Image Ambiguity Optimization for Object Extraction: A Soft Computing Approach

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Abstract

Object extraction is performed using soft computing tools (e.g., neural networks, genetic algorithms and fuzzy logic) and image ambiguity optimization. Hopfield type neural network is used for object background classification. Genetic Algorithms (GAs) are used to evolve such a Hopfield type optimum network architecture where each chromosome represents an architecture. Fuzzy sets are introduced into this Neuro-GA framework. Output status of neurons at the converged state of each network is viewed as a fuzzy image subset. Measures of image ambiguity, in terms of gray level as well as spatial, of this image subset are considered, in isolation and in combination, as the index of fitness of chromosomes. Gray level ambiguity measures take care of global information whereas spatial (geometrical) ambiguity measures use local and shape information. The best chromosome of the GA that corresponds to the least image ambiguity of the final generation represents the optimum network configuration for object extraction. It is seen that spatial ambiguity measure based algorithms have an edge over gray level ambiguity based ones, especially for highly corrupted and compact objects, so far as the preservation of shape and elimination of noise are concerned. For non-compact objects, gray level ambiguity based optimization techniques produce superior performance.

Keywords: Object extraction, image ambiguity, Hopfield type network, genetic algorithms, soft computing.

1. Introduction

Soft computing [9] is a consortium of methodologies, which work synergistically, and provides, in one form or another, flexible information processing capabilities for handling real life ambiguous situations. Its aim is to tolerate the imprecision, uncertainty etc. in order to achieve tractability, robustness, low solution cost, and close resemblance with human like decision making. At this juncture, artificial neural networks (ANNs), genetic algorithms (GAs) and fuzzy logic (FL) are the three principal components of soft computing; where FL provides algorithms for dealing with imprecision and uncertainty, ANN the machinery for learning & adaptation; and GA for optimization & searching.

ANNs [5] try to emulate the biological neural networks, especially, some aspects of human information processing system, with electronic circuitry and act as the machinery for learning and adaptation. They are designated by the network topology, connection strengths between pairs of neurons/nodes, node characteristics and rules for updating status. Since there are interactions among the neurons, the collective computational property inherently reduces the computational task and helps to produce output in real time and makes the system fault tolerant (with respect to noise and component failure). This assists the ANN models to be suitable for solving tasks requiring collective decision-making. Most of the image analysis operations are co-operative in nature and the recognition tasks mostly need formulation of complex decision regions. ANN models have the ability to achieve these properties.

The architecture/configuration of the network depends on the goal one is trying to achieve. (For example, in case of a Hopfield type neural network [6] performing object extraction [3], a neuron of the network corresponds to a pixel of the image.) In spite of the wide range of applicability of ANNs, there is no formal procedure to design an optimum network for a given problem. Finding out such a network for a given problem requires efficient searching in complex spaces in order to obtain optimal solutions in real time. This further makes the process computationally intensive.

GAs [4] are computational procedures modeled on the mechanics of natural genetic systems. They iteratively perform the following cycle of operations on a set of coded solutions/chromosomes, called a population, until some termination condition is achieved: selection (including fitness evaluation of each solution), reproduction (including crossover and mutation), and

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replacement of the old population with a new one.

GAs are viewed as efficient & adaptive search and optimization techniques. They are able to produce (near) optimal solutions having a large degree of implicit parallelism. Therefore, application of GAs for solving certain problems of image processing/pattern recognition appears to be appropriate and natural. Let us consider the example of finding out an optimum neural network architecture (say, Hopfield type network for object extraction, as mentioned earlier). This needs searching a complex space and thus GAs can find their application.

Fuzzy sets (FS) [12] are generalizations of conventional (crisp) sets. Conventional sets contain objects that satisfy precise properties required for membership. Fuzzy sets, on the other hand, contain objects that satisfy imprecisely defined properties to varying degrees. The deficiency of information available from incomplete/imprecise/noisy images suggests the use of fuzzy set theory to obtain solution (output) of the system with least uncertainty. Let us again consider the situation of object extraction using Hopfield type network, as mentioned earlier. The output status of the neurons, at the converged state, of each network can be viewed as a fuzzy image subset [10]; and thus objects can be extracted with least image ambiguity (uncertainty) measures [1,9].

