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Soldering formalism in noncommutative field theory: a brief note

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Abstract

In this Letter, I develop the soldering formalism in a new domain—the noncommutative planar field theories. The soldering mechanism fuses two distinct theories showing opposite or complimentary properties of some symmetry, taking into account the interference effects. The above mentioned symmetry is hidden in the composite (or soldered) theory. In the present work it is shown that a pair of noncommutative Maxwell–Chern–Simons theories, having opposite signs in their respective topological terms, can be consistently soldered to yield the Proca model (Maxwell theory with a mass term) with corrections that are at least quadratic in the noncommutativity parameter. We further argue that this model can be thought of as the noncommutative generalization of the Proca theory of ordinary spacetime. It is well known that abelian noncommutative gauge theory bears a close structural similarity with non-abelian gauge theory. This fact is manifested in a non-trivial way if the present Letter is compared with existing literature, where soldering of non-abelian models are discussed. Thus the present work further establishes the robustness of the soldering programme. The subtle role played by gauge invariance (or the lack of it), in the above soldering process, is revealed in an interesting way.

Keywords: Noncommutative gauge theory; Seiberg–Witten map; Soldering formalism; Maxwell–Chern–Simons theory

In recent years, noncommutative (NC) field theories [1,2] and in particular NC gauge theories have generated a lot of interest due to their appearance in the low energy limit in a system of open strings ending on D-branes, in the presence of a background field. The D-branes inherit the noncommutativity in the string boundaries. Thus the field theories living on the D-branes can be described by NC field theories, which can yield string theoretic results in certain limits. On the other hand, by itself NC field theory is a fascinating subject. Even though in NC quantum field theory, the basic computational scheme remains essentially the same as that of quantum field theory in ordinary spacetime, qualitatively distinct behavior is observed in the former. Some of the novel features of NC quantum field theories are UV/IR mixing [3] induced by non-locality, presence of solitons [4] in higher-dimensional scalar theories, dipole like elementary excitations [5], etc. This motivates further study of different aspects of field theories in the context of NC spacetime. In the present Letter, we will concentrate on some specific models in $(2+1)$-dimensional NC spacetime.

Investigations in the context of quantum field theories living in $(2+1)$-dimensional ordinary (commuta-

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tive) spacetime have proved to be rewarding in the past [6–8]. There exist several physically relevant models, such as Maxwell–Chern–Simons (MCS) model, self-dual (SD) models, fermions in interaction with gauge fields (massive Thirring model), that are closely interrelated and enjoy non-trivial duality (or equivalence) relations between operators of the respective theories. The MCS and SD models, along with their dual nature, have been studied exhaustively in [6]. Their connection with the fermion theories via bosonization (in the large fermion mass limit) was elucidated in [7]. A unified analysis of all these models can be found in [8].

A number of works, spanning all the above topics, pertaining to NC generalization, have appeared recently [9–12]. An NC generalization of the MCS model, obtained by exploiting the inverse Seiberg–Witten map [1] and its subsequent duality with the NC SD model was shown by the author in [9]. In the above mentioned NC SD model, the Chern–Simons term is structurally identical to the Chern–Simons term in ordinary spacetime. Bosonization of the NC massive Thirring model, in the large fermion mass limit, was carried through in [10], which reproduced a variant of the NC SD model, where the Chern–Simons term is the NC Chern–Simons term. Hence it was concluded (see Ghosh in [9]) that duality between the massive Thirring model and MCS model (in large fermion mass limit), a property valid in ordinary spacetime [6,7], is lost in NC spacetime. However, later it was shown in [12] that the above chain of duality can be maintained in NC spacetime as well, provided one considers an alternate version of the NC MCS model, proposed in [11], consisting of NC Maxwell and NC Chern–Simons terms. Interestingly, [12] shows that this NC MCS model is actually dual to the model obtained in [10] via bosonization, thereby completing the chain of dualities. All the above results are valid for the lowest non-trivial order in $\theta_{\mu\nu}$—the noncommutativity parameter.

