Some aspects of shock-like nonlinear acoustic waves in magnetized dusty plasma

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A nonlinear wave equation is derived in a magnetized plasma, comprising dust charged grains, to study the existence and propagation of solitary wave. The Sagdeev pseudopotential technique is applied to obtain the nonlinear wave equation in the dusty plasma. In contrast to the usual technique, the tanh-method, or to say the hyperbolic-method, to the nonlinear wave equation is used to derive the formation of soliton and its shock-like nature, as well. The nonlinear waves in dusty plasma, which could be of a greater interest in astrophysical plasmas, are also presented here to reflect significant advanced information on solitons.

I. INTRODUCTION

The plasma coexisting with finite micron-sized massive dust charged particles is quite common in the universe. During the last few years, the small amplitude wave propagation in the dusty plasma has received much attention to discover the salient features. The dusty plasma exists in astrophysical bodies and space environments, such as cometary tails, planetary ring systems, interstellar and circumstellar clouds, and asteroid zones,[1–7] as well as in laboratory plasmas, e.g., in tokamak and low-temperature glow discharges. The ubiquitous nature of the dusty plasma and its importance in the plasma environment have spurred many researchers to study the new features arising due to the addition of dust-charged grains. Enormous interest has been shown in the nonlinear waves, which have been studied extensively in various dusty plasma environments, and many of these problems are expected to be related with satellite observations. Normally the low-temperature plasma sustains negatively charged dust formed by the attachment of the electrons to the dust grains, while radiation, photoionization and field emission might yield the positive dust charged grains. The dust charges fluctuate for two reasons, one of which is turbulence and other is the temporal variation in the surrounding plasma properties. The phenomena can be significant when the plasma fluctuations are large and more rapid than the charging time of the dust grain, which is generally found in astrophysical plasmas. The dust charging phenomena could be modeled as a capacitor charging process and, accordingly, one can estimate the dust charging time scale (τ_c) and the maximum charged (q_d) on it, which are given by $\tau_c \sim \omega_{pi}(\lambda_D/a)$ and $q_d \sim C\phi_{f0}$ where ω_{pi} is the ion plasma frequency, λ_D is the Debye length, "a" is the size of the dust grains, C is the capacitance of the dust surface and ϕ_{f0} is the equilibrium floating potential of the dust surface. If we estimate the relative scaling of the wave time scale (τ_ω) and the dust charging time scale for a given plasma, we find

$$\frac{\tau_\omega}{\tau_c} \sim p\left(\frac{a}{\lambda_{Di}}\right)\frac{1}{k\lambda_{Di}}, \quad \text{for } k^2\lambda_{Di}^2 \ll 1, \tag{1}$$

where $p=[(m_d/m_i)(n_{i0}/n_{d0}Z_d)(1/Z_d)]^{1/2}$ and λ_{Di} is defined as the ion Debye length. It is to be noted that the above scaling has been derived for the dust acoustic wave (DAW), which is a natural normal sound wave in plasma consisting of dust charged grains. From the general linear charging equation one can see that for $\tau_\omega \gg \tau_c$ (which requires $p \gg 1$) the effect of the dust charge fluctuation could be nominal within the quasineutrality limit of plasma fluctuations, i.e., $(n_e/n_{e0} \sim n_i/n_{i0})$; because of which a constant dust charge model could be justified.[8,9] So under certain approximations, the plasma consisting of dust charged grains can be regarded as a multicomponent plasma. In the last few decades various aspects of linear and nonlinear waves have been studied extensively in generalized multicomponent plasmas.[10–15] However, interest in the nonlinear wave phenomena, augmented through the Korteweg–de Vries (KdV) and Sagdeev potential equations, has grown in parallel to relate the acoustic wave in laboratory and space plasmas. Das and Tagare[14] have studied the generalized multicomponent plasma, and the simplified plasma model with negative ions have been extensively studied by many authors,[13,16] which could have direct impact in dusty plasma.[17] Rao *et al.*[17] concluded by saying that the dust grains, in an acoustic wave, can be treated as similar to the multicomponent plasma with negative ions. In fact, the theoretical observations of Rao *et al.*[17] encouraged the experimental observations[18] in dusty plasma and confirmed the theoretical observations of the DAW. Later, Verheest,[19] Mamun *et al.*,[20] Ma and Liu,[21] and

