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A NOTE ON ZAREMBKA'S DUAL ECONOMY MODEL

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A vast literature has grown on dual economy models including attempts to generalise some of the earlier models in the area. The major purpose of the present paper is to point out certain formal difficulties associated with one such attempt, viz., that by Zarembka. In particular, the paper seeks to represent a slightly generalised version of Zarembka's model in an alternative framework. The advantage of this framework is that it enables one not only to highlight the 'error' in Zarembka's paper but to obtain the correct intertemporal and asymptotic solutions to the model. In the process it also helps generalise some of the results of Dixit's model to the case involving non-zero price elasticity of food demand. (JEL : Q40)

1. INTRODUCTION

A vast literature has grown on dual economy models. Some of these models are called classical and some, neoclassical[1]. One of the earliest models is a neoclassical model developed by Jorgenson (1961). Subsequently, attempts have been made to generalise the neoclassical results. One such attempt is by Zarembka (1970)[2]. In fact, Zarembka's model is quite often referred to as an important contribution in this field. The main purpose of the present paper is to point out some major 'error' in Zarembka's analysis of a neoclassical dual economy. In particular, the paper seeks to represent a slightly generalised version of Zarembka's model in an alternative framework. The advantage of this framework is that it enables one not only to highlight the 'error' in Zarembka's paper but to obtain the correct intertemporal and asymptotic solutions to the model.

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1. A survey of these models with some formalisations and extensions can be found in Dixit (1973).

2. Other attempts are Marino (1975) and Amano (1980). Apart from these typical dual economy models, there are some models which combine many of these dualistic features with a Keynes-type demand constrained industrial sector (Cordoso, 1981; Rakshit, 1982; Taylor, 1982). Rao (1992) develops a general framework to present these models.

Before developing the alternative framework let us note the main result of the neoclassical model. Jorgenson (1961) argues that when the per capita food production of the purely agrarian economy exceeds a certain critical level - i.e., the country satisfies the so-called 'viability' or 'productivity' condition - the supply of labour and food are sufficient to both initiate as well as sustain the growth of the industrial sector and that the proportion of labour in industry increases monotonically. Jorgenson derives these results on the assumption that when the industrial growth begins, per capita food demand remains constant, i.e., the price and income elasticities of demand for food by labourers are zero. Going beyond the asymptotic results of Jorgenson, Dixit (1970, 1973) derives the qualitative features of the development of a dual economy in finite time and finds that even when the income elasticity of demand for food is positive, Jorgenson's 'viability' condition remains valid. Zarembka (1970) goes for a further generalisation and introduces both non-zero income and price elasticities of demand for food. With the same 'viability' condition, he then tries to characterise the behaviour of a neoclassical dual economy, given a *constant* industrial capital output ratio. According to Dixit (1973, p. 348) and Marino (1975, p. 435), this amounts to analysing only the asymptotic characteristics of such an economy. However, as we shall show in this paper, Zarembka's results are not correct. In other words, what Zarembka derives as the "solution" of the model is in fact *neither* a solution which obtains at any finite time *nor* a situation to which the economy moves asymptotically.

The plan of the paper is as follows. Section 2 presents Zarembka's model (with a minor generalisation) in an alternative framework. Section 3 derives the intertemporal and the asymptotic behaviour of the model and in the process demonstrates why Zarembka's "solution" is not correct. Section 4 summarises the main findings of the paper.

