of $\mu_{x_i}, \sigma_{y_i}^2, \sigma_{x_i}^2$ and μ_{y_i} ($i = 1, 2, \ldots, 4$) in equation (1) the current proportion of proper fit between the ith pair of interacting dimensions was estimated. These estimates are shown in Table 6.

It was noted that the pair of dimension (y_3, x_3) was the most critical one. Only 75.58% of these two interacting dimensions were fitted properly. The percentage of proper fit between the pair of interacting dimensions (y_4, x_4) was also unacceptably low (96.12%). The proportion of proper fit of an NC contact base to a groove was estimated to be

$$0.99975 \times 0.99978 \times 0.75577 \times 0.96117 = 0.7261$$

This implied that incidents of 'tolerance stack-up' and misfit of the NC contact base was occurring 27.39% times.

Determination of Working Specifications for the Dimensions of the NC Contact Base

At first, the optimal means of the dimensions of the NC contact base were determined. Given μ_{y_i}, the mean $\mu_{x_i}^*$ of the dimension x_i, will be optimal if it leads to μ_c^*, which maximizes the proportion of proper fit between the ith pair of interacting dimensions. It is well known that, due to the symmetric property of the Normal distribution, the proportion of

Table 6. Current proportion of proper fit between interacting dimensions

Interacting dimensions (y_i,x_i)	Minimum clearance (c_i^L)	Maximum clearance (c_i^U)	Proportion of proper fit
(y_1, x_1)	0	0.40	0.99975
(y_2, x_2)	−0.15	0.30	0.99978
(y_3, x_3)	−0.35	−0.08	0.75577
(y_4, x_4)	−0.35	−0.08	0.96117

proper fit will be the maximum if μ_c^* is at the mid-value of the minimum and maximum clearances. This implied that

$$\mu^*_{c_i} = \frac{c_i^U + c_i^L}{2} \text{ or } \mu_{y_i} - \mu^*_{x_i} = \frac{c_i^U + c_i^L}{2} \text{ or } \mu^*_{x_i} = \frac{2\mu_{y_i} - (c_i^L + c_i^U)}{2} \tag{2}$$

Using the estimate of μ_{y_i} and known values of c_i^Uandc_i^L in equation (2), the optimal mean of a dimension x_i ($i = 1, \dots, 4$) was estimated. The calculated optimal mean values of x_i and the expected proportion of a proper fit between the interacting dimensions (y_i,x_i) at $\mu^*_{x_i}$ under the existing natural variation of y_i and x_i are given in Table 7.

It is noted that the percentage of proper fit of the dimensions (y_3,x_3) will increase by about 17% under the existing variations if the optimal mean for the dimensions x_3 is achieved. It might be worth noting here that this finding was contrary to the popular belief of the company personnel that a deviation by 0.06 mm would not have a significant effect. The percentage of proper fit of the dimensions (y_4,x_4) will also improve considerably if the optimal mean is achieved for the dimension x_4. It was estimated that achieving optimal means of x_i ($i = 1, 2, \dots, 4$) would lead to a reduction in incidents of 'tolerance stack-up' and misfit of the NC contact base from 27.39% to 10.01%.

For further reducing incidents of 'tolerance stack-up' and misfit of the NC contact base under the existing variability in the dimensions of the grooves, a reduction of variation in the dimension x_i (particularly for $i = 3, 4$) is essential. Demanding of the supplier too much reduction in variability is unfair and also unrealistic. Therefore, management set a target of proportion of proper fit for the mating dimensions (y_3,x_3) and (y_4,x_4) of 0.99, which would lead to an increase of the proportion of the proper fit of the NC contact base from 0.8999 to 0.98.