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Ma *et al.*[22] have extended the dusty plasma model with the dust charge fluctuation to show the characteristic behavior of dust acoustic wave and found the rarefactive soliton features similar to those observed in multicomponent plasmas. Further extensions have been made to show the propagation of dust-acoustic waves in magnetized plasma.[23,24] Mamun and his collaborators[25,26] also investigated the formation of dust acoustic solitary waves in magnetized dusty plasma by using the reductive perturbation technique. Now if one considers the constituents of electrons and ions to be Boltzmannian, playing a neutrality background to the dusty plasma, then the dynamics of the dust-acoustic mode might reveal some new features. Based on this intuition, we considered an ideal plasma in space environments with the dust charged grains, and aimed to revisit the soliton features in dusty plasma embedded in an applied magnetic field. First of all, we derive the multidimensional nonlinear wave equation in a plasma in the presence of a homogeneous magnetic field with a view to study the various forms of the nonlinear wave. Further, the derivation is extended to the higher-order nonlinear effects wherein, in each case, the wave equation has been solved by employing a proposed method, called tanh-method[27–29] or hyperbolic-method to yield the new findings on solitons.

II. BASIC EQUATIONS AND DERIVATION OF NONLINEAR WAVE EQUATION

To study the nonlinear wave phenomena, we have considered a homogeneous magnetized dusty plasma consisting of electrons, singly charged positive ions, and negatively charged micron-sized massive dust grains. The basic equations governing the plasma dynamics are the equations of continuity and motion. The electrons and ions are taken to be Boltzmannian defined through their densities as

$$n_k = n_{ko} \exp\left(-\frac{q_k \phi}{T_k}\right), \tag{2}$$

with $k = e, i$, respectively, for the electrons and ions and $q_e = -e$ and $q_i = e$. The equations governing the dust dynamics, under the fluid descriptions, are written in the following normalized equations:[30,31]

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_x) = 0, \tag{3}$$

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} = \frac{1}{\alpha^2}\frac{\partial \phi}{\partial x} - \frac{v_y \sin\theta}{\alpha}, \tag{4}$$

$$\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} = \frac{v_x}{\alpha}\sin\theta - \frac{v_z \cos\theta}{\alpha}, \tag{5}$$

$$\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} = -\frac{v_y \cos\theta}{\alpha}. \tag{6}$$

The applied magnetic field is at an angle θ with the x axis. ϕ is the normalized electrostatic potential linked to the charges through the Poisson equation, given as

$$\frac{\lambda_d^2}{\rho^2}\frac{\partial^2 \phi}{\partial x^2} = n_d + \delta_2 n_e - \delta_1 n_i, \tag{7}$$

where n_d is the density of dust particle moving with the velocity $v_d(v_x, v_y, v_z)$ normalized by c_d. The space, x and time, t are, respectively, normalized by $\rho(=c_d/\omega_d)$ and $(\alpha\omega_d)^{-1}$ with $c_d = \sqrt{T_e/m_i}$ being the ion-acoustic speed, and $\omega_d = eH_0/cm_d$ is the gyrofrequency. Following notations $\alpha^2 = (\delta_1 - \delta_2)/(\gamma\delta_1 + \delta_2)$ with $\delta_1 = n_{io}/n_{do}$, $\delta_2 = n_{eo}/n_{do}$, $\delta_1 - \delta_2 = 1$, $\gamma = T_e/T_i$ are also used in the basic equations. In order to use the quasipotential analysis,[32,33] the dependent variables are made to be the functions of a single independent variable $\xi = \beta(x - Mt)$; where M defines the Mach number, and because of which, the basic Eqs. (3)–(7) are then reduced to the following form:

$$-\beta M \frac{dn_d}{d\xi} + \beta \frac{d}{d\xi}(n_d v_x) = 0, \tag{8}$$

$$-\beta M \frac{dv_x}{d\xi} + \beta v_x \frac{dv_x}{d\xi} = \frac{\beta}{\alpha^2}\frac{d\phi}{d\xi} - \frac{v_y \sin\theta}{\alpha}, \tag{9}$$

$$-\beta M \frac{dv_y}{d\xi} + \beta v_x \frac{dv_y}{d\xi} = \frac{v_x}{\alpha}\sin\theta - \frac{v_z\cos\theta}{\alpha}, \tag{10}$$

$$-\beta M \frac{dv_z}{d\xi} + \beta v_x \frac{dv_z}{d\xi} = \frac{v_y\cos\theta}{\alpha}, \tag{11}$$

and

$$\beta^2 \frac{\lambda_d^2}{\rho^2}\frac{d^2\phi}{d\xi^2} = n_d + \delta_2 n_e - \delta_1 n_i. \tag{12}$$

After some straightforward mathematical manipulation with Eqs. (8), (9), and (12) we get

$$v_x = M\left(1 - \frac{1}{n_d}\right), \tag{13}$$

$$v_y = \frac{\beta}{\alpha\sin\theta}\left[1 + \frac{\alpha^2 M}{n_d^3}\frac{dn_d}{d\phi}\right]\frac{d\phi}{d\xi}, \tag{14}$$

$$v_z = M\cot\theta\left(\frac{1}{n_d} - 1\right) - \frac{\cot\theta}{\alpha^2 M}\int_0^{\phi} n_d d\phi, \tag{15}$$

and Eq. (10) finally derives

$$\beta^2 \frac{d}{d\xi}\left[A(n_d)\frac{d\phi}{d\xi}\right] = 1 - n_d - \frac{n_d\cos^2\theta}{\alpha^2 M^2}\int_o^{\phi} n_d d\phi, \tag{16}$$

where $A(n_d) = 1 + (\alpha^2 M^2/n_d^3)(dn_d/d\phi)$.