In the present article soft computing tools are used for extracting object regions from both binary and gray images. A preliminary attempt in this line is reported in [2]. Hopfield type network is used for object extraction. The optimum architecture of such a network is evolved using GAs where each chromosome of a GA represents a network architecture [7]. The output status of the neurons, in the converged state, of each network is taken as a fuzzy image subset. Image ambiguity measure of this subset is considered as a measure of error/energy of the network, which in turn reflects the fitness of the corresponding chromosome in GA. Here, instead of making crisp decisions for the output status of the neurons (either 0 or 1), they are scaled to lie in the range [0,1] and is viewed as a fuzzy set. This, in turn, mimics the human reasoning process for making decisions in labeling the class of the pixels as object or background. This sort of uncertainty (indefiniteness in deciding whether a pixel has a label 1 or 0) is known as grayness ambiguity [1,9] and it considers global information. On the other hand, the fuzzy geometrical properties [8,10,11] of this image subset reflect the spatial ambiguity (indefiniteness in shape/geometry of the object) of the image by incorporating local information.

For the present investigation, we considered index of fuzziness and entropy of a fuzzy image subset as gray level ambiguity measures. As spatial ambiguity, compactness measure is used. A weighted combination of them is also explored. From experimental results, it is seen that for compact objects spatial ambiguity measure performs better than the corresponding gray level ambiguity based methods for preserving shapes and removing noises; whereas for non-compact objects gray level ambiguity based techniques produce superior performance.

2. Image Ambiguity

A gray tone image possesses ambiguity within each pixel because of the possible multi-valued levels of brightness. If the gray levels are scaled to lie in the range [0, 1], we can regard the gray level of a pixel as its degree of membership in the set of high-valued *bright pixels* - thus a gray tone image can be viewed as a fuzzy image subset. Regions, features, primitives, properties, and relations among them, that are not crisply defined, can similarly be regarded as fuzzy subsets of images [8,9].

A L level image A (of size $M \times N$) can therefore be considered as an array of fuzzy singletons, each having a value of membership denoting its degree of possessing some property (e.g., brightness, darkness, texture etc.). In the notation of fuzzy sets one may therefore write that $A = \{\mu_A(x_{mn}) : m = 1,2,\ldots,M; n = 1,2,\ldots,N\}$; where $\mu_A(x_{mn})$ denotes the grade of possessing such a property μ by the (m,n) pixel. For convenience, we may use μ and μ_A interchangeably. Incertitude in an image may be explained in terms of grayness ambiguity or spatial (geometrical) ambiguity or both. Grayness ambiguity means indefiniteness in deciding whether a pixel is object or background. Spatial ambiguity refers to indefiniteness in the shape and geometry of a region within the image. Gray level ambiguity considers only global information whereas spatial ambiguity takes care of local information. Some commonly used grayness ambiguity measures are index of fuzziness, fuzzy entropy and fuzzy correlation [1]; the spatial ambiguity measures are extracted from fuzzy geometrical properties e.g., compactness, IOAC etc. [8,10,11].

2.1 Gray Level Ambiguity

A few gray level ambiguity measures [1,9] relevant to the present work are described here.

Index of Fuzziness: The index of fuzziness (fI) of a fuzzy image subset A having n supporting elements is defined as

$$\gamma_p(A) = \frac{2}{n^{\frac{1}{p}}} l^p(A, \bar{A})$$

$$= \frac{2}{n^{\frac{1}{p}}} \left[\sum_{i=1}^{n} \{\min(\mu_A(x_i), 1 - \mu_A(x_i))\}^p \right]^{\frac{1}{p}}, \quad (1)$$

when $l^p(A, \bar{A})$ denotes the distance between fuzzy set A and its nearest ordinary set $\bar{A}$. An ordinary set $\bar{A}$ nearest to the fuzzy set A is defined as:

$$\mu_{\bar{A}}(x) = \begin{cases} 0 & if \quad \mu_A(x) \le 0.5 \\ 1 & if \quad \mu_A(x) > 0.5 \end{cases} \quad (2)$$

We used quadratic index of fuzziness i.e., $p = 2$.