Let the standard deviations of the dimensions $x_i(i = 3, 4)$ be unknown and denoted as $\sigma^*_{x_i}(i = 3, 4)$. The probability of proper fit between the interacting dimensions (y_i, x_i) will be $(1 - \alpha_i)(i = 3, 4)$ if

$$\frac{c_i^U - (\mu_{y_i} - \mu^*_{x_i})}{\sqrt{(\sigma^2_{y_i} + \sigma^{*2}_{x_i})}} = Z_{\alpha_i/2}$$

or

$$\sigma^*_{x_i} = \sqrt{\left[\left\{\frac{c_i^U - (\mu_{y_i} - \mu^*_{x_i})}{Z_{\alpha_i/2}}\right\} - \sigma^2_{y_i}\right]} \tag{3}$$

Table 7. Expected proportion of proper fit between the interacting dimensions at $\mu^*_{x_i}$ under the existing natural variation

Dimension (x_i)	Optimal mean ($\mu^*_{x_i}$)	Interacting dimensions	Proportion of proper fit
x_1	7.980	(y_1,x_1)	0.9998
x_2	8.025	(y_2,x_2)	1
x_3	1.530	(y_3,x_3)	0.9233
x_3	1.615	(y_4,x_4)	0.9749

Putting $Z_{\alpha_i/2} = 2.576$ and the values of c_i^U and $\mu_{x_i}^*$, and estimates of μ_{y_i} and $\sigma_{y_i}^2$ in equation (3), the values of $\sigma_{x_3}^*$ and $\sigma_{x_4}^*$ were found to be 0.0249 mm and 0.0414 mm respectively.

Since, under the current variability, more than 99.98% proper fit would be achieved between the interacting dimensions $(y_i, x_i)(i = 1, 2)$ by achieving the optimal means of the dimensions x_1 and x_2, the working specifications for x_i $(i = 1, 2)$ were taken as $\mu_{x_i}^* \pm 3\hat{\sigma}_{x_i}(i = 1,2)$. On the other hand, to achieve 99% proper fit between the interacting dimensions $(y_i, x_i)(i = 3, 4)$, the mean and SD of x_i $(i = 3, 4)$ should be at their optimal and desired values respectively. Therefore the working specifications for x_i $(i = 3, 4)$ were taken as $\mu_{x_i}^* \pm 3\sigma_{x_i}^*(i = 3, 4)$. The proposed working specifications for the four dimensions of the NC contact base are shown in Table 8. The procured NC contact base should conform to these specifications to achieve 98% proper fit of the NC contact base into the grooves of plastic cases currently produced in the company. These working specifications can be viewed as 98% performance specifications.

Consideration of Measurement Error

On day-to-day operations, different inspectors will carry out the measurements for monitoring incoming lot quality. If measurement error is high relative to the specifications, the chance of misjudgement (i.e. accepting a non-conforming dimension and rejecting a conforming dimension) will be high. From a repeatability and reproducibility study carried out previously it was known that uncertainty (σ_e) in the measurement of dimensions by different inspectors with a digital vernier is only 8.1% of the overall variation. Montgomery & Runger (1994) recommended that if $6\sigma_e$ is 10% of (USL-LSL) the measurement process can be considered adequate. That is, when USL-LSL $= 6\sigma$ the measurement process can be considered adequate if $(\sigma_e/\sigma) \leq 0.1$, which implies that the current measurement process is adequate.

It is worth mentioning that, in the general situation when measurement error is large, the rate of both types of misclassifications will be high. In order to minimize the misclassifications, the test specifications or test limits (the limits that should be applied to judge the acceptability of a product based on a set of measured values) should be different from the performance specifications. Grubbs & Coon (1954) have provided three criteria for fixing test specifications, which can be used when measurement error is large. The three criterions for setting test limits are (i) ensuring both types of misclassifications are equal; (ii) the sum of the two types of misclassifications is minimized and (iii) the cost of making wrong

Table 8. Proposed working specifications for x_i $(i = 1, 2, \ldots, 4)$

	Working specifications	
Dimension	Lower limit (LWSL)	Upper limit (UWSL)
x_1	7.843	8.117
x_2	7.899	8.151
x_3	1.455	1.636
x_4	1.491	1.739

decisions is minimized. On the other hand, in a situation where failure of a component can be catastrophic, 100% inspection should be carried out and test limits should be inside the desired performance specifications by an amount that ensures no non-conforming units will be accepted as the result of a measurement error. Eagle (1954) has discussed the procedures for determining test specifications when a high reliability of conformance to performance specifications is required.