2. THE MODEL

The economy has two sectors - agriculture (sector 1) and industry (sector 2), used synonymously with primary and secondary sectors, respectively. The population (L) is assumed to grow at an exogenous rate[3] :

$$L = e^{vt} \tag{a}$$

where t is time and the initial population is taken to be unity. Population and labour will be used interchangeably (Zarembka, 1970, p. 108). Full employment is assumed. Let L_i and l_i be the absolute quantity and the proportion of labour engaged in sector i. Then we have

$$l_1 + l_2 = 1 \tag{1}$$

The production of agricultural output (Y_1), i.e., food, requires a single variable input, labour (with decreasing returns represented by the parameter β_1),), but experiences technological progress at an exogenously given rate b_1 . The per capita

3. Our notations will differ from those of Zarembka (1970) in many cases.

food production can therefore be written as[4]

$$Y_1/L = e^{\theta t}\, l_1^{\beta_1}, \qquad o < \beta_1 < 1 \qquad (2)$$

where the choice of units enables us to drop any possible constant term in the production function and

$$\theta = b_1 - v(1-\beta_1)$$

Note that the 'viability' condition mentioned in Section 1 is nothing but the requirement that θ is positive. (See Jorgenson, 1961, p. 342; Dixit, 1973, p. 332; Zarembka, 1970, p. 120).

The insustrial production function, a Cobb-Douglas one, relates output (Y_2) to inputs of capital (K_2) and labour (L_2) and exhibits technological progress :

$$Y_2 = K_2^{1-\beta_2}\,(e^{b_2 t} L_2)^{\beta_2}, \qquad 0 < \beta_2 < 1, \qquad \bullet \qquad (b)$$

Following the tradition of the literature on growth theory the technological improvement factor appears multiplicatively with labour[5]; $e^{b_2 t} L_2$ may be called (industrial) labour measured in *efficiency units* (Dixit, 1973, p. 337). Further, following Amano (1980) we introduce two inputs ratios involving labour in efficiency units - capital-labour ratio in industry (k_2) and industrial capital-population ratio (z_2)[6] :

$$k_2\, l_2 = z_2, \qquad o < l_2 \leq 1 \qquad (3)$$

where

$$k_2 = K_2/(e^{b_2 t} L_2) \quad \text{and} \quad z_2 = K_2/(e^{b_2 t} L)$$

It is assumed that the rural per capita income in terms of food (y_1) is the average agricultural product while the urban (per head) wage rate in terms of industrial goods (w_2) is the marginal product of labour in industry which is derived below in terms of k_2 :

$$y_1 = \frac{Y_1}{L_1} = \frac{Y_1/L}{l_1} \qquad (4)$$

4. This follows from (a) and the assumed production function :

$$Y_1 = e^{b_1 t} L_1^{\beta_1}$$

5. A change of notations here : $b_2\ \beta_2$ here equals Zarembka's b_2 (Zarembka, 1970, p. 110)

6. Note that z_2 is a datum at any t. We shall see later that the given value of z_2 determines values of endogenous variables k_2, l_2 etc. at a given t

$$w_2 = \delta Y_2/\delta L_2 = \beta_2 e^{b_2 t} k_2^{1-\beta_2} \tag{5}$$

Migration is assumed to ensure the rural per capita income remains a constant fraction (μ) of the urban wage rate, allowing for some income gap between the two sectors (Zarembka, 1970, p. 111) :

$$y_1 = \mu w_2/p \qquad 0 < \mu \leq 1 \tag{6}$$

where p is the price of food relative to the price of industrial good.

The per capita food demand in both sectors (denoted d_i for sector i) are assumed to be identical, each having constant (real) income elasticity (ε) and (relative) price elasticity (η). The per capita food demand for the economy as a whole (d) is, therefore, a weighted average of such sectoral demands :

$$d = l_1 d_1 + l_2 d_2 = cp^{-\eta} y_1^{\varepsilon} (l_1 + \mu^{-\varepsilon} l_2)$$

where c is a constant and $0 \leqq \varepsilon, \eta \leqq 1$. The equilibrium in the food market requires :

$$Y_1/L = c p^{-\eta} y_1^{\varepsilon} (l_1 + \mu^{-\varepsilon} l_2) \tag{7}$$

This is the same as the eq. (9) of Zarembka except for the bracketed term on the RHS[7].