Entropy: Entropy (DeLuca & Termini) of a fuzzy image subset (f2) using logarithmic gain function is given by

$$H(A) = \frac{1}{n \ln 2} \sum_{i=1}^{n} \{S_n(\mu_A(x_i))\}, \quad (3)$$

with

$$S_n(\mu_A(x_i)) = -\mu_A(x_i) \ln\{\mu_A(x_i)\} - \{1 - \mu_A(x_i)\} \ln\{1 - \mu_A(x_i)\}; \quad (4)$$

and that of Pal and Pal (f3) using exponential gain function is given by

$$H(A) = \frac{1}{n(\sqrt{e} - 1)} \sum_{i=1}^{n} \{S_n(\mu_A(x_i)) - 1\} \quad (5)$$

with

$$S_n(\mu_A(x_i)) = \mu_A(x_i) e^{1-\mu_A(x_i)} \{1 - \mu_A(x_i)\} e^{\mu_A(x_i)}. \quad (6)$$

Another definition of entropy, which involves the distance of the fuzzy image subset from its furthest ordinary set, is given by Bart Kosko (f4). It says

$$R_p(A) = \frac{l^p(A, \bar{A})}{l^p(A, \underline{A})}$$

$$= \frac{\left[\sum_{i=1}^{n} \{\min(\mu_A(x_i), 1 - \mu_A(x_i))\}^p \right]^{\frac{1}{p}}}{\left[\sum_{i=1}^{n} \{\max(\mu_A(x_i), 1 - \mu_A(x_i))\}^p \right]^{\frac{1}{p}}} \quad (7)$$

where $\underline{A}$ is an ordinary set furthest to the fuzzy set A, defined by

$$\mu_{\underline{A}}(x) = \begin{cases} 0 & if \quad \mu_A(x) > 0.5 \\ 1 & if \quad \mu_A(x) \le 0.5. \end{cases} \quad (8)$$

Note that, these entropy measures, first of all, compute the fuzziness related to individual pixel of the image and then make an average over all the pixels to get a quantification of the amount of average ambiguity, the image possesses.

2.2 Spatial Ambiguity

In image analysis we often want to measure geometrical properties (reflecting spatial ambiguity measure) of regions in an image that are not crisply defined. The concept of digital picture geometry is extended to fuzzy subsets and is called fuzzy geometry [10,11], which includes area, perimeter, compactness, height, width, length, breadth, etc. [8,10]. After generalizing the standard geometric properties, it becomes possible to use these definitions to construct image description without committing to a specific segmentation of an image. A few fuzzy geometrical properties relevant to the present work are described below.

Area: The area of a fuzzy image subset μ is defined as

$$a(\mu) = \sum_{i,j} \mu. \quad (9)$$

The area is therefore the weighted sum of the regions on which μ has constant value weighted by these values.

Perimeter: If μ is piece-wise constant, the perimeter of μ is defined as

$$P(\mu) = \sum_{i,j,k} |\mu(i) - \mu(j)| \times |Ar(i, j, k)|. \quad (10)$$

This is just the weighted sum of the lengths of the arcs $Ar(i, j, k)$ along which the regions having μ values $\mu(i)$ and $\mu(j)$ meet. In case of an image if we consider the pixels as the piece-wise constant regions, and the common arc length for adjacent pixels as unity then the perimeter of an image is defined by

$$p(\mu) = \sum_{i,j} |\mu(i) - \mu(j)| \quad (11)$$

where $\mu(i)$ and $\mu(j)$ are the membership values of two adjacent pixels.

