ENVIRONMETRICS
Environmetrics 2005; **16**: 149–165

Size distributions of suspended particles in open channel flow over bed materials

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SUMMARY

Controlled experiments have shown that the grain-size distribution of suspended sediments is related to bed material, flow velocity and height of suspension above the sand bed in an open channel flow. A theoretical model has been developed for computation of suspended grain-size distribution on the basis of continuity equations of sediment and water, using the computed bed-layer concentration as a reference. The proposed model includes the effect of suspension concentration into the mean velocity, turbulent and viscous shear stresses owing to the dynamic coupling between the flow and sediments in suspension. The effect of hindered settling due to the increased concentration in suspension is also taken into account. The model is considered to be a more general one than the existing models, and the results of the present model compare well with the experimental data. Copyright

1. INTRODUCTION

Transport of non-cohesive sediments such as sand, silt or gravel under hydraulic conditions has received much attention in terms of basic problems related to the process of grain-sorting. During transportation, movement of non-cohesive sediment is generally classified into two categories: bed load and suspended load. In erodible channels, these are the two processes of sediment transportation, which have the greatest influence on particle size distributions. Study of the grain-size distributions in suspension and deposition is important for the solution of practical problems in the field of sedimentation (both geologically ancient and modern), siltation of a reservoir, genesis of bed forms, etc. The problems of sediment-laden flows in open channels are of direct concern to river engineers, geomorphologists; and are also relevant to fields such as coastal sediment transport, environmental fluid mechanics, and two-phase flow.

The patterns of grain-size distribution have been studied by many investigators in different sedimentary environments. Bagnold and Barndorff-Nielsen (1980), Ghosh and Mazumder (1981), Wyrwoll and Smyth (1985), Fieller and Flenley (1992), Kothyari (1995), Purkait and Mazumder (2000), Bhattacharya *et al.* (2000) and others provide different statistical approaches to grain-size distributions in sediment suspension and depositional environments. Controlled experimental studies in laboratory flumes have shown that the grain-size distribution in suspension under unidirectional flow is related to flow velocity, height of suspension and the nature of the bed materials (Sengupta, 1979; Ghosh *et al.*, 1981; Sengupta *et al.*, 1991, 1999). These studies have shown that a sorting process is initiated immediately above the bed and that the grain sizes of the bed layer influence the size distribution of the suspended load above. Ghosh and Mazumder (1981) studied a mathematical model to explain the generation of uni-modality, symmetry and log-normality of grain size distribution in suspension even when the size distribution at the bed is not log-normal, and suggested that the suspended load will have a tendency to be log-normal for that size range of bed material for which the logarithmic transformation of the settling velocity is linearly distributed.

Thacker and Lavelle (1977), Woo *et al.* (1988), and Ni and Wang (1991) studied the effect of hindered settling on the diffusion equation to determine the vertical distribution of sand particles with high concentration. Mazumder (1994) developed a method for computing the grain-size distribution of suspended sediment from the texture of bed materials with the help of a diffusion approach. The method utilizes the idea of reduction of fall velocity of sand particles with an increase in sediment concentration in suspension.

However, the above-mentioned methods are not concerned with the effect of sediment concentration on the mean velocity distribution in the sediment-laden turbulent flow. The presence of suspended sediment causes the momentum exchange between the water and sediment particles. The local momentum exchange affects the turbulent eddy structure, and thus influences the turbulent fluctuations. Itakura and Kishi (1980), Coleman (1981), Umeyama (1992), Umeyama (1999) and Mazumder and Ghoshal (2002) proposed models to compute the vertical concentration distribution in which an additional component due to the effect of sediment suspension on the mean velocity profiles was included. Due to the presence of suspended concentration in the turbulent flow, a perturbation approach to the clear water flow was considered by Mendoza and Zhou (1995) to describe the physical process of the interaction between flow and the suspended sediment. They developed a general form of the velocity and the corresponding suspended sediment concentration profiles using the usual Rouse diffusion equation.

In order to achieve a more general mathematical model for the computation of grain-size distribution in suspension from bed materials, it is desirable to include the effect of sediment suspension concentration on the mean flow, through dynamic coupling between the turbulent flow and the sediment in suspension. The purpose of the present article is to generalize the model developed by Mazumder (1994) for the computation of the suspension distribution from the bed materials, taking into consideration the dynamic coupling effect between the flow and the suspension concentration and corresponding eddy diffusivity. Grain-size frequency distribution of suspended loads at different heights above the four sand beds of different size distributions have been computed in this article using the computed bed load equation as the reference concentration. The validity of the present model has been tested with the experimental data of suspended load samples collected in the laboratory flumes at Indian Statistical Institute (ISI), Calcutta, and Uppsala University under controlled conditions. Comparisons of the results of the present model with observed data show reasonably good agreement.

2. THEORETICAL MODELS

In a steady and uniform turbulent flow when the concentration C varies only with the vertical co-ordinate y throughout the depth, and the diffusion coefficients of sediments and water are assumed to be equal ($\in_s = \in_m$), the vertical distribution of suspended sediment concentration with particle settling velocity W in a fluid–sediment mixture can be described (Hunt, 1954) as

$$\in_s \frac{dC}{dy} + (1 - C)CW = 0 \tag{1}$$

which satisfies the continuity condition of sediment and water. Equation (1) is derived from the diffusion equations of sediment and water. The equality in diffusion coefficients in sediment and water is not strictly accurate but is a close approximation for small particles. Moreover, the concentration C being a volumetric proportion is small throughout the depth (outer flow region) except near the bed. Experiments on fluid–sediment mixtures have shown that a substantial reduction of particle fall velocity occurs due to the increased suspension concentration. If the density of the fluid mixture is increased by sediments, the buoyancy force is increased, decreasing the settling velocity. Another dominant effect reducing the settling velocity is the increase in viscosity and specific weight of the suspension. All these effects need to be incorporated in the calculation. According to Richardson and Zaki (1954), the effective settling velocity of sediment W varies with concentration C as a result of hindered settling and is given by

$$W = w_0(1 - C)^{\alpha} \tag{2}$$

where w_0 is the single fall velocity in clear water and α is the exponent of reduction of fall velocity, which varies from 2 to 4 depending on the particle Reynolds number and the size of non-cohesive sediment particles. This equation has shown the importance of the relation in modelling the reduction of particle fall velocity in sediment-laden flows (Woo *et al.*, 1988; Ni and Wang, 1991; Mazumder, 1994; Mazumder and Ghoshal, 2002).

When the turbulent flow carries sediments in suspension, the density ρ of the sediment–water mixture can be written as

$$\rho = \rho_f + (\rho_s - \rho_f)C \tag{3}$$

where ρ_f is the density of clear water, ρ_s is the density of sediments and C its instantaneous volume concentration. The total shear stress τ in the sediment-laden turbulent flow is balanced by the sum of the viscous shear stress τ_v and the turbulent shear stress τ_t, and is given by

$$\tau = \tau_v + \tau_t \tag{4}$$

In (4) the viscous shear stress τ_v takes into account the increase of viscosity due to suspended particulate near the bed.