In this case study, since measurement error variance was quite small and the failure of the component is not expected to be catastrophic, no different test specifications for the dimensions of the NC contact base were proposed. Further, maintaining two sets of specifications for a particular component creates confusion among personnel leading to implementation difficulty.

Follow-up Actions

The working specifications for the dimensions of the NC contact base (Table 8) were communicated to the supplier along with a request to take the necessary actions so that the working specifications were met. Accordingly, the supplier took some corrective measures.

Determining a Sampling Plan for Monitoring the Incoming Quality of the NC Contact Base

Acceptance sampling by variables was preferable since it would provide valuable information regarding the cause of any non-conformance, such as a large variance or shifted mean. Further, for a given sample size, sampling by variables is known to give better quality protection than sampling by attributes. The International Standard for sampling by variables, ISO 3951:1989, suggests in paragraph 1.2b that univariate plans be applied to each of the m quality characteristics separately when the quality of an item depends on m quality characteristics that are measured on a continuous scale. Implementation of multi-sampling plans simultaneously for monitoring the quality of a product is very difficult from practical perspective. In addition, in this approach, usually the combined consumer's risk (i.e. the probability of accepting a poor lot) is lower than the consumer's risk for one characteristic, and the combined producer's risk (probability of rejecting a very good lot) is larger than for one characteristic, which is unfair and detrimental to a good relationship with the supplier. Usage of multivariate acceptance sampling by variables (Hamilton & Lesperance, 1995; Baillie, 1987) developed by assuming that a multivariate normal distribution of the quality characteristics can alleviate the problem of applying multi-univariate plans separately. The multivariate sampling plans requires estimation of the overall proportion of non-conformance, which cannot be computed without computer support, and also it is difficult for the inspectors to comprehend.

Considering the statistical aspects of different types of sampling plans, education levels among the inspectors, current practice and resource constraints the management of the company opted for a single attribute sampling plan with the condition that sample size (n) should not exceed 6% of the lot size. The lot size of the incoming NC contact base is 1000, which implied that n should be less than or equal to 60.

We began with the assessment of the current non-conformance fraction of the NC contact bases. For this purpose, a sample of size 30 was collected from the new lots of the NC contact bases supplied after the supplier took corrective actions. The observed measurements on each dimension were plotted on normal probability paper and no plot indicated the presence of any outliers. The summary statistics of the four dimensions and the estimated non-conformance fraction of the dimensions are given in Table 9.

A NC contact base will be considered non-conforming if any of its dimensions violates the working specification. Thus, the proportion of non-conforming NC contact bases at the current state of the supplier's process was estimated as

$$(1 - 0.99907 \times 0.99883 \times 0.97687 \times 0.99922) = 0.0259 \cong 0.026$$

As a first step towards continuous improvement, the management of the company was willing to accept a non-conformance fraction of 0.026 as the acceptable quality level (AQL), i.e. the poorest level of quality for the supplier's process acceptable as a process average. The lot tolerance percent defective (LTPD), i.e. the poorest level of quality (fraction non-conformance) the company was willing to accept in an individual lot was 0.10.

It was desired that at least 95% of lots having true fraction non-conformance not exceeding 0.026 should be accepted (i.e. producer's risk (α) = 0.05), and not more than 8% of lots having true fraction non-conformance not less than 0.10 should be accepted (i.e. consumer's risk $\beta =$ 0.08). Since the sample size was restricted, attainment of desired protection for both producer and consumer was impossible. A plan could be devised in which α or β is controlled; however, this will unnecessarily penalize either the consumer or producer. With n fixed, therefore, the problem was to choose a best compromise plan. According to Golub (1953), under this situation the best plan is one that maximizes the sum of probabilities of accepting true quality p_1 (AQL) and rejecting lots of true quality p_2 (LTPD). Mathematically, the expression, which is to be maximized, may be written as

$$P = \Pr_{p_1}(A) + \Pr_{p_2}(R) \tag{4}$$

where, $\Pr_{p_1}(A)$ represents the probability of accepting lots of true quality $= p_1$ and $\Pr_{p_2}(R)$ represents the probability of rejecting lots of true quality $= p_2$.