Further, Eq. (16) can be expressed as

$$\frac{1}{2}\frac{d}{d\phi}\left[\beta A(n_d)\frac{d\phi}{d\xi}\right]^2 = A(n_d)\left[1 - n_d - \frac{n_d\cos^2\theta}{\alpha^2 M^2}\int_o^{\phi} n_d d\phi\right] \equiv -\frac{dV(\phi)}{d\phi}. \tag{17}$$

Now integrating Eq. (17), we derive

$$\beta^2 A^2 P = \phi - F(\phi) - \frac{BF^2(\phi)}{2} + \alpha^2 M^2 \left[\frac{(1 - BF(\phi))}{F'(\phi)} - \frac{1}{2(F(\phi))^2} - B\phi - \frac{1}{2} \right], \tag{18}$$

where

$$P \equiv \left(\frac{d\phi}{d\xi} \right)^2, \quad B = \frac{\cos^2\theta}{\alpha^2 M^2}, \tag{19}$$

$$F(\phi) = \int n_d d\phi. \tag{20}$$

Equation (12) with

$$f(\phi) = \int (\delta_2 n_e - \delta_1 n_i) d\phi = \left(\delta_2 e^{\phi} + \frac{\delta_1}{\gamma} e^{-\gamma\phi} \right) \tag{21}$$

derives

$$\frac{C\beta^2}{2} P = F(\phi) + f(\phi), \tag{22}$$

where $C = \lambda_d^2/\rho^2$ is used. P can be eliminated from Eqs. (18) and (22) and the process derives the differential equation in $F(\phi)$ and the pseudopotential $\psi(\phi)$, $(=-P)$ can be derived from Eq. (22). Now, since the differential equation for $F(\phi)$ is nonlinear, an analytic solution would be difficult to obtain. Nevertheless, one can expand $F(\phi)$ in a power series up to any order along with the coefficients of the series obtainable from Eqs. (18) and (22).

We now presume the expression for pseudopotential as

$$P = C_1\phi^2 + C_2\phi^3 + C_3\phi^4, \tag{23}$$

with

$$F(\phi) = b_1\phi + b_2\phi^2 + b_3\phi^3 + b_4\phi^4. \tag{24}$$

Substituting (23) and (24) into (22) the values of b_1, C_1, C_2, C_3 are obtained as follows:

$$b_1 = 1$$

and

$$C_1 = \frac{2}{C\beta^2} \left(b_2 + \frac{(\delta_2 + \gamma\delta_1)}{2} \right), \tag{25}$$

$$C_2 = \frac{2}{C\beta^2} \left(b_3 + \frac{(\delta_2 - \gamma^2\delta_1)}{6} \right), \tag{26}$$

$$C_3 = \frac{2}{C\beta^2} \left(b_4 + \frac{1}{24} (\delta_2 + \gamma^3\delta_1) \right). \tag{27}$$

Equation (18), after using Eqs. (25)–(27), appears as

$$\frac{A^2}{C} \left(b_2 + \frac{(\delta_2 + \gamma\delta_1)}{2} \right) + \left(b_2 + \frac{B}{2} \right) (1 + 2\alpha^2 M^2 b_2) = 0, \tag{28}$$

$$\frac{A^2}{C} \left(b_3 + \frac{1}{6} \frac{(\delta_2 - \gamma^2\delta_1)}{2} \right) + b_3 + Bb_2 - 2(4b_2^2 - 3b_3)\alpha^2 M^2 \left(b_2 + B\frac{b_3}{2} \right) = 0, \tag{29}$$

$$\frac{A^2}{C} \left(b_4 + \frac{1}{24} \frac{(\delta_2 + \gamma^3\delta_1)}{2} \right) + b_4 + B\frac{b_2^2}{2} + Bb_3 + \alpha^2 M^2 \left[\frac{9}{2} b_3^3 - 36 b_2^2 b_3 + 8 b_4 b_2 - 7 B b_2 b_3 - 3 b_4 - 4 b_2^3 - 40 b_2^4 \right] = 0. \tag{30}$$

First one has to solve Eqs. (28)–(30) for b_2, b_3, b_4 and then by using them in Eqs. (25)–(27), C_1, C_2, C_3 can be evaluated explicitly. In deriving the above results, we have used the boundary conditions, as $\phi \to 0, \psi(\phi) = 0, d\psi/d\phi = 0$. The expand of $\psi(\phi)$ up to the terms ϕ^3 writes

$$\frac{d^2\phi}{d\xi^2} = D_1\phi - D_2\phi^2, \tag{31}$$

with $D_1 = C_1, D_2 = -3C_2/2$.