In fully developed sediment-laden turbulent flow, the effect of the dynamic coupling between sediment suspension and the turbulent flow may be important. Consequently, the momentum diffusion coefficient $\in_s$ for sediment in sediment-laden flow is given by Einstein and Chien (1955) as

$$\in_s = \frac{\tau_t}{\rho_f(1 + AC)\frac{du_p}{dy}} \tag{5}$$

where u_p represents the mean velocity in the x-direction for the perturbed flow field due to the presence of sediments in suspension and $A = (\rho_s - \rho_f)/\rho_f$ is the relative density of sediment. According to

Mendoza and Zhou (1995), the perturbed mean velocity u_p is considered as the sum of the unperturbed mean velocity $\bar{u}$ of clear water and the perturbation of mean velocity $\bar{u}_{ps}$ due to the effect of suspension, i.e. $u_p = \bar{u} + \bar{u}_{ps}$. In this flow situation, the viscous shear stress due to higher concentration is defined as

$$\tau_v = \mu_c \frac{du_p}{dy} \tag{6}$$

where μ_c, the dynamic viscosity is a function of concentration and is given by Thomas (1965) as

$$\mu_c = \mu\left(1 + 2.5C + 10.05C^2 + 0.00273 \exp\left(16.6C\right)\right) = \mu g(C) \tag{7}$$

where μ is the coefficient of viscosity of clear water. The total shear stress component τ is given by

$$\tau = \int_y^d \rho g J \mathrm{d}y' \tag{8}$$

where d is the water depth, g is the acceleration due to gravity, and J is the energy slope. Equation (8) leads to the bed shear stress τ_0 as

$$\tau_0 = \rho_f u_*^2 (1 + A\bar{C}), \;\; \bar{C} = \int_0^1 C \mathrm{d}\xi \tag{9}$$

where u_* is the friction velocity. Accordingly, the total vertical shear stress distribution τ/τ_0 can be written as a function of dimensionless depth $\xi = y/d$, relative density A and the sediment concentration C:

$$\frac{\tau}{\tau_0} = \frac{1 - \xi + A\int_\xi^1 C \mathrm{d}\xi}{1 + A\bar{C}} \tag{10}$$

Using (4), (6), (7) and (10) in (5), the momentum diffusion coefficient $\in_s$ for sediment in the mixture is given by

$$\in_s = \frac{u_*^2\left(1 - \xi + A\int_\xi^1 C \mathrm{d}\xi\right)}{(1 + AC)\frac{du_p}{dy}} - \frac{\mu g(C)}{\rho_f(1 + AC)} \tag{11}$$

with $u_p = \bar{u} + \bar{u}_{ps}$. Equation (11) indicates that the momentum diffusion coefficient $\in_s$ is a function of concentration, effective dynamic viscosity and the perturbed mean velocity gradient due to the presence of suspension concentration.

In order to determine the mean perturbed velocity u_p, the mean velocity (unperturbed) distribution for clear-water flow in the form of log-wake-law is considered as

$$\frac{\bar{u}}{u_*} = \frac{1}{\kappa}\ln\xi + \frac{1}{\kappa}\ln(d/z_0) + \frac{\Pi}{\kappa}2\sin^2(\pi/2\xi) \tag{12}$$

where κ is the von-Karman constant, z_0 is the equivalent roughness height, and Π is the Coles wake strength parameter (Coles, 1956), which determines the deviations of velocity measurements from the usual log-law. For $\Pi = 0$, (12) reduces to the log-profile. The perturbation of mean velocity $\bar{u}_{ps}$ due to suspension concentration is approximated from the effect of dynamic coupling between the mechanism of turbulent flow velocity and suspended sediment concentration profiles in sediment-laden flow. According to Mendoza and Zhou (1995), the perturbation of the mean velocity $\bar{u}_{ps}$ is approximated as

$$\frac{\bar{u}_{ps}}{u_*} = -\frac{A\delta}{2\kappa}\left[D_1(1-\xi) + D_2(1-\xi)^2 + \cdots\right] + \left(\frac{1}{\kappa} + N_1\right)\ln\xi + N_2 \tag{13}$$

where D_1 and D_2 are dimensionless constants representing derivatives of concentration of different order at the upper surface; N_1, N_2 are dimensionless integrating constants; and δ is the reference volume concentration near the bottom of the channel.

Expanding the trigonometric term in (12) in terms of $(1-\xi)$ and combining with (13), Mendoza and Zhou (1995) suggested a unified expression for the perturbed mean velocity u_p as

$$\frac{u_p}{u_*} = \frac{1}{\kappa}\ln\xi + F_1(1-\xi) + F_2(1-\xi)^2 + \frac{1}{\kappa}\ln(d/z_0) \tag{14}$$

where F_1 and F_2 are dimensionless constants to be estimated from the velocity data. Explicit expressions of F_1 and F_2 are

$$F_1 = -\frac{\delta}{2\kappa}AD_1 - N_1, \; F_2 = -\frac{\Pi}{2\kappa}\pi^2 - \frac{\delta}{2\kappa}AD_2 - \frac{N_1}{2} \tag{15}$$

Furthermore, from (15) it appears that the contribution from suspension particulates to the perturbed mean velocity arises through the constants F_1 and F_2, whereas the contribution of Coles wake strength to the velocity profile is through F_2. If $F_1 = F_2 = 0$, (14) reduces to the usual logarithmic velocity profile. The mean velocity equation (14) is verified by Mendoza and Zhou using the widely used and most classical experimental data of Vanoni (1946), Einstein and Chien (1955) and Coleman (1981). For our present mathematical model, we have used the Mendoza and Zhou velocity profile (14).

Substituting (14) for the velocity gradient in (11), the expression for $\in_s$ is given by

$$\in_s = \frac{du_*(1-\xi+A\int_\xi^1 C\mathrm{d}\xi)}{(1+AC)\left\{\frac{1}{\kappa\xi} - F_1 - 2F_2(1-\xi)\right\}} - \frac{\mu g(C)}{\rho_f(1+AC)} \tag{16}$$

The vertical concentration distribution of sediments in suspension is obtained after substitution of $\in_s$ from (16) and particle settling velocity W from (2) in the diffusion equation (1). The modified expression for concentration gradient is given by

$$\frac{\mathrm{d}C}{\mathrm{d}\xi} = \frac{dw_0 f(C)\left\{\frac{1}{\kappa\xi} - F_1 - 2F_2(1-\xi)\right\}}{\nu_f g(C)\left\{\frac{1}{\kappa\xi} - F_1 - 2F_2(1-\xi)\right\} - du_*\left(1-\xi+A\int_\xi^1 C\mathrm{d}\xi\right)} \tag{17}$$

where $\nu_{\rm f}$ is the kinematic viscosity, $f(C) = C(1-C)^{\alpha+1}(1+AC)$ and $g(C)$ is a non-linear expression given by (7). The integro-differential equation (17) is more general than that obtained by Woo *et al.* (1988), Mendoza and Zhou (1995) and others.