As far as $O(\theta)$ computations are concerned, both the above definitions of NC MCS theory in [9] and [11] are equally viable but distinct alternatives, with [11] probably being the more popular one. On the other hand, the issue of NC extension of the SD model is tricky. If we follow the approach of the Seiberg–Witten map [1], our way of defining the NC SD model in [9] appears to be the natural one since in the absence of any (manifest) gauge invariance, the Seiberg–Witten map should not come in to the picture. Since the SD model in ordinary spacetime is a quadratic theory with no gauge symmetry, there will be no significant effects of noncommutativity. This is because the θ-dependent term will come only from the $*$-product of two operators. These contributions are ignored assuming that total derivative terms can be dropped from the action. Hence the model in question in [10,12], consisting of a mass term and NC Chern–Simons term, should not be thought of as NC SD model. In the present Letter, our reasoning will be corroborated further in the context of another model of a similar nature—the Proca model.

It is now time to put our work in its proper perspective. The soldering formalism [13–15], to be explained below, has been used extensively (see [16] for an updated review and references therein), in the context of theories in ordinary spacetime. We demonstrate that it is adaptable to noncommutative spacetime as well, leading to interesting and non-trivial results. The present work is concerned with $O(\theta)$ modifications only. The noncommutativity brings in additional features which can be directly related to similar behavior in soldering in non-abelian gauge theories in ordinary spacetime [17].

Furthermore, the noncommutative soldering, in the present case, generates a particularly simple model, which we would like to interpret as the Proca model in NC field theory framework. We will argue that the NC version of the Proca model should contain the mass term and *ordinary* Maxwell term (with possible $O(\theta^2)$ corrections, $\theta_{\mu\nu}$ being the noncommutativity parameter), and not the NC Maxwell term, for reasons exactly similar as above. In the NC theory considered here, the NC Proca model appears in an exactly analogous situation where the Proca model emerges in ordinary spacetime. Our framework in demonstrating the above will be the soldering formalism.

Let us briefly introduce the idea of soldering formalism [13]. Combination of two distinct models to construct a single composite model is interesting, especially if both the initial and final theories are physically relevant. The above result clearly shows a direct connection between the parent and daughter theories. However, pursuing this idea in a generic case, in a systematic way, is indeed non-trivial. The soldering formalism precisely does this job for a particular class of a pair of models, which manifest the *dual* aspects of some symmetry, such as chirality, self-duality [14], etc. The soldering procedure can be carried through for such a particular pair in a well defined manner and the resulting soldered model, in a certain sense, hides the above mentioned symmetry. In fact, the above mentioned equivalence between the parent and daughter theories is quite deep rooted. This can be established [15] in an alternative canonical transformation scheme, whereby in a Hamiltonian framework, the soldered model can be broken up into the parent dual models. Hence, the soldering formalism and the canonical transformation prescription are complimentary to each other.

I start by introducing the SD-MCS duality in conventional spacetime. For convenience we follow the notations and metric ($g^{\mu\nu} = \mathrm{diag}(1, -1, -1)$) of [6]. The self-dual (or anti-self-dual) Lagrangians, consisting of the ordinary and topological mass terms are

$$\mathcal{L}^{\pm}_{SD} = \frac{1}{2} f^{\mu} f_{\mu} \pm \frac{1}{2M} \epsilon^{\alpha\beta\gamma} f_{\alpha} \partial_{\beta} f_{\gamma}. \tag{1}$$

These two models can be generated via bosonization [7] (in the large fermion mass limit), from two distinct fermion theories of mass M, having opposite chiralities. On the other hand, the (corresponding dual) MCS models [6] are described by

$$\mathcal{L}^{\pm}_{MCS} = -\frac{1}{4} F^{\alpha\beta} F_{\alpha\beta} \pm \frac{M}{2} \epsilon^{\alpha\beta\gamma} A_{\alpha} \partial_{\beta} A_{\gamma},$$
$$F_{\alpha\beta} = \partial_{\alpha} A_{\beta} - \partial_{\beta} A_{\alpha}. \tag{2}$$

Note that total derivative terms in the Lagrangian will be dropped throughout the present (classical) discussion. Clearly, Eq. (2) is invariant under the gauge transformation

$$A_{\mu} \to A_{\mu} + \partial_{\mu}\lambda, \tag{3}$$

whereas no such manifest symmetry exists for (1). However, one can solve the equations of motion and constraints for both of the above models and show that there exists the identification [6],

$$f_{\mu} = \epsilon_{\mu\nu\tau} \partial^{\nu} A^{\tau},$$

that reduces one model to the other.