Golub derived that for fixed n, the values of c (acceptance number) which maximize (4) is the integer nearest to

$$c = -\frac{1}{2} + \frac{n}{\log(p_2/p_1)/\log(q_1/q_2) + 1} \tag{5}$$

Table 9. Summary statistics of dimensions x_i ($i = 1, \ldots, 4$) (after corrective measures taken by the supplier)

Dimension	Mean ($\bar{x}_i$)	SD ($\hat{\sigma}_{x_i}$)	% Nonconformance
x_1	7.976	0.0412	0.093%
x_2	8.029	0.0386	0.117%
x_3	1.528	0.0362	2.310%
x_4	1.619	0.0367	0.078%

where $q_1 = 1 - p_1$ and $q_2 = 1 - p_2$
If the value of equation (5) falls exactly midway between two integers then equal maxima exist at those integers.

Putting $n = 60$, $p_1 = 0.026$ and $p_2 = 0.10$ in equation (5) we found $c = 3$. Thus, the single sampling plan that minimizes $\alpha + \beta$ is $n = 60$; $c = 3$. In this sampling plan $\alpha = 0.071$; $\beta = 0.137$ and $\alpha + \beta = 0.208$. The OC curve of the sampling plan (see Appendix 3) indicates that the discriminatory power between good and bad lots of the plan is quite satisfactory.

It may be noted that, in the case of multi-characteristic inspection, the sequence of ordering the characteristics for inspection is very important with respect to the total expected cost of inspection. Daffuaa & Raouf (1990) have shown that given a component with N characteristics the sequence of inspection that minimizes the total expected cost of inspection is $[1, 2, \ldots, N]$ if the order of the characteristics is an increasing order of the ratio of the cost of inspection to the probability of rejection, i.e.

$$C_1/R_1 \leq C_2/R_2 \leq \cdots \leq C_N/R_N,$$

where C_i & $R_i (0 < R_i < 1)(i = 1, 2, \ldots, N)$ are the cost of inspection and probability of non-conformance of the ith characteristics respectively.

If $C_i = C$ for all i, the rule becomes

$$R_1 \geq R_2 \geq \cdots \geq R_N$$

This means that the characteristic with the highest rate of non-conformance is to be inspected first, followed by that with the next highest rate of non-conformance, until all characteristics are inspected.

The cost of inspection for all four dimensions of the NC contact base is the same and the probabilities of non-conformance of the dimensions x_1, x_2, x_3, x_4 were estimated to be 0.00093, 0.00117, 0.02313 and 0.00078 respectively. This implied that the optimal sequence of ordering the four dimensions for inspection would be x_3, x_2 x_1, x_4.

Recommendation

The following were recommended.

(1) For the purposes of procurement and evaluation of suitability of NC contact bases use the working specifications given in Table 8.

(2) For the purpose of sentencing incoming lots of NC contact bases use the single sampling plan $n = 60$; $c = 3$. While measuring the four dimensions of an NC contact base follow the following order of sequence: x_3, x_2, x_1, x_4.

(3) Continue to insist the supplier reduce the variability in the dimensions of NC contact bases, particularly in the dimension x_3. Also explore the possibility of reducing the variability in the dimensions of the plastic case so that even in the event of failure of the supplier to reduce further the variability in the dimensions of NC contact base, the percentage of proper fit of the NC contact base can be increased.