In particular, it $F_1 = F_2 = 0$, (17) directly reduces to that of Woo *et al.* (1988), who used the logarithmic velocity profiles for clear water flow. Equation (17) also reduces to that of Mendoza and Zhou (1995) if we consider the following assumptions:

1. The viscous shear stress $\tau_{\rm v}$ is negligible as compared with the turbulent shear stress $\tau_{\rm t}$, i.e. the viscous effect is negligible near the bed.
2. The effect of hindered setting α is neglected.
3. Only the diffusion equation for sediment is considered (Rouse type equation).

So under the given assumptions, the equation of Mendoza and Zhou with a little change in notation is given by

$$\frac{\mathrm{d}C}{\mathrm{d}\xi} = -\frac{w_0 C(1+AC)\left\{\frac{1}{\kappa\xi} - F_1 - 2F_2(1-\xi)\right\}}{u_*\left\{1 - \xi + A\int_\xi^1 C\mathrm{d}\xi\right\}} \tag{18}$$

If $F_2 = 0$ and $\tau_{\rm t}$ is independent of concentration, (18) reduces to the equation of Itakura and Kishi (1980) as

$$\frac{\mathrm{d}C}{\mathrm{d}\xi} = \frac{w_0 C(1-\kappa\xi F_1)}{\kappa u_*\xi(1-\xi)} \tag{19}$$

If we put $F_1 = F_2 = 0$, (18) reduces to

$$\frac{u_*\kappa\xi}{1+AC}\left[1 - \xi + \int_\xi^1 C\mathrm{d}\xi\right]\frac{\mathrm{d}C}{\mathrm{d}\xi} + w_0 C = 0 \tag{20}$$

in which the velocity gradient is derived only from log-law of clear water flow. Again, if $\tau_{\rm t}$ is used for clear water flow, and the term $C^2 \approx 0$ for low concentration, (20) reduces to the famous Rouse equation,

$$\kappa u_*\xi(1-\xi)\frac{\mathrm{d}C}{\mathrm{d}\xi} + w_0 C = 0 \tag{21}$$

2.1. *Numerical solution of the problem*

As the direct analytical solution of (17) including non-linear expressions of $f(C)$ and $g(C)$ is difficult to handle, a numerical approach based on Runge–Kutta method is adopted to solve the equation. Equation (17) can be rewritten as

$$\frac{\mathrm{d}C}{\mathrm{d}\xi} = \frac{\mathrm{d}w_0 f(C)\frac{\mathrm{d}u_{\rm p}}{\mathrm{d}\xi}}{\nu_{\rm f} g(C)\frac{\mathrm{d}u_{\rm p}}{\mathrm{d}\xi} - \mathrm{d}u_*^2\left(1 - \xi + A\int_\xi^1 C\mathrm{d}\xi\right)} \tag{22}$$

Differentiating (22) with respect to ξ to remove the integration sign from the term $\int_\xi^1 C\mathrm{d}\xi$, one gets the following second-order non-linear differential equation as

$$\frac{\mathrm{d}^2C}{\mathrm{d}\xi^2} = H(\xi)\frac{\mathrm{d}C}{\mathrm{d}\xi} - \left[\frac{\nu_\mathrm{f}}{\mathrm{d}w_0}\frac{g(C)}{f(C)}H(\xi) + \frac{u_*^2(1+AC)}{w_0 f(C)G(\xi)} - \frac{f'(C)}{f(C)}\right]\left(\frac{\mathrm{d}C}{\mathrm{d}\xi}\right)^2 - \frac{\nu_\mathrm{f}}{\mathrm{d}w_0}\frac{g'(C)}{f(C)}\left(\frac{\mathrm{d}C}{\mathrm{d}\xi}\right)^3 \quad (23)$$

where

$$G(\xi) = \frac{\mathrm{d}u_\mathrm{p}}{\mathrm{d}\xi}, \quad H(\xi) = \frac{\mathrm{d}^2u_\mathrm{p}}{\mathrm{d}\xi^2}\Big/\frac{\mathrm{d}u_\mathrm{p}}{\mathrm{d}\xi}$$

The functions $f'(C)$ and $g'(C)$ represents derivatives of the function $f(C)$ and $g(C)$ with respect to C.

To solve (23), we use the boundary conditions at the reference level $\xi = \xi_\mathrm{a}$ near the bed as

$$C = C_{\xi_\mathrm{a}} \quad \text{at} \quad \xi = \xi_\mathrm{a}$$
$$\left.\frac{\mathrm{d}C}{\mathrm{d}\xi}\right|_{\xi=\xi_\mathrm{a}} = \frac{\mathrm{d}w_0 f(C_{\xi_\mathrm{a}})G(\xi_\mathrm{a})}{\nu_\mathrm{f}g(C_{\xi_\mathrm{a}})G(\xi_\mathrm{a}) - \mathrm{d}u_*^2\left(1-\xi_\mathrm{a}+A\int_{\xi_\mathrm{a}}^1 C\mathrm{d}\xi\right)} \quad (24)$$

where C_{ξ_a} is the reference concentration at the reference level $\xi = \xi_\mathrm{a}$. Using the velocity profile (14), the analytical solution of Rouse's diffusion equation has been set up in a modified form as

$$C = C_{\xi_\mathrm{a}}\exp\left[\frac{w_0}{u_*}\left\{\frac{1}{\kappa}\ln\left(\frac{1-\xi}{\xi}\frac{\xi_\mathrm{a}}{1-\xi_\mathrm{a}}\right) + F_1\ln\left(\frac{1-\xi_\mathrm{a}}{1-\xi}\right) - 2F_2(\xi_\mathrm{a}-\xi)\right\}\right] \quad (25)$$

When $F_1 = F_2 = 0$, we have the well-known Rouse formula as

$$\frac{C}{C_{\xi_\mathrm{a}}} = \left(\frac{1-\xi}{\xi}\frac{\xi_\mathrm{a}}{1-\xi_\mathrm{a}}\right)^{\frac{w_0}{\kappa u_*}} \quad (26)$$

which is the solution of the Rouse sediment balance equation (21).