Before discussing the role of soldering in the present context, let me introduce the soldering mechanism [14] in an explicit way for a generic situation. Here an iterative Noether procedure is exploited to lift the global symmetries of the constituent models to a local symmetry in the composite model. The general prescription is to express the variations of the actions of the models, $\mathcal{A}^{+}(A)$ and $\mathcal{A}^{-}(B)$ (that are to be soldered), in the following form:

$$\delta\mathcal{A}^{+}(A) = \int d^3x\, J^{+}_{\mu}(A) \partial^{\mu} a,$$
$$\delta\mathcal{A}^{-}(B) = \int d^3x\, J^{-}_{\mu}(B) \partial^{\mu} a, \tag{4}$$

where $J^{+}_{\mu}(A)$ and $J^{-}_{\mu}(B)$ denote the generic Noether currents corresponding to the global invariances under,

$$\delta A = \delta B = a. \tag{5}$$

Next, one introduces an auxiliary field C, with a particular local transformation

$$\delta C \approx f(a), \tag{6}$$

such that the following action

$$\mathcal{A}(A, B, C) = \mathcal{A}^{+}(A) + \mathcal{A}^{-}(B) + \mathcal{W}(A, B, C), \tag{7}$$

is invariant under the local transformations given in Eqs. (5), (6). The last term in the action, $\mathcal{W}(A, B, C)$, incorporates the interference effects. It is of such a form that the variational equation of motion for C is an algebraic one for C. Thus C can be eliminated classically from (6), thereby yielding the cherished action $\mathcal{A}^{S}(G)$ for the soldered model, where the fields A and B occur in a gauge invariant combination $G = A - B$, $\delta G = 0$.

In ordinary spacetime, application of the soldering mechanism for the self and anti-self-dual models (or there MCS versions), induces the Proca model [14],

$$L(G) = -\frac{1}{4} F^{\mu\nu}(G) F_{\mu\nu}(G) + \frac{m^2}{2} G^{\mu} G_{\mu},$$
$$G_{\mu} = \frac{1}{\sqrt{2}} (A_{\mu} - B_{\mu}). \tag{8}$$

As mentioned before, we will concentrate on the analogous phenomenon in the context of noncommutative field theory.

The NC spacetime is characterized by

$$[x^{\rho}, x^{\sigma}]_{*} = i\theta^{\rho\sigma} \tag{9}$$

The $*$-product is given by the Moyal–Weyl formula

$$p(x) * q(x) = pq + \frac{i}{2} \theta^{\rho\sigma} \partial_{\rho} p \partial_{\sigma} q + O(\theta^2). \tag{10}$$

All our discussions will be valid up to the first non-trivial order in θ.

The NC extension of the Chern–Simons action has been derived in [18]. The NC MCS model is defined in the following way [11]:

$$\hat{\mathcal{A}}^{\pm}_{MCS} = \int d^3x \left[-\frac{1}{4}\hat{F}^{\mu\nu} * \hat{F}_{\mu\nu} \pm \frac{M}{2}\epsilon^{\mu\nu\lambda}\left(\hat{A}_\mu * \partial_\nu \hat{A}_\lambda + \frac{2}{3}\hat{A}_\mu * \hat{A}_\nu * \hat{A}_\lambda\right)\right], \quad (11)$$

where

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - i\hat{A}_\mu * \hat{A}_\nu + i\hat{A}_\nu * \hat{A}_\mu.$$

The structural similarity with corresponding expressions of non-abelian gauge theory is very much apparent. It will be revealed subsequently that the connection goes deeper.