Short-run Equilibrium

The system of equations (1) - (7) provide 7 equations in an equal number of unknowns, namely l_1, l_2, Y_1/L, k_2, y_1, w_2 and p. To show the determination of short-run equilibrium of the model, the above system is reduced to one of two equations in two endogenous variables, l_2 and k_2 - eq. (3) given earlier and eq. (8) derived below.

First, using (2) and (4) - (6), we rewrite (7) as follows[8] :

$$l_1^{\lambda} k_2^{\eta(1-\beta_2)} = \lambda e^{-\alpha t} (\mu^{\varepsilon} l_1 + l_2) \tag{8}$$

or, since $l_1 = 1 - l_2$ by (1) hence $\mu^{\varepsilon} l_1 + l_2 = \mu^{\varepsilon} + (1 - \mu^{\varepsilon} l_2$, this equation may be rewritten once more as :

7. Zarembka (1970, p. 112) has ignored the term on the ground that the agricultural population L_1 would be much larger than the industrial population L_2 so that $L_1 + \mu^{-\varepsilon} L_2 + L$, i.e., $l_1 + \mu^{-\varepsilon} l_2 = 1$. However, as we shall see in Section 3, once the economy moves through time, L_2 would grow faster than L_1 and $l_2 = L_2/L$ and would approach unity. Thus Zarembka's assumption is not tenable in the long-run. Of course, the term equals 1, if either $\mu = 1$ or $\varepsilon = 0$.

8. In the eq. (7) substitute (6) for p, (4) for y_1, (2) for Y_1/L and (5) for w_2. This derivation is shown in the Appendix.

$$k_2^{\eta(1-\beta_2)} = A\ e^{-\alpha t}\ u(l_2) \tag{8}$$

where

$$A = o\mu^{-(\varepsilon+\eta)}\beta_2^{-\eta} \qquad > 0,$$

$$\lambda = \beta_1 + (1-\beta_1)(\varepsilon+\eta) \qquad > 0,$$

$$\alpha = \eta b_2 + \theta\{1-(\varepsilon+\eta)\}, \text{ and}$$

$$u(l_2) = [\mu^{\varepsilon} + (1-\mu^{\varepsilon})\, l_2]/l_1^{\lambda}, \quad (l_1 = 1 - l_2).$$

We assume that $\alpha > 0$ – a sufficient condition guaranteeing this is $\theta > 0$ and $\varepsilon + \eta < 1$.

The two equations (3) and (8) determine two variables k_2 and l_2, given t and z_2. This can be easily shown graphically. We just give here an outline of this diagram. On a $k_2 - l_2$ plane, (3) gives downward sloping curve (with an asymptote to the k_2-axis and with k_2 tending to the value z_2 when $l_2 \rightarrow 1$).

On the other hand, (8) gives an upward-rising curve showing that k_2 has a finite value at $l_2 = 0$ but tends to infinity when l_2 tends to one[9]. The two curves intersect at positive values of l_2 and k_2.

3. INTERTEMPORAL BEHAVIOUR AND LONG-RUN EQUILIBRIUM OF THE MODEL

Let us first describe how z_2 changes over time. The profit in the industrial sector is $K_2\ (\delta Y_2/\delta K_2)$ which is assumed to be reinvested entirely in this sector. Let the (proportional) rate of growth of a variable, say X, be denoted by $\hat{X}$ (*i.e.*, $\hat{X} = (dX/dt)/X$). Assuming depreciation to be a constant fraction (δ_2) of the existing capital stock, we have

$$\hat{K}_2 = \frac{1}{K_2}\frac{dK_2}{dt} = \frac{\delta Y_2}{\delta K_2} - \delta_2$$

$$= (1-\beta_2)\ k_2^{-\beta_2} - \delta_2 \qquad \text{(from } (b) \text{ and definition of } k_2) \tag{9}$$

9. As l_2 rises (and hence l_1 falls), $u(l_2)$ rises so that k_2 has to rise to satisfy the equation. Further, as $l_2 \rightarrow 1$, (i.e., $l_1 \rightarrow 0$), $u(l_2) \rightarrow \alpha$ and hence $k_2 \rightarrow \alpha$, Again, as $l_2 \rightarrow 0$, (*i.e.*, $l_1 \rightarrow 1$), $u(l_2) \rightarrow \mu^{\varepsilon}$ and hence k_2 tends to a finite value.