Compactness: The compactness of a fuzzy image subset μ is defined as

$$comp(\mu) = \frac{a(\mu)}{p^2(\mu)}. \quad (12)$$

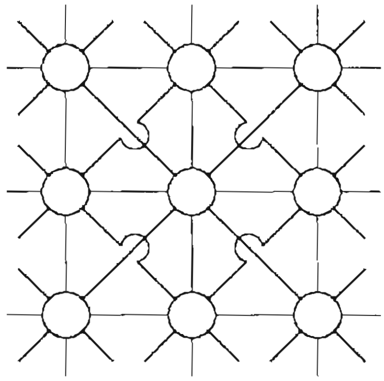


Figure 1. Topology of the network

Table 1. *pcc* for the synthetic images

pcc with fitness evaluation based on ambiguity measure	Image with					
	$\sigma = 15$		$\sigma = 24$		$\sigma = 32$	
	NE	*E*	*NE*	*E*	*NE*	*E*
Quadratic index of fuzziness (*f*1)	98.56	98.61	96.46	96.29	90.33	90.16
Entropy (DeLuca & Termini) (*f*2)	98.95	98.63	96.95	95.80	90.45	90.45
Entropy (Pal & Pal) (*f*3)	98.32	98.68	95.32	96.70	90.67	92.31
Entropy (Kosko) (*f*4)	98.34	98.78	95.61	95.53	89.65	90.36
Fuzzy compactness (f6)	99.15	98.39	98.29	98.07	96.90	95.78
Combination of f6 & f1 (f7)	98.76	98.88	95.61	96.46	91.46	89.67
Combination of f6 & f2 (f8)	99.00	98.90	97.71	97.80	90.97	91.09
Combination of f6 & f3 (f9)	98.88	98.83	97.27	97.02	90.43	91.60
Combination of f6 & f4 (f10)	98.97	98.73	96.17	96.97	90.38	90.14

In the present article we will use grayness and spatial ambiguity measures both individually and in a combined way to compute the error or energy of a Hopfield type network, which in turn will reflect the fitness of each of the chromosome while executing GA.

3. Object Extraction

As mentioned earlier, object extraction is performed using Hopfield type neural network. GA evolves such optimum Hopfield networks to produce segmented output. The fitness evaluation of each chromosome of GA is done using various types of image ambiguity measures. Let us describe the whole process in the following subsections.

3.1 Hopfield Type Neural Network for Object Extraction

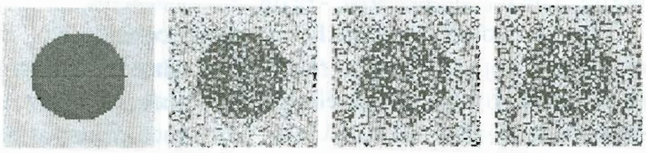
To use a Hopfield type neural network for object background classification, a neuron is assigned corresponding to every pixel. Each neuron can be connected to its neighbors (over a window) only. The connection can be full (a neuron is connected to all of its neighbors) or can be partial (a neuron may not be connected with all of its neighbors). The network topology for a fully connected third order neighborhood is depicted in Figure 1. Here, the maximum number of connections of each neuron with its neighbors is 8. In practice, all these connections may not exist. Again different neurons may have different connectivity configuration within its neighbors to produce optimum segmented output. There are various ways to design such an optimum architecture. One way to design such an optimum network is the use of genetic algorithms. The status updating rules of these neurons are similar to those of Hopfield's model [6]. The energy function of this model consists of two parts. The first part is due to the local field or local feedback and the second part corresponds to the input bias of the neurons. The first part is viewed as the impact of gray levels of the neighboring pixels. The total energy contributed by all pixel pairs will be $-\sum_i \sum_j W_{ij} V_i V_j$, where V_i, V_j are the status of the *ith* and *jth* neurons, respectively and W_{ij} represents the connection strength between these two neurons. In our experimental study W_{ij} is either 0 or 1 (connection is absent or present) and the value of W_{ij} is evolved by GA.

For every neuron i there is an initial input bias I_i which is taken to be proportional to the actual gray level of the corresponding pixel. If the gray value of a pixel is high (low), the corresponding intensity value of the scene is expected to be high (low). The input bias value is taken in the range [-1,1]. Under this framework an ON (1) neuron corresponds to an object pixel and the OFF (-1) one to background. So the threshold between object and background can be taken as 0. Thus the amount of energy contributed by the input bias values is $-\sum_i I_i V_i$.