Now it is necessary to estimate the reference level $\xi = \xi_\mathrm{a}$ at the lowest elevation near the sand bed of known grain-size distribution laid down on the flume base. Ghosh *et al.* (1981), Dyer and Soulsby (1988) and Mazumder (1994) assumed the reference level $\xi_\mathrm{a} = \xi_0$, with the bed roughness height as the lower bound. Their choice of reference level at the bed roughness ξ_0 was used to compute the suspension concentration at any height above the bed through the bed-load layer, where the grain–grain and grain–bed collisions were important. The roughness parameter ξ_0 is much smaller than the bed layer thickness. So, it would be more appropriate to choose the reference level elevation ($\xi = \xi_\mathrm{a}$) at the top of the bed load layer. The bed load thickness ξ_a is estimated according to Wiberg and Rubin (1989) as

$$\xi_\mathrm{a} = \frac{0.68T_*D_{65}}{(1+A_1T_*)D} \quad (27)$$

where $T_* = \tau_0/\tau_\mathrm{c}$ is the transport stage, in which τ_0 is the bed shear stress and τ_c is the critical shear stress for sediment size D_{65} in the sand bed mixture, and $A_1 = 0.0204(\ln D_{65})^2 + 0.022\ln D_{65} + 0.0709$

(D_{65} in cm). If a sediment size D_{65} and transport stage T_* are specified, it is possible to calculate ξ_a from (27). Furthermore, τ_c is estimated from the quasi-analytical method proposed by Wiberg and Smith (1985).

Following the approach suggested by Smith and McLean (1977), the bed layer concentration at the reference level ξ_a is assumed as

$$C_{\xi=\xi_a} = \frac{\gamma_0 C_b S}{1 + \gamma_0 S} \tag{28}$$

where $\gamma_0 = 0.004$, $S = T_* - 1$ is the normalized excess shear stress and C_b is the relative bed concentration as a function of size ϕ, where $\phi = -\log_2 D(\text{in mm})$.

To compute the vertical concentration distribution at any height ξ, the fourth order Runge–Kutta method has been adopted to solve the non-linear second order differential equation (23) subject to the boundary conditions (24), using the computed reference concentration C_{ξ_a} from (28) at the reference level ξ_a from (27) for different values of α. Once the suspension concentrations of different sizes ϕ at any height are known from (23), (24), (27) and (28), the relative suspension concentration $C'_\xi(\phi)$ may be obtained as

$$C'_\xi = C_\xi \Big/ \sum_\phi C_\xi(\phi) \tag{29}$$

Equation (29) is used to calculate the relative suspension concentration $C'_\xi(\phi)$ of a given grain size with a settling velocity w_0 (in clear water) at any height above the bed for a given relative bed concentration $C_b(\phi)$. Using (27) and (28), it is possible to determine the relative bed-layer concentration $C'_{\xi_a}(\phi)$ of sediment of different sizes at the reference level $\xi = \xi_a$, which is the bed load height.

3. EXPERIMENTAL STUDIES

The theoretical models have been verified by comparing the computed suspension distributions with those observed in the laboratory flume. The experiments were conducted in a closed circuit laboratory flume (Sengupta *et al.*, 1991) designed at the Indian Statistical Institute (ISI) Calcutta. Both the experimental and recirculating channels of the flume are of the same dimensions (10 m × 50 cm × 50 cm). The experimental walls of the flume are made of transparent plastic windows for a length of 8 m, affording a clear view of the sediment movements. Two non-clogging types of centrifugal pumps providing the flow are located outside the main body of the flume. The intake and outlet pipes are freely suspended from an overhead structure to allow tilting of the flume. Both the outlet pipes (pumps 1, 2) are fitted with bypass pipes and valves, so that by adjusting these valves in the outlet and the bypass pipes, the flow can be set at any desired speed up to 1.30 m/s for a water depth (d) of 35 cm. The upstream bend of the channel is divided equally into three sub-channels and a honeycomb cage is placed at either end of the sub-channels in order to ensure smooth, vortex free uniform flow of water through the experimental channel. A schematic diagram of the flume is given in Figure 1.

A series of experiments had been performed over sand beds of two grain size distributions (bed–10C1 and 10C2). Both the sand beds used for the experiments have the same modal size with the peak