Further, from (3), we have

$$\hat{z}_2 = \hat{K}_2 - (b_2 + \hat{L})$$
$$= (1-\beta_2) k_2^{-\beta_2} - \delta_2 - (b_2 + \nu) \text{ [by (a) and (9)]}$$
$$= (1-\beta_2) k_2^{-\beta_2} - \rho \tag{10}$$

where

$$\rho = \delta_2 + b_2 + \nu$$

Now, first observe that

$$\hat{z}_2 \gtreqless 0 \quad \text{according as} \quad k_2 \lesseqgtr k_2^* \tag{11}$$

where k_2^* is given by

$$(k_2^*)^{\beta_2} = (1-\beta_2)/\rho$$

Thus a horizontal line $k_2^* S$, drawn at a height of $k_2 = k_2^*$, in the $k_2 - l_2$ plane in Figure 1 divides the first quadrant into two regions $- z_2$ will be falling or rising depending on whether the existing k_2 is in the region above or below this line.

Next, to locate the regions in Figure 1 in which k_2 or l_2 will be falling or rising, consider eqs. (3) and (8). Taking logarithms on both sides of these equations and then differentiating with respect to t, we get two equations to solve for rates of growth of k_2 and l_2 and the solutions are as follows (see the Appendix for derivation) :

$$\{\eta (1-\beta_2) + x\} \quad \hat{l}_2 = \eta (1-\beta_2) \hat{z}_2 + \alpha \tag{12}$$

$$\{\eta (1-\beta_2) + x\} \quad \hat{k}_2 = x \hat{z}_2 - \alpha \tag{13}$$

where x, a function of l_2 is given by

$$x = \lambda \frac{l_2}{1-l_2} + \frac{(1-\mu^{\varepsilon}) l_2}{\mu^{\varepsilon} + (1-\mu^{\varepsilon}) l_2} > 0, \tag{14}$$

and hence,

$$x' = \frac{dx}{dl_2} = \lambda \frac{1}{(1-l_2)^2} + \frac{\mu^{\varepsilon}(1-\mu^{\varepsilon})}{\{\mu^{\varepsilon} + (1-\mu^{\varepsilon}) l_2\}^2} > 0$$

Note that $x \to 0$ or α according as $l_2 \to 0$ or 1.

Since the bracketed term on the LHS of each of (12) and (13) is positive, the sign of $\hat{l}_2$ or $\hat{k}_2$ will be the same as that of the RHS of the corresponding equation. Consider (12) first. Obviously, $\hat{l}_2 > 0$ whenever $\hat{z}_2 \geq 0$. Further, in view of (10), the RHS of (1) and hence $\hat{l}_2$ is positive, if $\alpha \geq \eta\ (1 - \beta_2)\rho$. If, on the other hand, $\alpha < \eta\ (1 - \beta_2)\rho$. there is a value of k_2, say $\bar{k}_2$, given by

$$\bar{k}_2^{\beta} = [\eta(1-\beta_2)^2/\{\eta(1-\beta_2)\rho - \alpha\}] \tag{15}$$

such that

$$\hat{l}_2 \gtreqless 0 \text{ according as } k_2 \lesseqgtr \bar{k}_2$$

Obviously, $\bar{k}_2$ is higher than k_2^* defined in (11). This case is shown in Figure 1 where $\bar{k}_2 R$ is the horizontal line (drawn at a value of $\bar{k}_2$) on which $\hat{l}_2 = 0$.