Utilizing the Seiberg–Witten map, to the lowest non-trivial order in θ,

$$\hat{A}_\mu = A_\mu + \theta^{\sigma\rho} A_\rho \left(\partial_\sigma A_\mu - \frac{1}{2}\partial_\mu A_\sigma\right),$$
$$\hat{F}_{\mu\nu} = F_{\mu\nu} + \theta^{\rho\sigma}(F_{\mu\rho}F_{\nu\sigma} - A_\rho \partial_\sigma F_{\mu\nu}),$$
$$\hat{\lambda} = \lambda - \frac{1}{2}\theta^{\rho\sigma} A_\rho \partial_\sigma \lambda, \quad (12)$$

(where $\hat{\lambda}$ and λ are infinitesimal gauge transformation parameters in NC and ordinary spacetimes), we arrive at the following $O(\theta)$ modified form of the NC MCS theory, expressed in terms of ordinary spacetime variables,[1]

$$\hat{\mathcal{A}}^{\pm}_{MCS} = \int d^3x \left[-\frac{1}{4}\left(F^{\mu\nu}F_{\mu\nu} + 2\theta^{\rho\sigma}\left(F^{\mu}{}_{\rho}F^{\nu}{}_{\sigma}F_{\mu\nu} - \frac{1}{4}F_{\rho\sigma}F^{\mu\nu}F_{\mu\nu}\right)\right) \pm \frac{M}{2}\epsilon^{\mu\nu\lambda}A_\mu \partial_\nu A_\lambda \right], \quad (13)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. It should be remembered that under the Seiberg–Witten map, the NC Chern–Simons term exactly reduces to the Chern–Simons term in ordinary spacetime [18] to all orders in θ. In 2 + 1 dimensions, (13) further simplifies to

$$\hat{\mathcal{A}}^{\pm}_{MCS} = \int d^3x \left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \pm \frac{M}{2}\epsilon^{\mu\nu\lambda}A_\mu \partial_\nu A_\lambda - \frac{1}{8}\theta^{\rho\sigma}F_{\rho\sigma}F^{\mu\nu}F_{\mu\nu}\right]. \quad (14)$$

The equations of motion are

$$\partial_\mu (F^{\mu\nu} \pm M\epsilon^{\mu\nu\lambda}A_\lambda) = O(\theta), \quad (15)$$

where explicit form of the $O(\theta)$ term in the right-hand side is not required for our present analysis. The change in A_μ

$$\delta A_\mu = a_\mu, \quad (16)$$

induces the following changes in the actions:

$$\delta\hat{\mathcal{A}}^{\pm}_{MCS} = \int d^3x \left[(J^{\pm}_{\mu\nu} + J^{(\theta)}_{\mu\nu})\partial^\mu a^\nu \right], \quad (17)$$

where

$$J^{\pm}_{\mu\nu} \equiv -F_{\mu\nu} \pm M\epsilon^{\mu\nu\lambda}A_\lambda, \quad (18)$$

$$J^{(\theta)}_{\mu\nu} \equiv -\frac{1}{4}(F^2\theta_{\mu\nu} + 2(\theta \cdot F)F_{\mu\nu}). \quad (19)$$

A short-hand notation $\theta^{\mu\nu}F_{\mu\nu} = \theta \cdot F$ has been adopted.

Now the auxiliary variable $C_{\mu\nu}$ is introduced in the action via the background interaction and contact terms

$$\tilde{\mathcal{A}} = \hat{\mathcal{A}} - \int \frac{1}{2}\left[(J + J^{(\theta)}) \cdot C + \frac{1}{2}C^2\right]. \quad (20)$$

The transformation[2] of $C_{\mu\nu}$,

$$\delta C_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - \delta J^{(\theta)}_{\mu\nu}, \quad (21)$$

together with transformation of A_μ in (16), changes the actions by

$$\delta\hat{\mathcal{A}}^{\pm}_{MCS} = \int d^3x \left[\pm\frac{M}{2}C^{\mu\nu}\epsilon_{\mu\nu\lambda}a^\lambda + \frac{1}{2}(-F^{\mu\nu} \pm M\epsilon^{\mu\nu\lambda}A_\lambda)\delta J^{(\theta)}_{\mu\nu}\right]. \quad (22)$$

The idea behind introduction of the fine tuned $C_{\mu\nu}$-dependent counterterms in the two actions (that are to

[1] It should be kept in mind that in the conventional Hamiltonian form of the field theory in (13), only spatial noncommutativity is allowed. But this is not of direct concern to us in the present analysis.