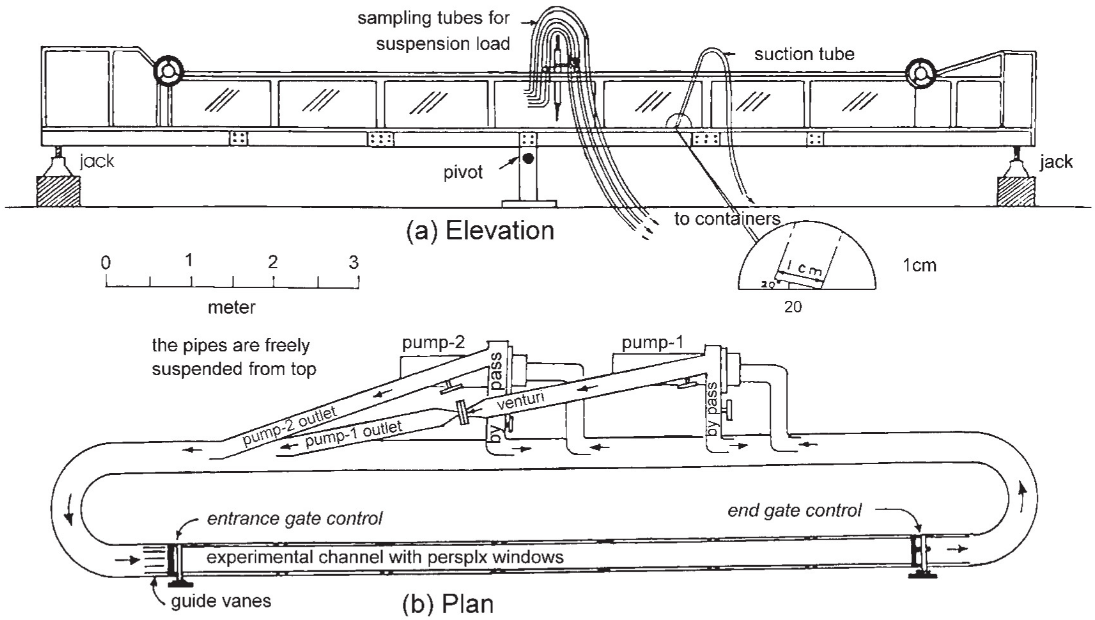

Figure 1. Schematic diagram of the flume

value at 2.0ϕ, but different values of roughness (Figures 2a,b). For each experiment a sand bed of known grain-size distribution was laid down on the flume base to produce a uniform bed, generally 2–3 cm thick. The experiments were always started with smooth and flat sand beds. The water depth was kept at a constant height of 35 cm above the flume base for all the experiments. Bed forms were generated when the sediment transport started. The dimension of bedforms depended not only on the flow velocity but also on the grain-size distribution of the bed materials. After a run time about 30 min, the process of sediment movement in suspension seemed to reach a steady state, with the suspension concentration reaching a saturated point. This was confirmed by the results of *repeated* sampling of the suspended sediments from different heights, which showed little change in proportion of grain sizes, irrespective of the presence or absence of ripples in the bed immediately below the sampling point. Hence, while developing the theoretical model, it was felt that the influence of ripples can be safely ignored when the steady state has been reached in suspension. This ensured a stable value for the measured quantity in view of the variability of the ripple dimensions. Over each of these sand beds, experiments were conducted at different maximum velocities ($u_{\max}$), varying from 0.70–1.20 m/s. At each maximum velocity ($u_{\max}$) when the state of equilibrium was reached, samples of suspended sediments were collected from different heights above the flume base. A rack and pinion arrangement of siphon pipes allowed for suction of suspended sediments with measured volume of sediment-laden water from any desired height at a distance of 7.5 m downstream from the honeycomb cage. During the collection of suspended sediment samples, a record of bed conditions was maintained and the effective bed height (h') was obtained by averaging the recorded multiple number of ripples passing over the period of sampling. After the evaporation of water, the sediments were sieved by an electrically operated sieve shaker using micro-sieves of $\phi/2$ intervals. We weighed the amount of each size class by an electronic digital balance.

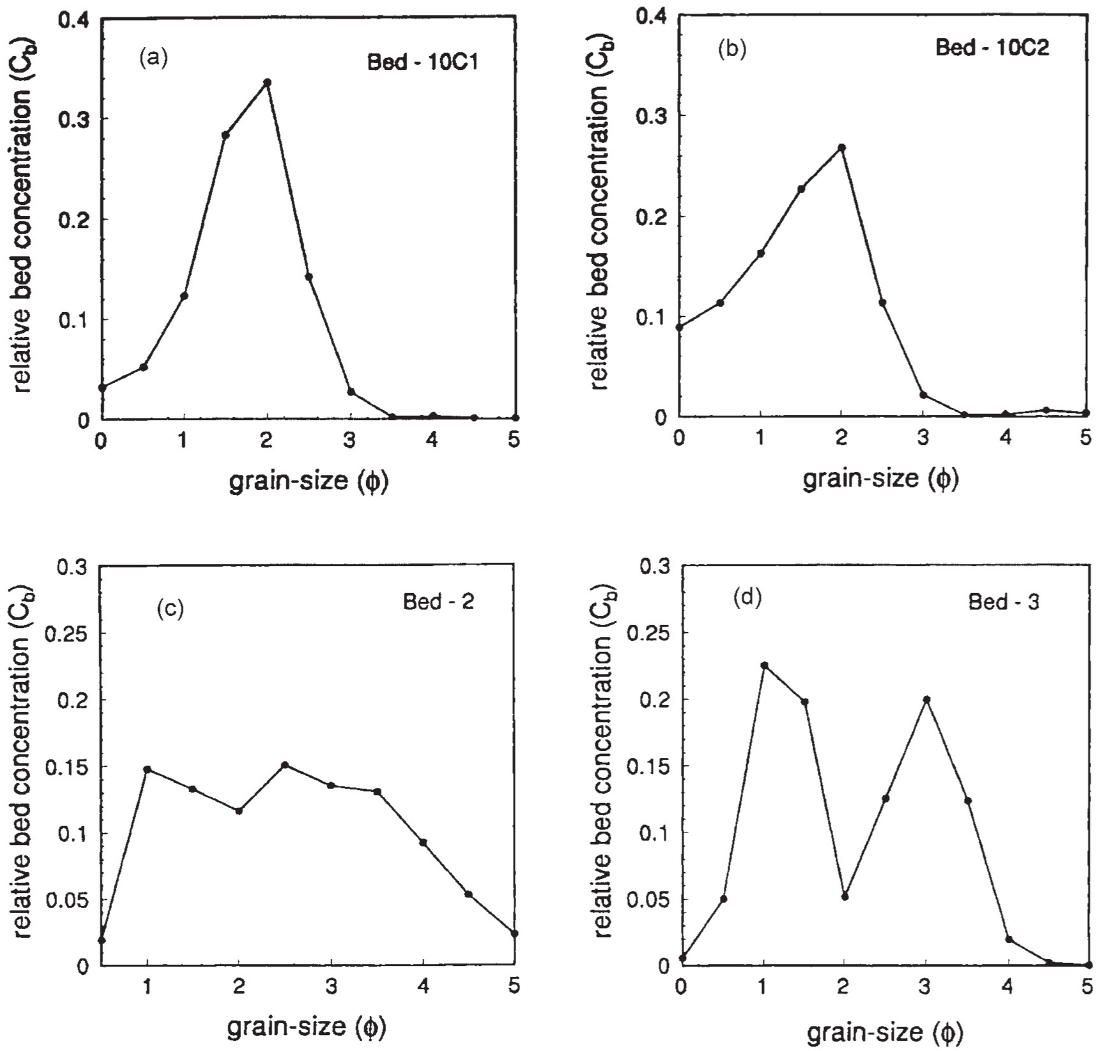

Figure 2. Grain-size distributions of sand beds–10C1, 10C2, 2 and 3

The flow velocities were measured over more than 85% of the flow depth by inserting the propeller (2.5 cm diameter) of an Ott laboratory current meter to the sampling heights (H) immediately after collection of suspended load samples. The mean values of at least five velocity measurements, made over a period of 3 min in the same height, were recorded. The way in which the velocity was measured ensured that the variability of ripple dimensions with respect to the velocity was not effective. The data included measurements of water surface slope, flow depth, water temperature, bed samples, and bed configurations including lengths and heights of bed forms (Table 1).

Suspended sediment data collected from the Uppsala University flume experiments under controlled conditions (Sengupta, 1979; Ghosh *et al.*, 1981) were also used for verification of the present model. Bed materials of six different grain-size distributions were used in their experiments.

Table 1. Flow parameters used for experimental verifications

Bed no.	Run	d (cm)	H (cm)	h' (cm)	k_s (cm)	y (cm)	$u_{\max}$ (cm/s)	T (°C)	J	Vol. (lit)
10C1	1	35	—	2.00	0.0420	—	68.13	29	0.0015	—
10C1	2	35	10	1.72	0.0420	8.28	99.00	30	0.0018	5
	2	35	20	1.72	0.0420	18.28	99.00	30	0.0018	5
	3	35	10	1.47	0.0420	8.53	115.31	30	0.0020	5
10C2	2	35	5	1.58	0.0517	3.42	106.13	29	0.0019	5
	2	35	10	1.58	0.0517	8.42	106.13	29	0.0019	5
2	III 2	30	15	1.7	0.0297	13.3	121.30	19	0.0020	5
	III 2	30	25	1.7	0.0297	23.3	121.30	19	0.0020	5
3	VII 1	30	10	2.5	0.0450	7.5	97.80	19	0.0018	10
	VII 1	30	20	2.5	0.0450	17.5	97.80	19	0.0018	10

d = depth of water, H = sampling height, h' = effective average bed height during experiment, k_s = bed roughness (D_{65}), $y = H - h'$ = prediction height, $u_{\max}$ = maximum velocity, T = temperature of water, J = slope.