Consider now (13) according to which $\hat{k}_2 < 0$ whenever $\hat{z}_2 < 0$. Thus $\hat{k}_2$ could be zero or positive only below the horizontal line $k_2^* S$. Set the RHS of (13) equal to zero to get all configurations of k_2, l_2 at which $\hat{k}_2 = 0$:

$$x\ \hat{z}_2 = \alpha \tag{16}$$

Since x is an increasing function of l_2 (by 14)) and $\hat{z}_2$ is a decreasing function of k_2 (by 10)), the e.g. (16) may be represented by an upward-rising curve below the line k_2^* S in Figure 1. The other features of this curve are that as $l_2 \to 0$, $x \to 0$ and hence $\hat{z}_2 \to \alpha$ which is possible only when $k_2 \to 0$, while as $l_2 \to 1$, $x \to \alpha$ and hence $\hat{z}_2 \to 0$ which occurs only when $k_2 \to k_2^*$. Thus (16) gives an upward-rising curve which starts at the origin and ends at the point *S*. *OeFs* is one such curve in Figure 1 and $\hat{k}_2$ is positive and negative in regions below and above this curve, respectively.

Suppose now the economy starts from a position like *c*. Since at this point $\hat{l}_2 > 0$ and $\hat{k}_2 < 0$, the economy will move in the south-east direction. One possibility is that the economy moves along a path like cd'S, i.e., k_2 will be falling and l_2 will be rising all the time approaching asymptotically k_2^* and 1, respectively.

The other possibility is that the economy moves along a path like cde. The point e has $\hat{k}_2 = 0$ but $\hat{l}_2 > 0$ so that the economy will not stay here but move to the right whence $\hat{k}_2$ would be positive. Thus from here the economy may move along a path like efS.