[2] We have checked that keeping $\delta C_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ and introducing more interference terms in the action does not solve the problem at hand.

be soldered) is that the variations in the two actions will be of opposite signature. This feature will give rise to the invariance in the soldered action. In the present instance, this is true for $\theta_{\mu\nu}=0$ but this does not quite happen for non-zero $\theta_{\mu\nu}$ since the $F_{\mu\nu}$-term appears with the same sign in variations of both the actions. However, notice that considerations of the equations of motion (15) reveals that the factor $(-F^{\mu\nu}\pm M\epsilon^{\mu\nu\lambda}A_\lambda)$ itself in (22) is of $O(\theta)$ and so $\frac{1}{2}(-F^{\mu\nu}\pm M\epsilon^{\mu\nu\lambda}A_\lambda)\delta J^{(\theta)}_{\mu\nu}$ in (22) will contribute to $O(\theta^2)$ correction. Thus, this term in the variations of the actions in (22) can be ignored in our present discussion. The rest of the variations in (22) are of the form that is amenable to soldering. In fact, the variations are same (up to $O(\theta)$) as their ordinary spacetime counterpart.

Let us pause for a moment to appreciate the closeness of our analysis with existing results in the context of soldering of non-abelian self dual models [17]. First of all, the Euler kernel, that is the Noether current receives non-linear contributions as a result of noncommutativity (or non-abelian nature [17]), which is quite natural. However, it is striking that the auxiliary field $C_{\mu\nu}$ ceases to transform in the conventional way and the more involved transformation (21) is clearly "identical" to its counterpart in the non-abelian context [17].

We introduce the auxiliary $C_{\mu\nu}$ field and write down the total action as

$$\hat{A}=\tilde{A}^+(A,C)+\tilde{A}^-(B,C)$$
$$=\hat{A}^+(A)+\hat{A}^-(B)$$
$$-\frac{1}{2}\int\big[C^2+\big[J^+(A)+J^-(B)+J^{(\theta)}(A)+J^{(\theta)}(B)\big].C\big]. \quad (23)$$

The $C_{\mu\nu}$ field is constrained by the relation

$$C_{\mu\nu}=-\frac{1}{2}\big(J^+_{\mu\nu}(A)+J^-_{\mu\nu}(B)+J^{(\theta)}_{\mu\nu}(A)+J^{(\theta)}_{\mu\nu}(B)\big). \quad (24)$$

This allows us to replace $B_{\mu\nu}$ in favor of the basic dynamical variables and we obtain the soldered Lagrangian,

$$\hat{\mathcal{L}}^{(S)}=-\frac{1}{8}F^{\mu\nu}(A-B)F_{\mu\nu}(A-B)$$
$$+\frac{M^2}{4}(A-B)^\mu(A-B)_\mu$$
$$-\frac{1}{16}\big[\big(-F(A)^{\mu\nu}+M\epsilon^{\mu\nu\lambda}A_\lambda\big)+\big(-F^{\mu\nu}(B)-M\epsilon^{\mu\nu\lambda}B_\lambda\big)\big]$$
$$\times\big[F^2(A)\theta_{\mu\nu}+2\big(\theta.F(A)\big)F_{\mu\nu}(A)+F^2(B)\theta_{\mu\nu}+2\big(\theta.F(B)\big)F_{\mu\nu}(B)\big]. \quad (25)$$

It should be mentioned that the above step is somewhat formal since the theory is obviously not Gaussian. However, this is legitimate as far as classical considerations go. Similar steps have also been performed in [17].

Note that the θ-independent part of the action in (25) is in the form that was advertised at the beginning. In fact, this part is identical to the ordinary spacetime result [14]. The remaining part of the action $\hat{\mathcal{L}}^{(S)}$ in (25) is once again dropped since it is of $O(\theta^2)$. The argument is exactly the same as the one given below (22). Hence, the soldered theory is the Proca term with $O(\theta^2)$ modification,

$$\hat{\mathcal{L}}^{(S)}=-\frac{1}{8}F^{\mu\nu}(A-B)F_{\mu\nu}(A-B)+\frac{M^2}{4}(A-B)^\mu(A-B)_\mu+O(\theta^2)$$
$$=-\frac{1}{8}F^{\mu\nu}(G)F_{\mu\nu}(G)+\frac{M^2}{4}G^\mu G_\mu+O(\theta^2), \quad (26)$$

where, in keeping with our earlier notation in (8), $G\equiv A-B$. Thus, our major observation is that, to the lowest non-trivial order in θ, soldering of the NC MCS models is indeed possible and consistent. The whole process is clearly reminiscent of the similar phenomenon [14] in ordinary spacetime, with the noncommutativity introducing a non-abelian flavor.