The size distributions of two different sand beds, nearly uniform (bed-2) and bimodal (bed-3), are used in this article for computational purposes (Figures 2c,d). A description of the equipment, techniques of velocity measurement and sample collection have been given in Sengupta (1979). Hydraulic parameters arising from the experiments used for verification of the proposed models are given in Table 1.

4. COMPARISON WITH EXPERIMENTAL DATA

Mean velocity (14) for sediment-laden turbulent flow is plotted against $(\xi = y/d)$ for various values of maximum velocities $(u_{\max})$ over more than 85% of the flow depth above the four sand beds in Figures 3(a–f). It is observed that the agreement between the measured and computed velocities is very close throughout the vertical height. The velocity profile provides a best fit with correlation coefficient $r = 0.95$. Values of $F_1,\ F_2$ and the constant term $\frac{1}{\kappa}\ln(d/z_0)$ obtained from the regression analysis are presented in Table 2. The velocity equation (14) is more realistic for sediment laden flow, because it accounts for the interaction between the turbulent flow and the suspended sediments. From the experimental data, it may be noted that the maximum velocities occur below the water surface. This phenomenon may occur due to secondary circulation produced by air–water resistance or some other effect in the flume. The present velocity distribution does not include the effect of air–water resistance, despite the fact that this model includes the effect of sediment suspension in the flow. The method developed by Mendoza and Zhou (1995) for studying the flow perturbation due to sediment suspension is still valid.

Both bed load and suspended load concentrations are computed for four different sand beds (numbered 10C1, 10C2, 2 and 3) using different methods discussed in the previous section. Computations based on (29) using (23), (24), (27), (28) have been performed for different maximum velocities ($u_{\max} = 98.7,\ 106.1,\ 115.3,\ 121.3\,\text{cm/s.}$), different heights and $\alpha = 2$, 3 and 4 for all four sand beds. According to Smith and McLean (1977) and Wiberg and Rubin (1989), size distributions of bed load have been computed for different sand beds. In our calculation, the influence of bedforms on the velocity and suspension concentration has been spatially averaged. Suspended load concentrations computed by different methods (present method, modified Rouse equation, and Mendoza and Zhou) above each of these beds are obtained, using the computed bed layer concentration as a reference.

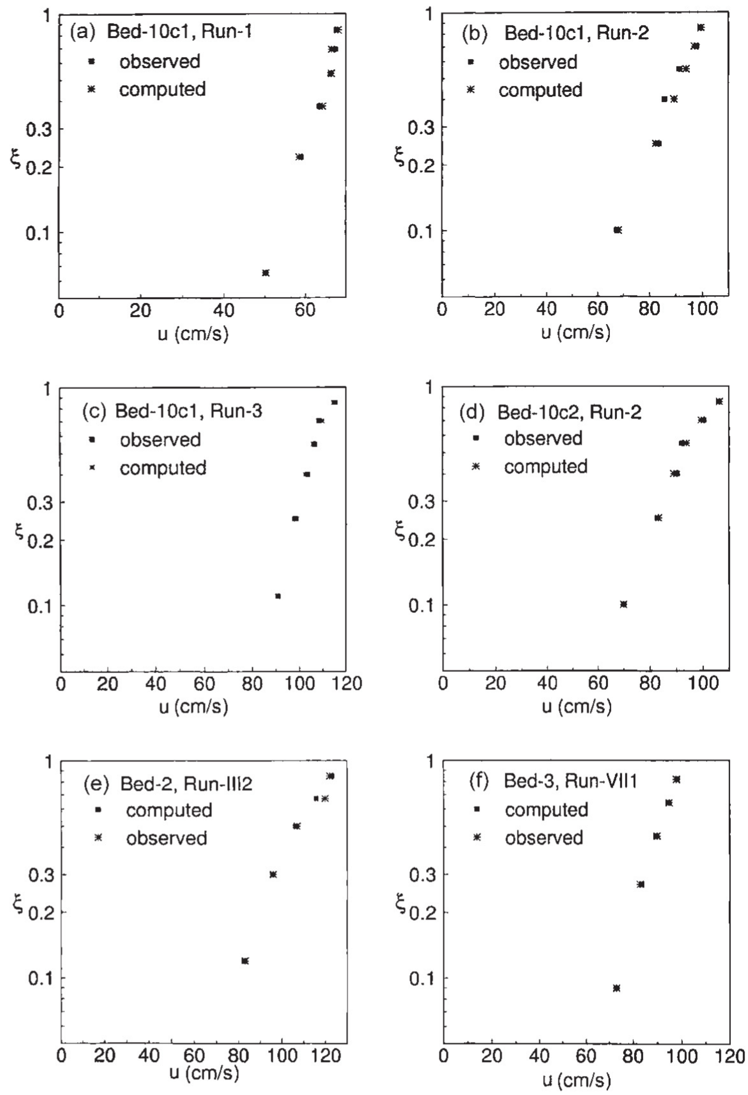

Figure 3. Observed and computed velocity distributions using (14) for different maximum velocities above the sand beds (see Table 1)

Observed and computed values of relative suspension grain-size distribution above four sand beds with different velocities and heights have been plotted in Figures 4(a–g) for $\alpha = 3$. Values of concentration profiles are almost the same for $\alpha = 3$, 4. Hence a graphical comparison is omitted here for $\alpha = 4$. It is clearly observed from the figures that for a given sand bed and fixed u_{max}, suspended load decreases with increase in height and the mean particle size at the upper level is smaller than that of the lower level.