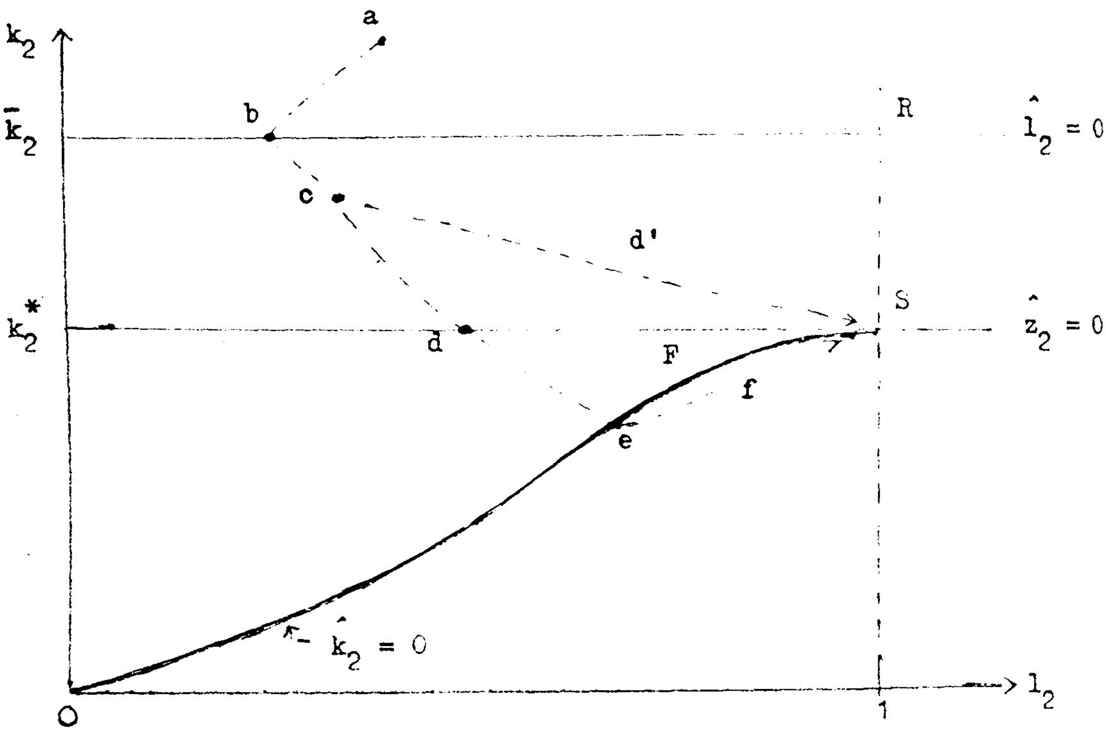

Figure 1 : Overtime Movements of k_2, l_2, l_1

Another possibility arises regarding the initial point. As we have seen earlier, if $\alpha \geq \eta\ (1-\beta_2)\rho$, $l_2 > 0$ if at any point in the diagram, (the horizontal line $\bar{k}_2\ R$ does not exist), in which case two possible paths are those indicated above. If, however, $\alpha < \eta\ (1-\beta_2)\rho$, then $\hat{l}_2 < 0$ above the line $\bar{k}_2\ R$. Thus, if the economy starts from a point like a, k_2 will be falling as before, but initially l_2 will also be falling so that it moves along a path like ab. At the point b, the fall in l_2 is arrested, but the economy moves below this line since $\hat{k}_2 < 0$. Afterwards it moves towards S following any of the two alternative paths indicated before. Along *ab*, however, $L_2 < \nu$ and the proportion of labour employed in industry is falling. This is a possibility not pointed out by Zarembka and not present in Dixit's model (1973, pp. 343-348) where the value of η is zero.

The upshot of the foregoing analysis is that irrespective of the alternative initial situations and the alternative time paths, the economy will approach the point S - the point at which $l_2 = 1$ and $k_2 = k_2^*$, i.e., a situation where L_2 is growing at the same rate as population, and K_2 at the rate $b_2 + \nu$. We shall discuss more about this later, but let us first examine Zarembka's solution.

Zarembka's solution of the model (given in his equations (14) - (18) on (pp. 114-5) is derived on the basis of the assumption that the rate of growth of industrial employment ($\hat{L}_2$) could be arbitrarily fixed at a *given* value, in which case, Zarembka argues that the industrial capital-output ratio will approach asymptotically a value given by

$$K_2 / Y_2 = (1 - \beta_2)/(b_2 + \delta_2 + \hat{L}_2) \quad (17)$$

Zarembka's solution can now be easily obtained using this value of the capital output ratio. It is easy to check that at this value of K_2 / Y_2, $\hat{k}_2 = 0$[10] i.e., one is on some point on the curve OeFS where the point in question depends on the given value of $\hat{L}_2$. In particular, there is no reason why the point should be identically the same as *S*. Converting now the eq. (8) into rates of growth, and noting that in Zarembka's model $\mu^{\epsilon} l_1 + l_2 = \mu^{\epsilon}$, (see our footnotes 8 and 7), we have $\lambda \hat{l}_1 + \eta (1 - \beta_2) \hat{k}_2 = - \alpha$, or since $\hat{k}_2 = 0$,we have

$$\hat{l}_1 = - \frac{\alpha}{\lambda} \quad (18)$$

or, since $\hat{l}_1 = \hat{L}_1 - \hat{L}$ and $\hat{L} = \nu$, we have

$$\hat{L}_1 = \nu - \frac{\alpha}{\lambda} \quad (18.1)$$

Further, from (18) and (1) we get $\hat{l}_2 = - (l_1/l_2) \hat{l}_1$ and

$$\hat{L}_2 = \hat{L} + \frac{l_1}{l_2} \hat{l}_1 = \nu + \frac{L_1}{L_2} \frac{\alpha}{\lambda} \quad (18.2)$$