This is the mathematical technique to derive the pseudopotential without using the quasineutrality condition in the plasma and, under the assumption $\phi \ll 1$, it derives as

$$\beta^2 A \frac{d^2\phi}{d\xi^2} = A_1\phi - A_2\phi^2. \tag{32}$$

Where the coefficients are derived as

$$A_1 = (\gamma\delta_1 + \delta_2) - \frac{\cos^2\theta}{\alpha^2 M^2} (\delta_1 + \delta_2),$$

$$A_2 = (\gamma\delta_1 - \delta_2) - \frac{\cos^2\theta}{\alpha^2 M^2} \left[(\gamma\delta_1 + \delta_2) - \frac{1}{2} (\gamma^2\delta_1 + \delta_2) \right],$$

and

$$A = 1 - M^2.$$

Now Eq. (32) expects the solution in the form of sech (ξ), tanh (ξ) or any other hyperbolic function and hence we introduce a transformation $\phi(\xi) = W(z)$ with $z = \text{sech}\,\xi$ into Eq. (32). As a result of which Eq. (32) reduces to a Fuchsian-like nonlinear ordinary differential equation as

$$\beta^2 A z^2 (1 - z^2) \frac{d^2 W}{dz^2} + \beta^2 A z \frac{dW}{dz} - A_1 W + A_2 W^2 = 0. \tag{33}$$

The regular singularities at $z = 0$ demands that Eq. (33) could be solved by the Frobenius series solution method and to employ it the solution $W(z)$ is expanded as a power series in z as

$$W(z) = \sum_{0}^{\infty} a_r z^{\rho + r}. \tag{34}$$

The problem then reduces to finding the values of a_r and ρ, which in turn would reveal the nature of the solutions. We modify the earlier procedure[28,29] by truncating the series into a finite series along with $\rho=0$. Thence the substitution of series (34) into the Eq. (33) determines the coefficients of the series. The leading order analysis determines the number of the terms in the series, which equals to 3, and enables us to write W(z) as

$$W(z)=a_0+a_1z+a_2z^2. \tag{35}$$

Again the symmetry of Eq. (33) finds $a_1=0$, and the solution of Eq. (35) is then substituted into the differential equation. The collection of different orders from the recurrence relation finally finds the solution as

$$\phi(x,t)=\left(\frac{A_1}{2A_2}\right)\mathrm{sech}^2\left(\frac{x-Mt}{\delta}\right) \quad \text{with} \quad \delta=\sqrt{\frac{4A}{A_1}}, \tag{36}$$

along with a shock-wave solution

$$\phi(x,t)=\frac{A_1}{2A_2}\left[1+\tanh^2\left(\frac{x-Mt}{\delta}\right)\right]. \tag{37}$$

We see that the solution depends on the coefficients A_1 and A_2, which are functions of plasma parameters, as well as of θ; where θ appears due to the applied magnetic field in the dynamical system. For the typical values of γ related to the laboratory plasmas along with a reasonable value of the Mach number M, the variation of the nonlinear coefficients, A_1 and A_2 are shown in Fig. 1(a), with the dust-charged concentrations where the value of θ is taken to be small. The variation always maintains $A_2>A_1$, even with the variation of temperature ratio, γ [Fig. 1(b)]. But with increase of dust concentrations, the nonlinear effect increases while it decreases with the temperature ratio. So it is essential to control the plasma parameters in the laboratory to get the desired features of the solitons in plasma-acoustic wave. The nature of soliton variation not only depends on different plasma configurations but also depends on θ. But the nature, for large θ [Fig. 1(c)], is quite different from that for small θ as shown in Fig. 1(b). For small θ, the dust-concentration variation shows three regions [Fig. 2(a)] having either compressive or rarefactive solitons. The amplitude of the solitary wave is positive, resulting in the existence of a compressive solitary wave ($\phi>0$) and the solitary wave turns over to be a rarefactive solitary wave ($\phi<0$) as and when the amplitude is negative.[20] In the presence of a small percentage of dust concentrations, the compressive soliton with decreasing amplitude becomes a rarefactive soliton at a certain critical concentration. Again, for a higher percentage of dust, there will be always compressive solitons in the plasma-acoustic waves. However, in the case of higher field strength, there exist two regions. In the region of a small percentage of dust-concentrations, the rarefactive soliton yields, which finally turns to the compressive soliton due to the higher dust-concentrations. Again the nature does not change appreciably with the variation of temperature-ratio, γ [Fig. 2(b)]. The overall observations, in the presence of applied magnetic field, lead to the conclusion that the controlling of the plasma parameters is required to obtain the desired solitons in laboratory plasmas. For a very small percentage of the dust-grains, both A_1 and A_2 are of the same sign and a compres-