The calculated values of vertical concentration distribution are also compared with observed suspension data of particle size D_{65} above the sand beds (bed nos.–10C1, 10C2, 2, 3) for different u_{max}

Table 2. Values of F_1 and F_2

Bed no.	Run	F_1	F_2	u_* (cm/s)	$\frac{1}{\kappa}\ln(d/z_0)$
10C1	1	6.90	9.00	2.00	33.41
10C1	2	0.72	0.00	6.47	15.66
10C1	3	−5.24	6.44	5.83	20.72
10C2	2	−4.70	6.18	9.10	12.66
2	III 2	−0.26	−5.87	6.00	20.46
3	VII 1	0.23	−9.9	2.50	39.68

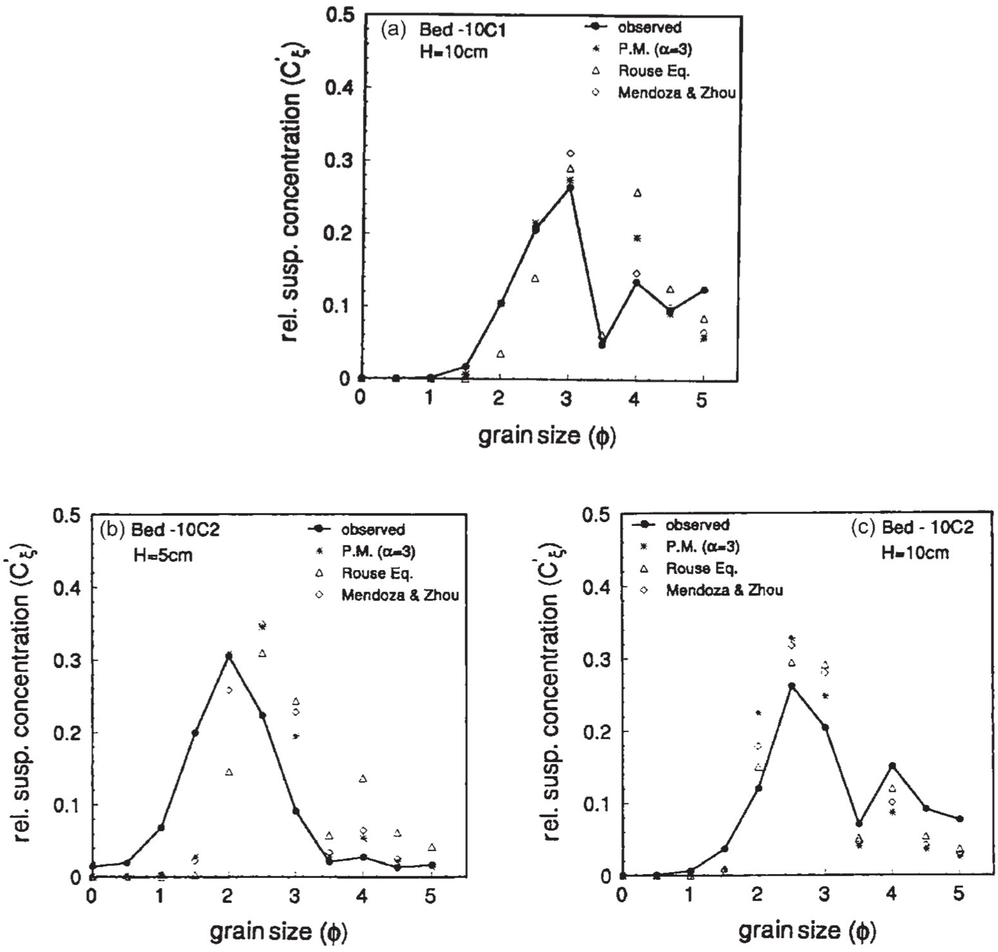

Figure 4. Computed and observed grain-size distributions in suspension at different heights and different velocities. Rel. susp. concentration (C'_ξ): (a) above the bed 10C1, run-3, at $y = 8.50$ cm; (b) above the bed 10C2, run-2 at $y = 3.42$ cm; (c) above the bed 10C2, run-2 at $y = 8.42$ cm; (d) above the bed 2, run-III2 at $y = 13.30$ cm; (e) above the bed 2, run-III2 at $y = 23.30$ cm; (f) above the bed 3, run-VII 1 at $y = 7.5$ cm; (g) above the bed 3, run-VII 1 at $y = 17.50$ cm

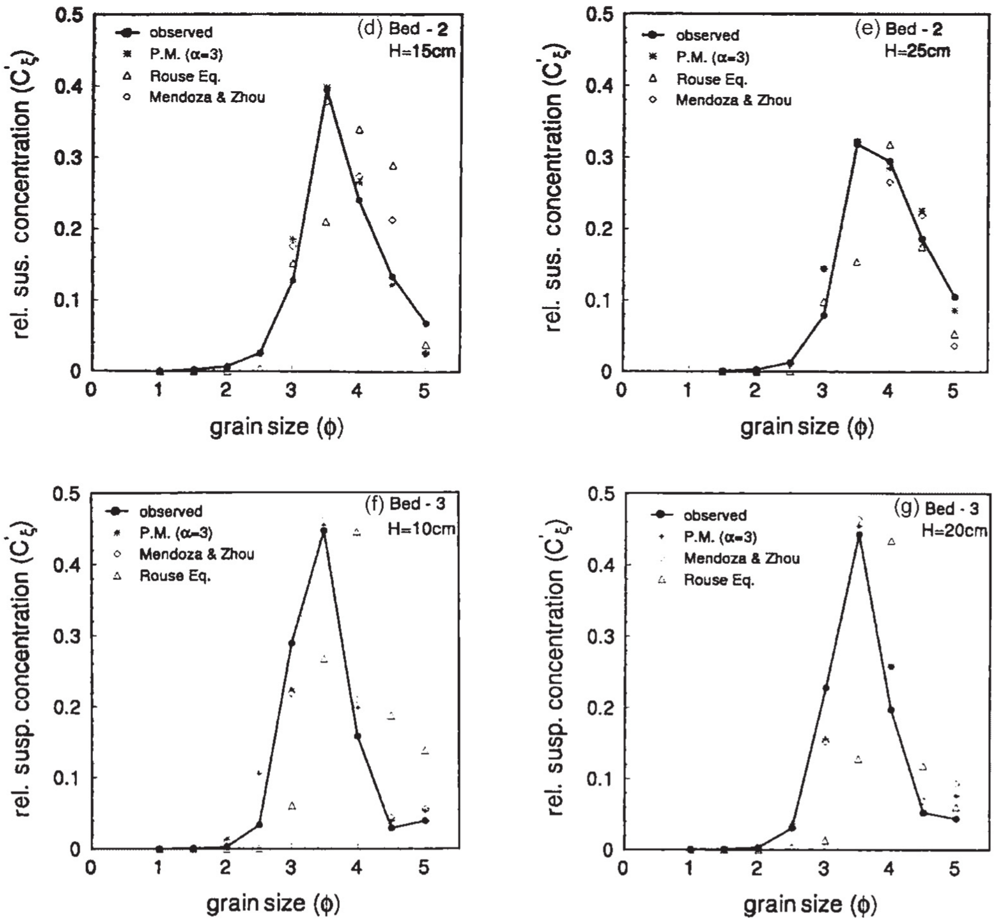

Figure 4. Continued

and $\alpha = 3$ in Figures 5(a–d), using the concentration (C_a) at the reference level $a = 5$ cm above beds–10C1, 10C2 and at the level $a = 10$ cm above beds-2, 3. The trends of the observed and computed suspended load distributions, as seen in the figures, generally agree well. The weighted relative errors between the observed and computed values have been computed by the following formula:

$$E = \sqrt{\left[\sum \frac{(C'_c - C'_0)^2}{C'^2_0} C'_0\right]} = \sqrt{\left[\sum \frac{(C'_c - C'_0)^2}{C'_0}\right]} \tag{30}$$

where C'_c is the computed relative suspension concentration and C'_0 is the observed relative suspension concentration. The relative errors between the observed and computed values are shown in Table 3 for the present method and the methods developed by modified Rouse and by Mendoza and Zhou (1995). It is clear from Table 3 that the smallest error is obtained by the present method for all the beds for