We are now faced with the question that how far is it justified to identify the above massive vector model, whose kinetic part is *identical to the ordinary spacetime Maxwell term*, and not of the NC Maxwell form, as the NC generalization of the Proca model, up to $O(\theta)$. Our views, favoring the above identification are presented below.

There are some points in the present Letter that need to be stressed in the perspective of the recent paper [12]. First of all, in the present Letter, we have taken the noncommutative generalization of MCS theory that has been suggested in [11] and advocated in [12]. To $O(\theta)$, we recover the action that is same as that of the ordinary spacetime Proca theory. We claim this to be the *noncommutative generalization of the Proca theory* as well. Our contention is that since there is no gauge invariance in the Proca model which is a free theory having only quadratic terms in the action, there will be no effects of noncommutativity, at least to $O(\theta)$, assuming that total derivative terms in the action need not be taken in to account. This ties up very nicely with our previous work [9], where similar reasonings were put forward for the noncommutative extension of the self-dual model. This idea is firmly based on the basic premises of the Seiberg–Witten map where non-trivial nature of the mapping comes in to play *only in the presence of gauge invariance*.

This means that, to $O(\theta)$, the model in (26) can be elevated to the action of the corresponding NC model,

$$\hat{S}^{(S)} = \int d^3x\, \hat{\mathcal{L}}^{(S)} = \int d^3x \left(-\frac{1}{8}\hat{F}^{\mu\nu}(\hat{G}) * \hat{F}_{\mu\nu}(\hat{G}) + \frac{M^2}{4}\hat{G}^{\mu} * \hat{G}_{\mu}\right), \tag{27}$$

with the identification $\hat{G} = G$, $\hat{F}^{\mu\nu}(\hat{G}) \equiv \partial^{\mu}\hat{G}^{\nu} - \partial^{\nu}\hat{G}^{\mu}$, since there is no non-trivial Seiberg–Witten map for non-gauge theories. Thus θ-dependent terms in (27) can only appear from the $*$-products. However, the theory being quadratic, the $O(\theta)$ contributions coming from the $*$-products are total derivatives and will vanish in the action and we are left with

$$\hat{S}^{(S)} = \int d^3x \left(-\frac{1}{8}\hat{F}^{\mu\nu}(\hat{G})\hat{F}_{\mu\nu}(\hat{G}) + \frac{M^2}{4}\hat{G}^{\mu}\hat{G}_{\mu}\right) + O(\theta^2) = \int d^3x \left(-\frac{1}{8}F^{\mu\nu}(G)F_{\mu\nu}(G) + \frac{M^2}{4}G^{\mu}G_{\mu}\right) + O(\theta^2), \tag{28}$$

where in the last step we have recovered (26). Hence, Eq. (27) (or effectively (26)) is the cherished form of the noncommutative self-dual model.

Furthermore, from the soldering point of view, the above criterion corroborates with the ordinary spacetime result where self-dual and anti-self-dual models can also be soldered to generate the Proca model. This has to be the case since the self-dual models are dual to the MCS models, respectively. In the present Letter, we have demonstrated the noncommutative counterpart of MCS–Proca soldering. On the other hand we can consider the noncommutative generalization of the soldering of NC SD models to generate NC Proca model. But, as we have argued, to $O(\theta)$ there will be no changes in *any* of the models participating in the soldering process, since all of them are quadratic theories and none of them are gauge theories. Hence, we will reach the result, identical to the ordinary spacetime one, which also agrees with the conclusion presented here. This demonstrates the robustness of the soldering programme as well as consistency in our way of defining noncommutative generalizations of self-dual or Proca theories.

Indeed, it would be interesting if the noncommutative soldering can be performed consistently for the parent and daughter models where *all* of them are gauge theories. Such a problem has been discussed in [14] in the context of electromagnetic duality. Its NC extension is under study.

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