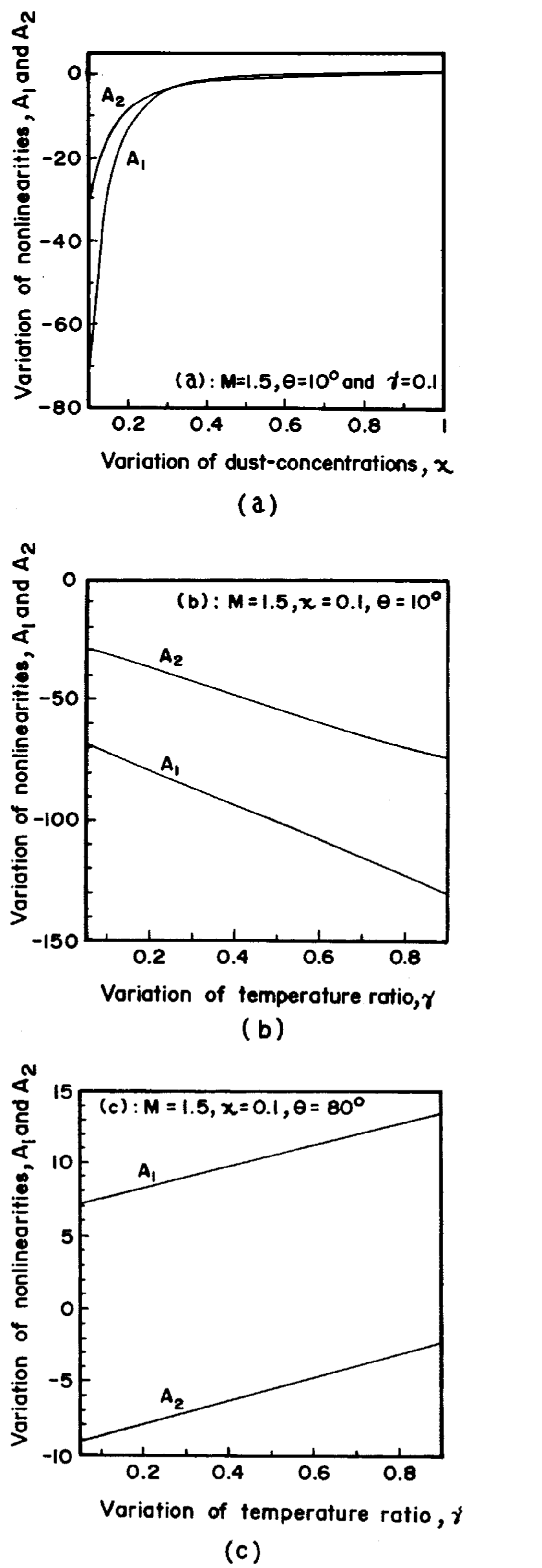

FIG. 1. Variation of nonlinearities with different plasma parameters: (a) with dust concentrations; (b) with temperature ratio; (c) with temperature ratio γ and oblique magnetic field.

sive solitary wave solution resulted. Further, with the addition of more dust concentration, the nonlinearities vary to opposite signs, the amplitude becomes negative and a profile of rarefactive wave propagation is exhibited. But for a