Implementation and Results

Comparison of the results of snap studies carried out before and after usage of the proposed specifications and the sampling plan revealed that incidents of 'tolerance stack-up' between the two components reduced from 19.2% to 3.9%. There was no scope to estimate the reduction in the percentage misfit of NC contact bases. However, it is expected that the percentage misfit of NC contact bases has also reduced significantly, which will lead to a reduction of customer complaints. It was reported that productivity in the assembly shop had improved considerably.

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Appendix 1

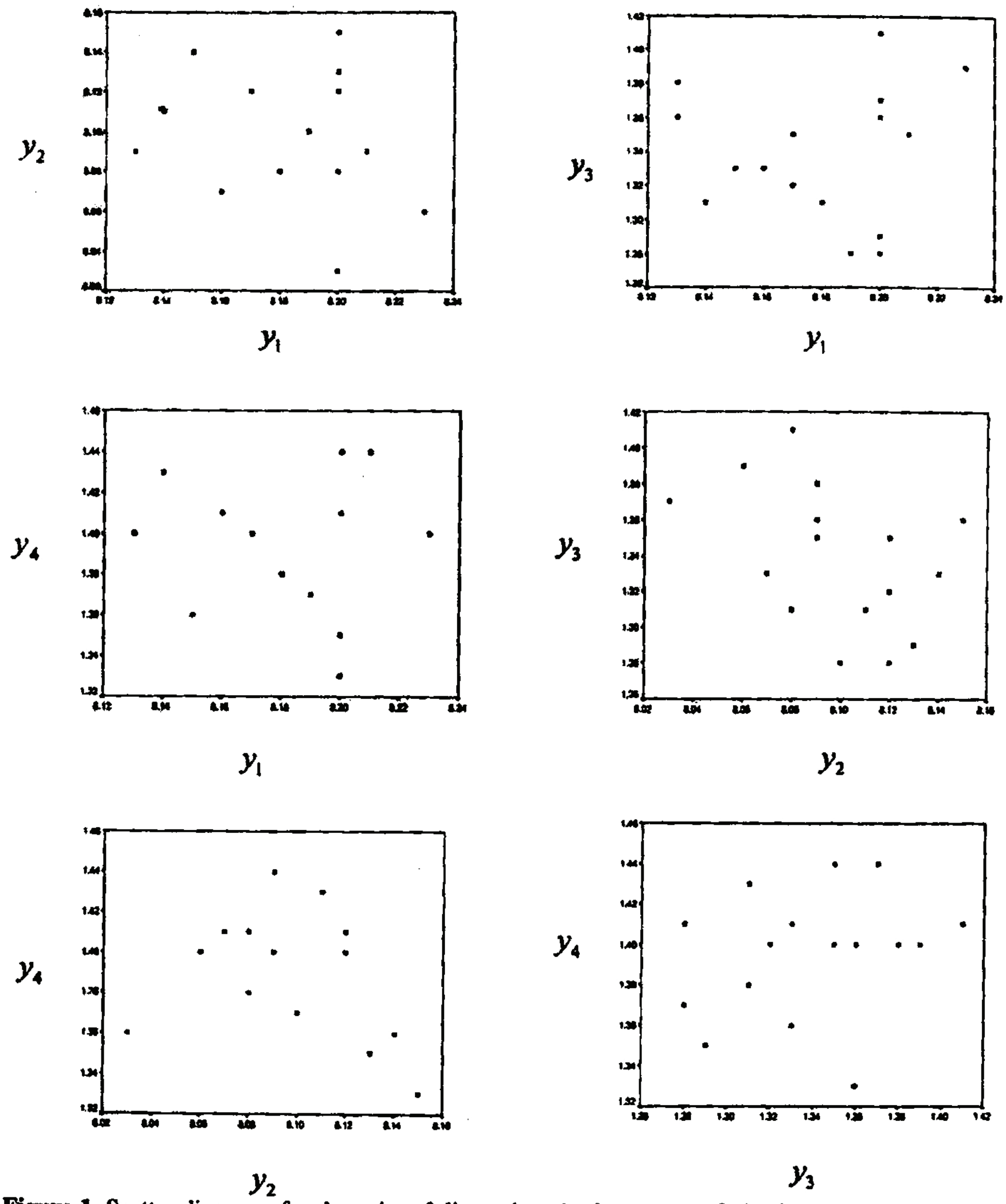
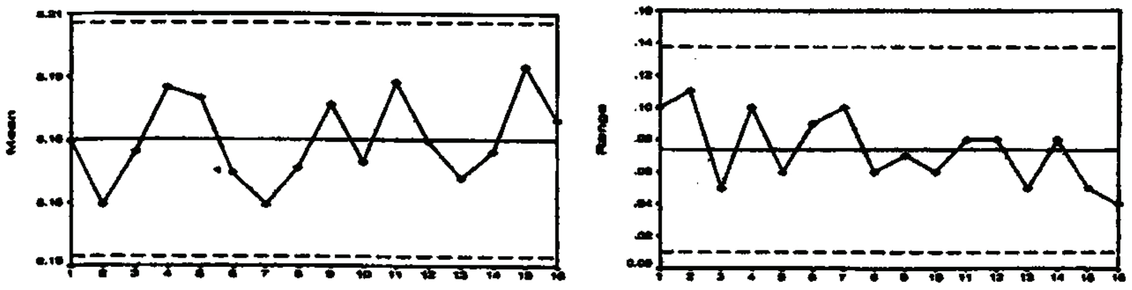
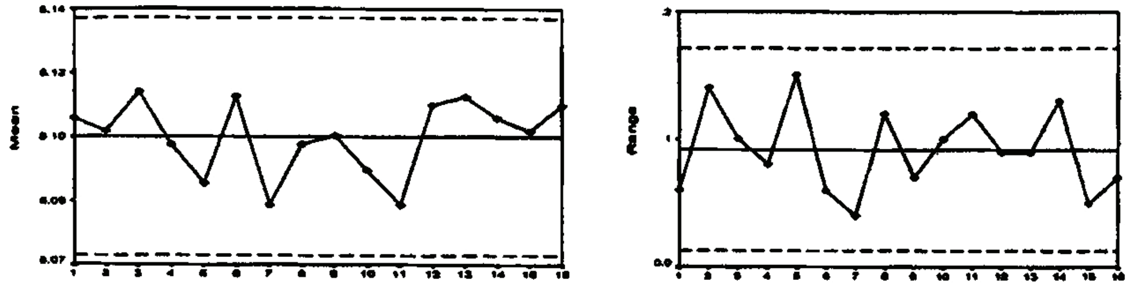


