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## Fault tolerant permutation mapping in multistage interconnection network

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### Abstract

An efficient scheme for fault tolerant mapping of permutations is designed. The proposed algorithm uses extra passes through the network, instead of additional hardware.

*Keywords:* Fault tolerant routing; Interconnection network; Permutation; Stuck-at fault

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### 1. Introduction

Permutation in multistage interconnection network (MIN) [1] is defined as a one to one exhaustive mapping from the set of inputs to the set of outputs. Presence of stuck-at faults in the switching elements of a MIN results in degradation of its permutation mapping capability. Research in this area has till now dealt with fault tolerant message routing from single source to

single destination [2,3]. On the contrary, in this article we propose an algorithm for routing of permutations in presence of stuck-at faults by performing multiple passes. For generalized faults, a combination of the method proposed in this article and the one developed in [2] may be adopted as long as the network possesses dynamic full access capability (DFA).

### 2. Permutation mapping in faulty networks

A switch, represented by an ordered pair  $\langle r, s \rangle$  where  $r$  and  $s$  represent the stage and the switch

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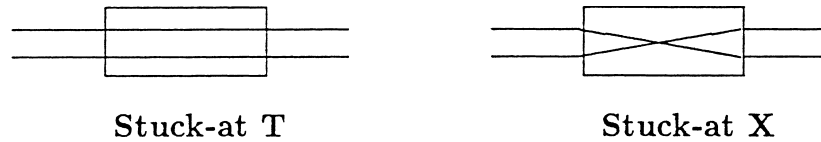


Fig. 1. Stuck-at-faults.

number, respectively, can be kept in either straight ( $T$ ) or exchange ( $X$ ) configuration. A faulty switch is assumed to be stuck at any one of these two configurations (see Fig. 1). Properties of the MIN utilized for designing the algorithm are as follows:

**Property 1 (Buddy property).** *A pair of switches in a stage are called output buddies if the outputs of both these switches go to the same pair of switches in the next stage. Again, a pair of switches in a stage are called input buddies if the inputs of both these switches come from the same pair of switches in the previous stage.*

**Property 2.** *It can be easily proved that under fault free condition, a submapping from some input processor  $i_1$  to an output processor  $\theta_1$  can always be achieved in two passes. In the first pass, all the switches in stage 0 to  $k$  are kept in their required configuration, i.e., those required to map  $i_1$  to  $\theta_1$ , while the remaining switches in stage  $(k + 1)$  to the last one along the route are set to  $T$ . In the second pass, the switches in stage 0 through  $k$  are kept in  $T$  while the rest are kept in their required configuration.*

### 2.1. In presence of stuck-at faults

Let *Fault set*  $F$  be the set of stuck-at faulty switches in the network. Multiple stuck-at faults in the network can be treated as a set of disjoint single faults if (i)  $F$  does not contain any switch from stage 0, (ii) for each of the switching elements in  $F$ , its input buddy is not in  $F$  and (iii) for each pair of switching elements  $S_i$  and  $S_j$  in  $F$  located in

stages  $i$  and  $j$ , respectively, such that  $i < j$ , neither  $S_i$  nor its input buddy is an element of the input tree of  $S_j$ , where input tree is the binary tree generated on the input side taking  $S_j$  as the root.

*Algorithm for fault tolerant routing:* Given a network and its fault set  $F$ , let  $l_{f,s}$  be the last stage in which a fault occurs. In Pass 1, for each switch in  $F$ , its input buddy,  $ib$ , is set to  $X$  and the corresponding output buddies in the previous stage,  $ob_1$  and  $ob_2$ , are set to a configuration opposite to one required for mapping  $\pi$  (or, *opp config*). All the other switches from stage 0 to  $l_{f,s} - 1$  are set to the configurations required for mapping  $\pi$  (or, *reqd config*), while those in the stages  $l_{f,s}$  to the last stage are set to  $T$ .

In Pass 2, for each faulty switch in  $F$ ,  $ob_1$  and  $ob_2$  are set to  $X$ . If the faulty switch is stuck-at- $X$ , then  $ib$  is set to *opp config*, otherwise  $ib$  is set to *reqd config*. All the other switches from stage 0 to  $l_{f,s} - 1$  are set to  $T$ , while those in the stages  $l_{f,s}$  to the last stage are set to *reqd config*.

### Algorithm Fault-tolerant-routing( ):

- Pass 1: For each switch in  $F$  set its  $ib$  to  $X$  and  $ob_1$  &  $ob_2$  to *opp config*.  
 Set other switches from 0 to  $(l_{f,s} - 1)$  to the *reqd config*.  
 Set other switches from  $l_{f,s}$  to the last stage to  $T$ .
- Pass 2: For each switch in  $F$  do  
 If (stuck-at  $T$ ) then set its  $ib$  to *reqd config*  
 else set its  $ib$  to *opp config*.  
 Set its  $ob_1$  and  $ob_2$  to  $X$ .

Set other switches from 0 to  $(l\_f\_s - 1)$  to  $T$ .

Set other switches from  $l\_f\_s$  to the last stage to  $reqd\ config$ .