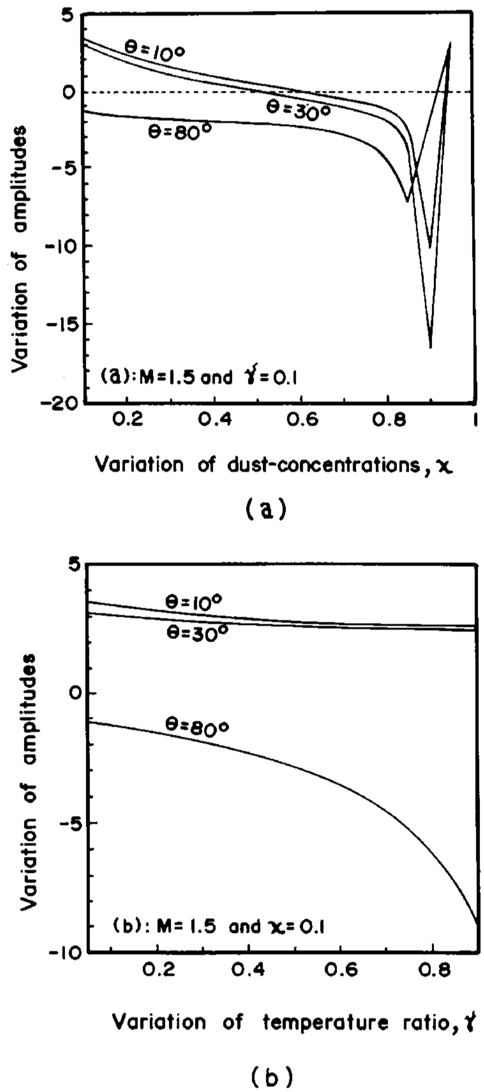

FIG. 2. Variation of amplitude with dust concentration and temperature ratio with oblique magnetic field.

higher percentage of dust grain, the nonlinearities again become positive and a compressive solitary wave is yielded. However, for much higher θ, such features vanish, and the signs of A_1 and A_2 remain different from each other. Thus, both compressive and rarefactive modes are possible in magnetized plasma, in contrast to the case of unmagnetized plasma, wherein only the compressive mode is possible. Thus, here the magnetic field plays the role to show the compressive and rarefactive solitary waves in the plasma. The acoustic modes are also controlled by the variation of the temperature ratio γ and these are shown in Fig. 1(b). For $\theta = 10°$, we see that there is a critical concentration of dust-charged grain for which A_2 vanishes and A_1 is nonzero. At this critical concentration of the dust-charged grains, the soliton solution shows large amplitude dispersive waves and the nonlinearity fails to form the soliton features in the dusty plasma. Again, at the neighborhood of the critical density, the soliton could be of explosive nature and this, in turn, depends on the conservation of the energy within the wave profile in the plasma. Otherwise, a bursting soliton is expected, as and when the energy grows with the growth of amplitude. It indicates the breaking of the double layer soliton solution and the reductive perturbation technique[34,35] might not be applicable as a whole. However, it was mentioned earlier that the possibility of having a large amplitude wave is not permissible, as the Sagdeev potential equation in unmagnetized plasma suggests that there is a barrier introduced by the positive ions from which the acoustic mode gets reflected before it grows to a high amplitude acoustic wave. Thus, we should find alternate arguments for finding the solitary wave propagation at this critical density. To extend the study at the critical concentration, we include next higher order term in the expansion of the Sagdeev potential and derive the wave equation as

$$\beta^2 A \frac{d^2\phi}{d\xi^2} = A_1\phi - A_2\phi^2 + A_3\phi^3, \tag{38}$$

with $A_3 = \frac{1}{6}(\delta_1\gamma^3 + \delta_2) + (\cos^2\theta/\alpha^2 M^2)[\frac{1}{2}(\gamma^2\delta_1 - \delta_2) - \frac{1}{2}(\gamma\delta_1 + \delta_2)^2 - \frac{1}{6}(\gamma^2\delta_1 - \delta_2)]$.

Equation (38), with a suitable linear transformation $F = \nu\psi + \mu$ with $\nu = 1$ and $\mu = A_2/3A_3$, reduces to a Duffing equation as

$$\beta^2 A \frac{d^2 F}{d\xi^2} - B_1 F + B_2 F^3 = 0, \tag{39}$$

where $B_1 = A_1 - 2A_2\mu + 3A_3\mu^3$ and $B_2 = A_3$ along with a restriction imposed on the coefficients given by $A_1 - A_2\mu + A_3\mu^2 = 0$.

This equation has special characteristic features, especially at $B_2 = 0$, at which the Duffing equation yields a stable or unstable soliton solutions depending, respectively, on $B_1 > 0$ or $B_1 < 0$. If the higher-order effect is taken off through $B_2 = 0$, the Duffing equation reduces to a linear wave equation. Now all the coefficients are plotted in Fig. 3(a). to support the analytical results as predicted. It is seen that the direction of the applied magnetic field plays an important role in exhibiting the stable soliton solution in the plasma acoustic wave. For small typical values of $\theta(= 10°, 15°)$ the coefficients B_1 and B_2 are of opposite signs (with $B_1 < 0$) due to which the soliton solution is supposed to be unstable [Figs. 3(a) and 3(b)]. Same feature appears in the case of large θ too showing [Figs. 3(c) and 3(d)] that both the cases have unstable soliton solutions. This case lies beyond the scope of the present study. However, for $\theta \equiv 45°$ [see Fig. 3(c)], B_1 is positive and one thus expects the stable soliton solution. Again it has been shown that the temperature effect does play a role in exhibiting the different features of the acoustic modes whereas magnetic field explains their coexistence of different nature in the plasma [Fig. 3(e)]. So the present study finds that the applied magnetic field is the root cause of showing a stable or a unstable soliton propagation in the dusty plasma studied through the Duffing equation. Moreover, this appears because of the higher order nonlinearity in the dynamics too. Now to solve the Duffing equation, we employ the tanh-method[27–29] for a stable soliton solution. Use of the tanh ξ- transformation reduces the Duffing Eq. (39) to a standard Fuchsian-like ordinary nonlinear differential equation as