Figure 1. Scatter diagrams for the pairs of dimensions in the groove of plastic case

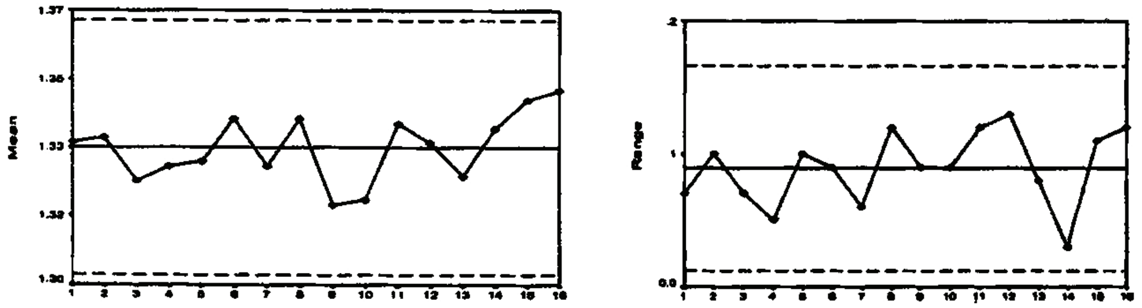
Appendix 2



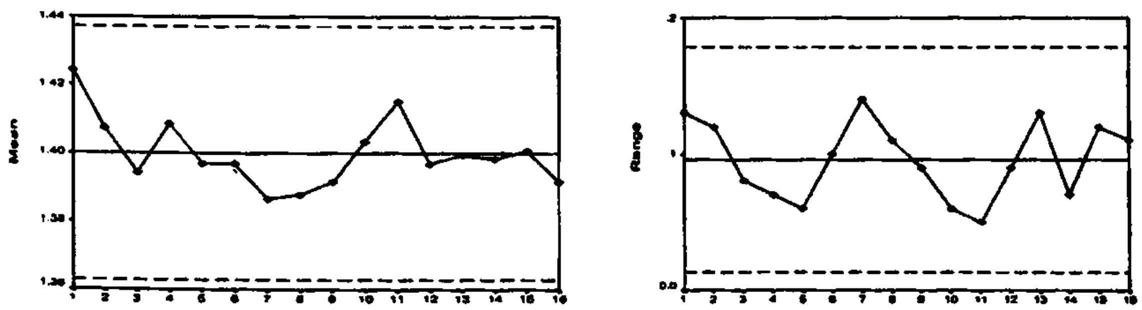
(a) $\overline{X}-R$ charts for y_1



(b) $\overline{X}-R$ charts for y_2



(c) $\overline{X}-R$ charts for y_3



(d) $\overline{X}-R$ charts for y_4

Figure 2. $\overline{X}-R$ charts for $y_1, \ldots, y_4$

Appendix 3

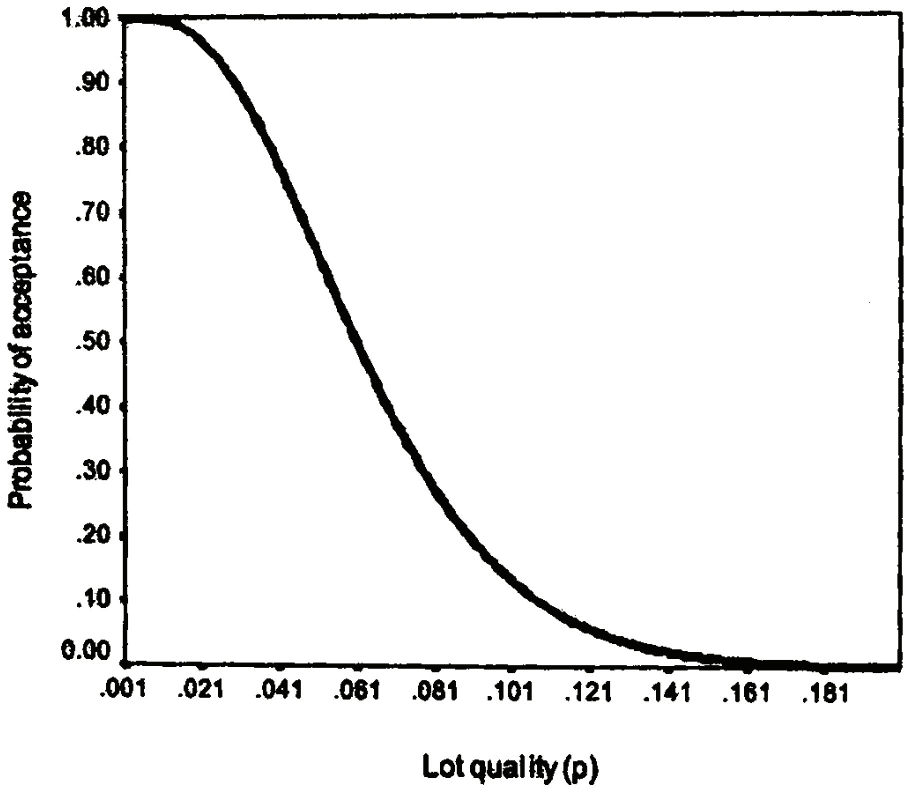


Figure 3. OC curve of the Sampling Plan