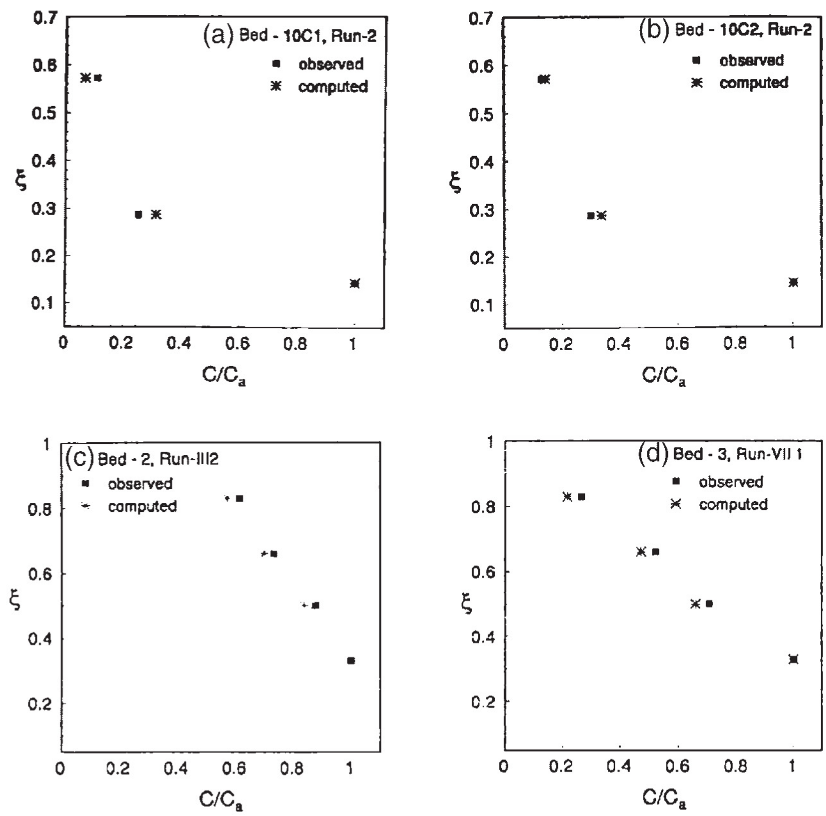

Figure 5. Calculated and measured veritcal sediment concentration of a representative size (d_{65}): (a) above the bed 10C1, run-2 using reference concentraton C_a at $a=5$ cm; (b) above the bed 10C2, run-2 using C_a at $a=5$ cm; (c) above the bed 2, run-III2 using C_a at $a=10$ cm; and above the bed 3, run-VII 1 using C_a at $a=10$ cm

$\alpha=3$ or 4. The present method is more realistic because it accounts for the dynamic coupling between the turbulent flow and sediment in suspension and the effect of hindered settling. Specifically, a correction due to sedimcnt suspension on the mean velocity, and viscous and turbulent shear stresses in addition to the hindered settling effect is included, whereas the other methods did not consider the effect of hindered settling and Hunt's diffusion equation. The traditional Rouse equation does not include the effect of sediment suspension on the turbulent mean flow. For comparison, the Rouse equation (26) is computed using only the mean velocity profile (14) which has the dynamic coupling between the flow and suspended sediments. Mendoza and Zhou developed a general form of velocity profile that included the interaction between the flow and suspended sediments to study the suspended sediment concentration, but they did not consider Hunt's diffusion equation as well as the effect of hindered settling due to increased concentration in suspension.

If the present method is used for computation of suspended loads directly from the bed's grain-size distribution $C_b(\phi)$ as a reference, the error increases. Separate computations show that the error in computation of the suspension concentration at different heights from a computed bed-layer concentration ($C_{\xi a}$) as a reference is much smaller than the error of computation of suspension load directly from the bed. The likely cause of error in computing suspension concentration near the bed is that the diffusion equation is not valid in this zone.

Table 3. Errors between computed and observed suspended loads above the beds

Bed no.	Run	Height (cm)	Present method			Rouse eqn.	Mendoza and Zhou (1995)
			$\alpha=2$	$\alpha=3$	$\alpha=4$		
10C1	3	10	0.39	0.34	0.32	0.49	0.38
10C2	2	5	0.65	0.60	0.50	1.14	0.80
10C2	2	10	0.19	0.18	0.18	0.37	0.43
2	III 2	15	0.24	0.23	0.25	0.59	0.31
2	III 2	25	0.27	0.26	0.26	0.36	0.32
3	VII 1	10	0.48	0.47	0.48	1.38	0.52
3	VII 1	20	0.36	0.27	0.26	0.91	0.33

5. CONCLUSIONS

A method of computation of size distribution of suspended materials has been developed in the course of the present study. The method utilizes the effect of concentration in the viscous and turbulent shear stresses in addition to the hindered settling due to the increased concentration in suspension applied to the Hunt's diffusion equations for sediment and water. Using a general form of velocity profile which shows the interaction between the flow and sediment concentration, Hunt's diffusion equation has been solved with a view to computing the suspended sediment concentration at any height above the beds, using the computed bed load equation as the reference concentration. The methods due to the modified Rouse equation (26) and to Mendoza and Zhou are also compared for suspended-load computation. The accuracy of each of these methods has been tested by comparing the computed data with observations on the grain-size distribution of suspended load samples collected in laboratory flumes (Indian Statistical Institute, Calcutta and Uppsala University). The results obtained from four different sand beds have been discussed. Quantitative estimates of the errors between the observed and the computed values indicate that the present method is superior to the others, because the present method accounts better for coupling between the suspended sediment and the fluid flow. Consequently, the derived concentration profile is more general. Although the present formulation for computation of suspended load is complex in nature, it aims at providing an insight into the problem for studying the various physical aspects which were not considered earlier.

ACKNOWLEDGEMENTS

The authors are grateful to Professor Supriya Sengupta, Dept. of Naval Architecture and Ocean Engineering, Indian Institute of Technology, Kharagpur, for providing us with suspended sediment data collected from Uppsala University. We record our thanks to Ms. Koeli Ghoshal for plotting some of the figures in the text and the Department of Science and Technology (DST), New Delhi for providing the financial grant (No. ESS/23/VES/109/2000) for conducting this research.

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