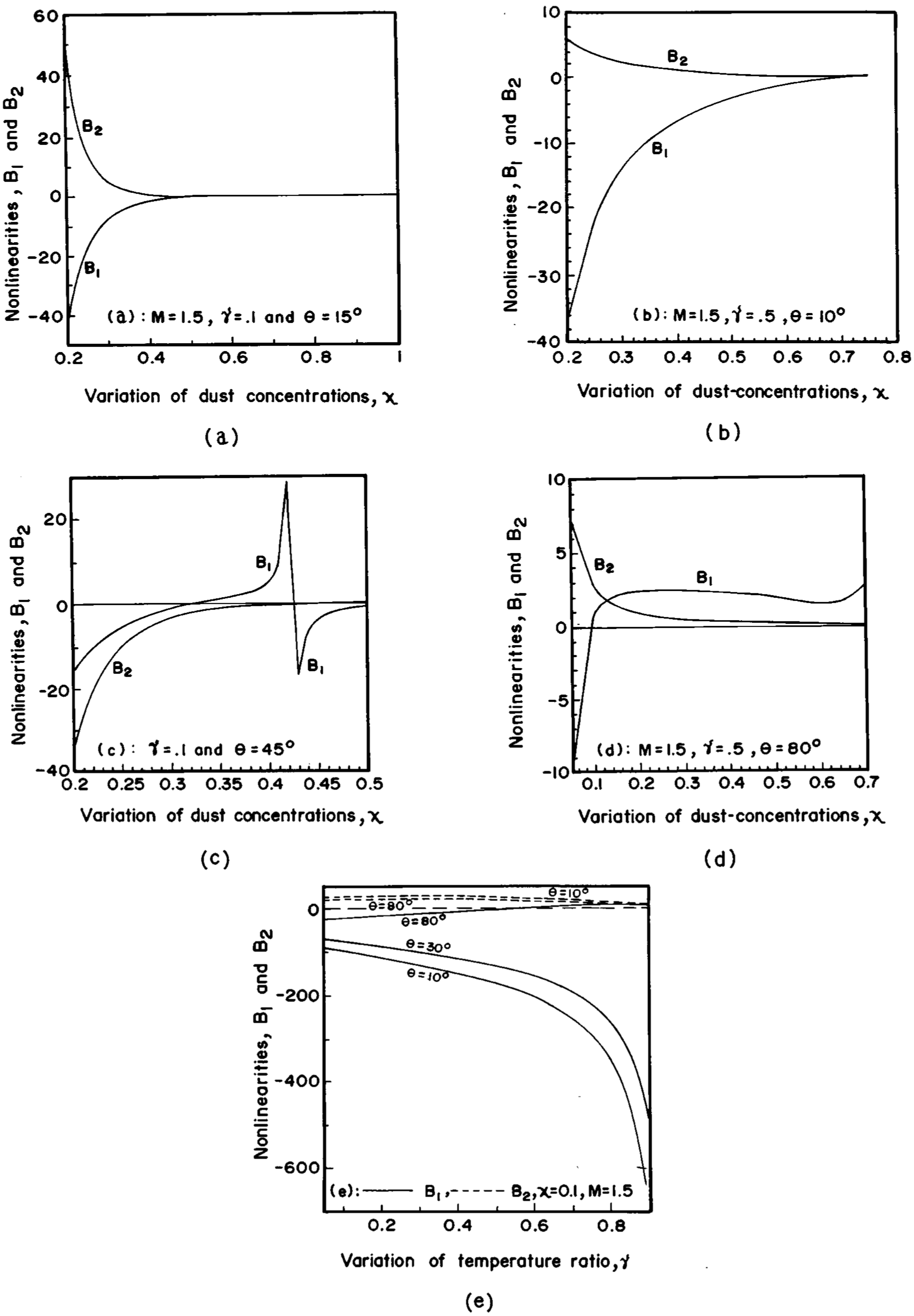

FIG. 3. Variation of higher-order nonlinearities with different plasma parameters as described in Fig. 1.

$$A\beta^2(1-z^2)^2\frac{d^2F}{dz^2}-2A\beta^2 z(1-z^2)\frac{dF}{dz}-B_1F+B_2F^3=0. \tag{40}$$