From two production functions

$$\hat{Y}_1 = b_1 + \beta_1 \hat{L}_1 , \quad (18.3)$$

10. Industrial production function (b) can be written as $Y_2 = K_2 k_2^{-\beta_2}$, so that at the asymptotic value of the capital-output ratio given in (17),

$$(1 - \beta_2) = (b_2 + \delta_2 = \hat{L}_2) (K_2/Y_2) = (b_2 + \delta_2 + \hat{L}_2) k_2^{\beta_2}$$

Now from the definition of k_2 given in (3),

$\hat{k}_2 = \hat{K}_2 - (b_2 + \hat{L}_2) = (1 - \beta_2) k_2^{-\beta_2} - \delta_2 - (b_2 + \hat{L}_2)$, by (9) and hence $\hat{k}_2 = 0$, in view of the preceding relation.

$$\hat{Y}_2 = (1-\beta_2)\hat{K}_2 + \beta_2(b_2 + \hat{L}_2) = \hat{K}_2 = b_2 + \hat{L}_2, \tag{18.4}$$

(since $\hat{k}_2 = \hat{K}_2 - (b_2 + \hat{L}_2) = 0$). Finally, using (2), (4), (5) and the condition $\hat{k}_2 = 0$, we get from (6).

$$\hat{p} = \hat{w}_2 - \hat{y}_1 = b_2 - \{\theta + (1-\beta_1)\frac{\alpha}{\lambda}\}$$

$$= b_2 - \frac{(1-\beta_1)\,\eta\, b_2 + \theta}{\lambda} \tag{18.5}$$

where use has been made of the values of θ and α given in (2) and (8).

Equations (18.1) - (18.5) are the same as eqs. (14) - (18) of Zarembka (1970, p. 114) expressed in different notations. It should be amply clear that Zarembka's assumption of a constant $\hat{L}_2$ is not tenable. As we have observed earlier, in this model l_1 falls and l_2 rises steadily (if not initially, but surely after sometime) so that L_1 / L_2 goes on falling over time. Hence $\hat{L}_2$, as derived in (18.2), cannot remain constant unless, of course, the economy is at the point S where $l_2 = 1$ and $\hat{L}_2 = \nu$. Thus eqs. (18.1) - (18.5) fail to give any "solution" of the model. These equations describe neither the asymptotic nor the intertemporal behaviour of the economy. For, as argued before, starting from a position like c the economy moves along either the path cd'S or the path cdefS in Figure 1. Therefore, a point like e which satisfies (18.1) - (18.5) and hence represents Zarembka's "solution", is just a point where the economy might be located at one particular moment of time only.

Before concluding this section we may refer to the analysis of the neoclassical model by Dixit [4, pp. 343-348] who characterises the overtime behaviour of such an economy with $\eta = 0$ but $\varepsilon > 0$ It may be easily shown that most of his results remain unaltered in the present model with $\eta > 0$.

4. CONCLUDING OBSERVATIONS

In this paper we have tried to present a neoclassical dual economy model with non-zero price and income elasticities of demand for food, i.e., Zarembka's model with a minor generalisation in an alternative framework - a framework which helps us to derive very easily the intertemporal and asymptotic properties of the model. Our findings may be summarised as follows :

(i) What Zarembka has derived as the "solution" of the model describes neither the asymptotic behaviour of the model, nor its behaviour in any finite time, but just its behaviour in one particular point of time.

(ii) The model admits of one possibility which does not arise in Jorgenson - Dixit models (with zero price elasticity of demand for food) and which has not been pointed out by Zarembka. With reasonable values of various parameters it is possible that the rate of growth of industrial employment ($\hat{L}_2$) may initially be lower than that of population so that the proportion of labour employed in industry (l_2) may fall initially for some time.

(iii) The major results of the Jorgenson-Dixit models, however, remain unaltered. For instance, as we have seen earlier, starting from any point in Figure 1 the economy will always move towards the point S and to start at some such point, the economy needs only some capital from outside. Thus, even a small positive initial capital gives rise to sustained growth (Jorgenson, 1961, p. 326). Moreover, l_2 will go on rising, (if not from the beginning then surely after sometime) and will approach 1 asymptotically (Dixit, 1973, p. 343). Again, industrial capital-labour ratio (k_2) falls either throughout the entire course of development or at least for a long time and since $\hat{K}_2$ is inversely related to k_2 (*vide* the e.q. (9)), during this period industrial capital experiences accelerated growth (Dixit, 1973, p. 345).