Therefore, the expression of energy takes the form

$$E = -\sum_i \sum_j W_{ij} V_i V_j - \sum_i I_i V_i . \tag{13}$$

From a given initial state, the status of a neuron is modified iteratively to attain a stable state. The stable states of the network (local minima of the energy function) are made to correspond to the partitioning of a scene into compact regions of object and background.



(a) (b) (c) (d)

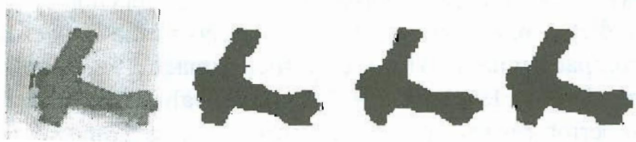
Figure 2. Synthetic image: (a) Original
Noisy versions: (b) $\sigma = 15$ (c) $\sigma = 24$ (d) $\sigma = 32$

(a) (b) (c)

Figure 3. Output images for the input image of Figure 2(b) using NE and (a) *f*2, (b) *f*6, (c) *f*8

(a) (b) (c)

Figure 4. Output images for the input image of Figure 2(c) using NE and (a) *f*2, (b) *f*6, (c) *f*8



(a) (b) (c) (d)

Figure 5. Biplane image: (a) Input
Output image using elitism and (b) *f*2, (c) *f*6, (d) *f*8

3.2 Optimum NN Evolution by GAs

Each chromosome of the GA represents a network architecture, as described above, where each bit of the chromosome represents W_{ij}. For an $m \times n$ image, each pixel (neuron) being connected to at most k of its neighbors, the length of the chromosome is $m \times n \times k$ bits. If a neuron is connected to any of its neighbors, the corresponding bit of the chromosome is set to 1, else 0. The initial population is generated randomly. Each network is then allowed to run for object extraction till it attains a stable state (converges).

Image ambiguity measure is taken as an index of fitness of the corresponding chromosome for its selection for the next generation. Crossover and mutation operations are performed on these selected chromosomes to get new offspring (architectures). The whole process is continued for a number of generations until the GA converges. The best chromosome of the final population represents a (sub) optimum architecture w.r.t. image ambiguity measures for object extraction.

3.3 Image Ambiguity as Fitness

In the stable states of the network, output value of each neuron will be in [0,1] representing its degree of belonging to the object class. Each pixel of the extracted output image (the output status of the neurons of the converged network) can thus be considered as an element of the fuzzy image subset *object*. Different types of image ambiguity measures of this subset are considered as measure of fitness of each network (chromosome).

a) Gray level ambiguity (I_g)

- Quadratic index of fuzziness, *f*1 (Eq. (1)).
- Entropy given by DeLuca and Termini, *f*2 (Eq. (3)).
- Entropy given by Pal and Pal, *f*3 (Eq. (5)).
- Entropy given by Bart Kosko, *f*4 (Eq. (7)).

b) Spatial ambiguity (I_s)

- Compactness, f6 (Eq. (12)).

c) Combination of gray level ambiguity and spatial ambiguity (I_a)

- Quadratic index of fuzziness and compactness, *f*7.
- Entropy of DeLuca & Termini and compactness, *f*8.
- Entropy of Pal & Pal and compactness, *f*9.
- Entropy of Bart Kosko and compactness, *f*10.

For the present investigation, we used a linear weighted combination of gray level and spatial ambiguity measures as follows.

$$I_a = w_1 I_g + w_2 (1 - I_s), \qquad (14)$$

where $w_1, w_2 > 0$.

4. Experimental Results

The effectiveness of the proposed technique has been demonstrated using some synthetic images, which are generated by adding $N(0, \sigma^2)$ noise $(\sigma = 15, 24, 32)$ to each pixel of the binary (two-tone) image shown in Figure 2(a); which has a highly compact object. The corresponding noisy versions are shown in Figures 2(b)-2(d). Size of each image is 64×64. The range of pixel value is [1, 32].