The Frobenius series solution method with finite series expansion as prescribed earlier derives a trivial solution. Thus, in this case it requires the consideration of an infinite series, which enables one to find the solution in the form

$$F(z)=a_0(1-z^2)^{1/2}, \tag{41}$$

and ultimately the final solution turns out to be

$$\phi(\xi)=\frac{A_2}{3A_3}\pm\left(\frac{2B_1}{B_2}\right)^{1/2}\operatorname{sech}\left(\frac{\xi}{\delta}\right). \tag{42}$$

This solution represents the profile of solitary wave or shock-like structure in dusty plasma depending fully on the plasma parameter variation. Now to find the other mode of the wave propagation, we integrate the wave Eq. (38) along with the boundary condition $d\phi/d\xi=\phi=0$ at $\xi\rightarrow\infty$, and find

$$\frac{1}{2}A\left(\frac{d\phi}{d\xi}\right)^2=\frac{1}{2}A_1\phi^2-\frac{1}{3}A_2\phi^3+\frac{1}{4}A_3\phi^4, \tag{43}$$

which admits the soliton solution[30]

$$\phi(\xi)=\left[\frac{A_2^2}{3A_1^2}\pm\left(\frac{A_2}{9A_1}-\frac{A_3}{2A_1}\right)^{1/2}\cosh\left(\frac{A_1}{A}\right)^{1/2}\xi\right]^{-1}, \tag{44}$$

with a suitable transformation, the soliton solution is now reducible to a well-known form

$$\phi(\xi)=\frac{A_1\operatorname{sech}^2 k\xi}{a_{\mp}+a_{\pm}\tanh^2 k\xi}, \tag{45}$$

with $a_{\pm}=\pm(4A_2^2/9A-4A_1A_3/2A^2)^{1/2}-(2A_2/A)$ and $k=\sqrt{A_1/2A}$.

Again Eq. (43) is reduced to a Sagdeev potential equation in the following form:

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2+V(\phi)=0. \tag{46}$$

The required condition for the existence of a double layer from Eq. (46) is $d\phi/d\xi=V(\phi)=0$ at $\phi=0$ and $\phi=\phi_m$, and we then transfer the equation as

$$\beta\frac{d\phi}{d\xi}=k\phi(\phi_m-\phi). \tag{47}$$

The new parameters are defined $k=\sqrt{A_3/4A}$ *and* $\phi_m=4A_2/3A_3$ along with the double layer condition $9A_1A_3=8A_2^2$. By applying the proposed tanh-method, the wave equation derives the double layer solution

$$\phi(\xi)=\frac{1}{2}\phi_m\left[1\pm\tanh\left(\frac{\xi}{\delta}\right)\right], \tag{48}$$

where $\delta=3\sqrt{AA_3}/A_2$.

Now because of the singularity at certain concentration one might need the inclusion of further higher-order terms in the Sagdeev potential. However, one has to be cautious here as, due to the inclusion of higher-order effect, the viscosity or the collisional effects might play a comparable effect. But to know the ordering effect in isolation one can take any number of terms in the expansion of pseudopotential. Thence it could be solved by the tanh-method[29] to get different kinds of nonlinear ion acoustic wave propagation in plasma.

III. CONCLUSION

The present study describes the formation of various nonlinear phenomena of dust acoustic waves in relation to the space and astrophysical environments. The Sagdeev pseudopotential technique is applied to derive the nonlinear wave equations to study the different acoustic waves in a magnetized dusty plasma, without assuming the quasineutral condition. The pseudopotential could not be obtained in the usual way, but a mechanism was found by which one can obtain a power series in ϕ for the pseudopotential. It has been seen that the different ordering gives rise to different types of nonlinear wave phenomena, which could be of interest in laboratory and space plasma. The parameter θ, the angle between the direction of wave propagation and magnetic field, and the temperature ratio γ, play important roles in determining the nature of various solitary waves. For different angles of applied magnetic field, the stable or unstable structure of the nonlinear wave in dusty plasma has been observed. For $\theta=45°$, both compressive and rarefactive solitons are possible and a shock-like wave structure exists under certain conditions, controlled by the plasma parameter, as well. The new approach of employing the tanh-method helps to one successfully obtain, the profiles of different solitary waves. The observations, predicting compressive and rarefactive soliton through the wave equation, could be related to those made by Freja scientific satellite[36] and by other spacecraft.[37] However, the additional observations made in this paper might motivate one to seek new findings in space plasmas through the satellite observations. Moreover, there are other various modes, which are yet known by satellite observations in astrophysical problems and, thus, the present results lead to significant advance in understanding on plasma acoustic mode for space plasma. The stability of the soliton has been also analyzed through the derivation of a Duffing equation, showing explicitly the dependence of the plasma variation arising owing to the variation of nonlinearity. Finally, the proposed mathematical technique has the merit that it finds successfully all kinds of soliton features in a magnetized dusty plasma.

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