APPENDIX

We show here derivation of a couple of equations used in the text.

Derivation of Equation (8)'

Eq. (7) can be rewritten as follows :

$$\frac{Y_1}{L} = cp^{-\eta}\ y_1^{\varepsilon}\ (l_1 + \mu^{-\varepsilon}\ l_2)$$

$$= c\left(\mu\ \frac{w_2}{y_1}\right)^{-\eta}\ y_1^{\varepsilon}\ (l_1 + \mu^{-\varepsilon}\ l_2) \qquad \text{by (6)}$$

$$= c_{\mu}^{-(\varepsilon+\eta)}\ w_2^{-\eta}\ y_1^{\varepsilon+\eta}\ (\mu^{\varepsilon}\ l_1 + l_2)$$

$$= c_{\mu}^{-(\varepsilon+\eta)}\ w_2^{-\eta}\left(\frac{Y_1}{L}\right)^{\varepsilon+\eta} l_1^{-(\varepsilon+\eta)}\ (\mu^{\varepsilon}\ l_1 + l_2), \qquad \text{by (4)}$$

Hence, transferring $(Y_1/L)^{\varepsilon+\eta}$ to the LHS and then using (2) to eliminate (Y_1/L), we get

$$e^{\theta\{1-(\varepsilon+\eta)\}t}\ l_1^{\beta_1\{1-(\varepsilon+\eta)\}}$$

$$= c_{\mu}^{-(\varepsilon+\eta)} w_2^{-\eta} l_1^{-(\varepsilon+\eta)} (\mu^{\varepsilon} l_1 + l_2)$$

$$= c_{\mu}^{-(\varepsilon+\eta)} (\beta_2 e^{b_2 t} k_2^{1-\beta_2})^{-\eta} l_1^{-(\varepsilon+\eta)} (\mu^{\varepsilon} l_1 + l_2), \text{ by (5)}$$

which can be simplified to yield :

$$e^{\alpha t} l_1^{\lambda} = A k_2^{-\eta(1-\beta_2)} (\mu^{\varepsilon} l_1 + l_2)$$

where A, λ, α are defined in the text in (8). The above relation then yields (8)'.

Derivations of Eqs. (12) - (14)

Taking logarithms on both sides of the eq. (8) we get

$$\lambda \log(1 - l_2) - \log\{\mu^{\varepsilon} + (1 - \mu^{\varepsilon}) l_2\} + \eta(1 - \beta_2) \log k_2 = \log A - \alpha t$$

Differentiating now w.r.t. time we have

$$-\frac{\lambda}{1 - l_2} \frac{d l_2}{d t} - \frac{1 - \mu^{\varepsilon}}{\mu^{\varepsilon} + (1 - \mu^{\varepsilon}) l_2} \frac{dl_2}{dt} + \eta(1 - \beta_2) \hat{k}_2 = -\alpha$$

or, using $\hat{l}_2 = (dl_2/dt)/l_2$, we get

$$\eta(1 - \beta_2) \hat{k}_2 - x \hat{l}_2 = -\alpha \quad \text{(i)}$$

where

$$x = \frac{\lambda l_2}{1 - l_2} + \frac{(1 - \mu^{\varepsilon}) l_2}{\mu^{\varepsilon} + (1 - \mu^{\varepsilon}) l_2}$$

Further, converting eq. (3) into rates of growth, we get

$$\hat{k}_2 + \hat{l}_2 = \hat{z}_2 \quad \text{(ii)}$$

Solving now (i) and (ii) for $\hat{l}_2$ and $\hat{k}_2$, we get (12) and (13).

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