A chromosome is represented as a binary string of length $64 \times 64 \times 8$ (8 neighbors are taken). The population size is kept fixed at 30. Generational replacement technique and linear normalization selection procedure are adopted. Number of copies produced by the *ith* chromosome with linear normalized fitness value f_i in a population of size n is taken as round (c_i); where $c_i = \frac{n \times f_i}{\sum_{i=1}^{n} f_i}$. Both the elitist model (E) and the standard GA (non-elitist model, NE) are implemented. The crossover and mutation probabilities are taken as 0.8 and 0.002, respectively. To attain the stable states (convergence of the network) each network is allowed to iterate for 20 times. GAs have been run for 200 iterations. Since the target values of pixels for these synthetic images are known, one can measure the percentage of correct classification of pixels (*pcc*) of the converged (evolved) network. The *pcc* of the best chromosome in the final generation using grayness ambiguity, spatial ambiguity and their weighted combination (we used $w1 = 450, w2 = 1$, for *f*8, *f*9 and $w1 = 45, w2 = 1$ for $f7, f10$ in Eq. (14); this choice maps the gray level and spatial ambiguity values in a similar range) based fitness evaluations for different noisy versions of the synthetic image is depicted in Table 1. It is seen from Table 1 that the *pcc* is more for *f*6 (where compactness is maximized); whereas *pcc* is less for *f*1-*f*4 (minimization of gray level ambiguity); and as expected, *f*7-*f*10 (combination of the above two) produced intermediate *pcc* values. It is also found that non-elitist model evolved networks with less connectivity than the corresponding elitist version. Various image ambiguity optimization based methodologies produced similar architectures.

For visual illustration let us consider the results of input image of Figure 2(b) ($\sigma = 15$). The output images obtained for the non-elitist model using entropy of DeLuca & Termini (*f*2), compactness (*f*6) and their combination (*f*8) are depicted in Figures 3(a)-3(c), respectively. For the input image of Figure 2(c) ($\sigma = 24$), the corresponding output images are shown in Figures 4(a)-4(c). By analyzing the results we can infer that spatial ambiguity optimization (compactness maximization) (Figures 3(b), 4(b)) produced the best segmented output, in terms of preserving shape and eliminating noise. This is more evident when the noise level is high (Figure 4(b)). This is possibly due to the fact that compactness optimization takes care of local information and works better for compact/circular objects. From the results it is noticed that non-elitism has an edge over the corresponding elitist version so far as shape preservation and noise removal are considered. Gray level ambiguity optimization techniques produced comparatively inferior results (Figures 3(a), 4(a)) compared to other techniques, especially, for higher level of noise, as it considers only global information. As expected, a combination of spatial and gray level ambiguity measures produced an intermediate performance (Figures 3(c), 4(c)).

The proposed approach is also considered for extracting objects from a real image namely, 'Biplane' (Figure 5(a)) that has an elongated object. The size of this image is also 64x64. Again, for typical illustration, the corresponding objects extracted using *f*2, *f*6, and *f*8 for elitist models are shown in Figures 5(b)-5(d). It is seen from these figures also that the proposed technique is able to segment the real objects satisfactorily. In this case we noticed that gray level ambiguity optimization (Figure 5(b)) showed superior performance over spatial ambiguity based one (Figure 5(c)) and the combination of them produced intermediate results (Figure 5(d)). Since the object in this image is not compact and has no noise, global information based techniques performed better.

5. Conclusions

Soft computing methodologies are used for extracting object regions from gray images. Object extraction is done using Hopfield type networks whose architectures are evolved by GAs and image ambiguity measures. Both gray level ambiguity and spatial ambiguity measures are considered, in isolation and in combination, as fitness measuring criteria for GAs. A synthetic image, corrupted by different levels of noise, and a real image are used for object extraction. It is found that spatial ambiguity based fitness evaluation has an edge over grayness ambiguity based ones for preserving shapes and eliminating noise when the objects in the images are compact in nature (for the synthetic image). On the other hand, gray level ambiguity based evaluation produced superior results for non-compact objects (for the real image). Combination of both spatial and gray level ambiguity based optimization mostly produced intermediate results. Moreover, non-elitist model is seen to provide better results (as far as number of connections and shape of the objects are considered) than the corresponding elitist version. It is to be noted that, in the present work the connection strengths are taken as either 0 or 1 (i.e., connection is absent or present). Investigation involving weighted connection may constitute a part of future study.

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