**Example.** Consider a faulty  $8 \times 8$  Omega network shown in Fig. 2. The stuck-at  $T$  and stuck-at  $X$  faults in stage 1 (switches 1 and 2) are indicated by ‘ $T$ ’ and ‘ $X$ ’, respectively. Note that the input buddies of  $\langle 1, 1 \rangle$  and  $\langle 1, 2 \rangle$  are  $\langle 1, 0 \rangle$  and  $\langle 1, 3 \rangle$ , respectively and the corresponding output buddies are  $\langle 0, 0 \rangle$  and  $\langle 0, 2 \rangle$  and  $\langle 0, 1 \rangle$  and  $\langle 0, 3 \rangle$ , respectively. The settings shown inside each switch are those required for mapping

$$\pi = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 4 & 2 & 7 & 0 & 1 & 6 \end{pmatrix}.$$

The entries in the form ‘ $x/y$ ’,  $x, y \in \{X, T\}$ , shown below each switch indicate that the corresponding switch is set to  $x$  in Pass 1 and to  $y$  in Pass 2. The permutations realized in the two passes are

$$\pi_1 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 3 & 0 & 5 & 2 & 7 & 6 & 1 \end{pmatrix}$$

and

$$\pi_2 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 6 & 7 & 5 & 3 & 2 & 1 & 0 \end{pmatrix},$$

respectively, thereby effectively realizing  $\pi$ .

### 2.2. In presence of generalized faults

Here, we generalize the results obtained in the previous section to include the case of unusable switches. For a single pass mappable  $N \times N$  permutation, the  $N$  routes  $\Sigma = \{\sigma_1, \sigma_2, \dots, \sigma_N\}$  may be divided into three pairwise disjoint classes  $\Sigma_1 = \{\sigma_{i_1}, \sigma_{i_2}, \dots, \sigma_{i_M}\}$ ,  $\Sigma_2 = \{\sigma_{j_1}, \sigma_{j_2}, \dots, \sigma_{j_O}\}$ , and  $\Sigma_3 = \{\sigma_{k_1}, \sigma_{k_2}, \dots, \sigma_{k_{N-M-O}}\}$ , such that  $\Sigma_1 \cup \Sigma_2 \cup \Sigma_3 = \Sigma$ . Here  $\Sigma_1$  are the paths unaffected by the faults,  $\Sigma_2$  are the paths affected by stuck-at faults and  $\Sigma_3$  are the paths that are blocked by the unusable faulty switches. Note that the paths of  $\Sigma_1$

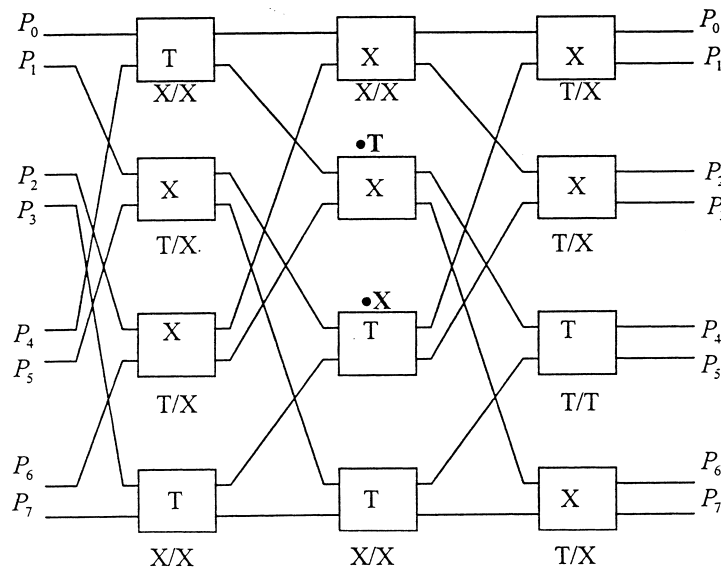


Fig. 2. Setting with in faulty  $8 \times 8$  omega network.

can be mapped in a single pass and the paths of  $\Sigma_2$  can be mapped in two passes using the algorithm described here. Paths of  $\Sigma_3$  can be mapped utilizing the methodology described in [2], as long as the MIN possesses DFA property.

Consider Fig. 2 again for illustration. Let  $\langle 1, 1 \rangle$  be stuck-at  $T$  and  $\langle 1, 2 \rangle$  be unusable. As earlier, the permutation to be mapped is

$$\pi = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 4 & 2 & 7 & 0 & 1 & 6 \end{pmatrix}.$$

Here

$$\Sigma_1 \equiv \begin{pmatrix} 0 & 1 & 6 & 7 \\ 3 & 5 & 1 & 6 \end{pmatrix}, \Sigma_2 \equiv \begin{pmatrix} 2 & 4 \\ 4 & 7 \end{pmatrix}, \text{ and}$$

$$\Sigma_3 \equiv \begin{pmatrix} 3 & 5 \\ 2 & 0 \end{pmatrix}.$$

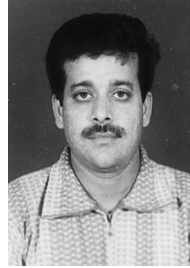
Obviously,  $\Sigma_1$  can be mapped in a single pass and  $\Sigma_2$  can be mapped in two passes. Each path in  $\Sigma_3$ ,  $3 \rightarrow 2$  and  $5 \rightarrow 0$ , may be realized in two passes as  $3 \rightarrow 6 \rightarrow 2$  and  $5 \rightarrow 6 \rightarrow 0$  respectively. Note that none of these routes pass through the faulty switches.

### 3. Conclusion

The scheme for fault tolerant mapping of permutations designed in this article can be used for any MIN possessing buddy property as long as the network retains DFA property. The routing algorithm designed here can also be easily extended to map multiple pass permutations.

### References

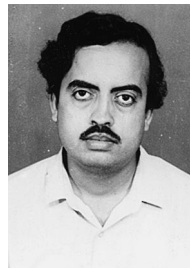
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