INDIAN STATISTICAL INSTITUTE

TWENTYSEVENTH CONVOCATION ADDRESS

THE STATISTICAL DYNAMICS of TURBULENCE

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Director

National Aeronautical Laboratory

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Mr President, Mr Chairman, Graduating Students, Ladies and Gentlemen,

It is a great privilege for me to have been asked to address you on an occasion that marks for the graduating students an important milestone in their careers, and for the rest of the Institute a major event in its annual calendar. Let me right away congratulate all those who are getting their degrees today : you have had the opportunity to live and learn in a great institution, founded and nurtured by some of the most distinguished scientists of this country ; and that very knowledge should give you a confident pride that is one of the major objectives of every educational process.

It is a wonderful tradition at this Institute that the convocation address is supposed to be a brief scientific talk, rather than a pious homily delivered by a supposedly wise man. In this gathering of distinguished statisticians, mathematicians and physicists, and of those who are going to join their ranks in coming years, the only appropriate thing for me to do, it seemed to me, was to talk about a physical problem that is basically statistical in nature and has defied solution for more than a hundred years.

I refer to the phenomenon of turbulence in fluid flows. As everyday observation—say of smoke rising from a cigarette, or water issuing from a tap—shows, a fluid flow may be in one of two states : smooth and regular (and often but not necessarily steady), called laminar, or chaotic and inherently time-dependent, called turbulent. More specifically, a turbulent flow is characterised by chaotic vorticity (vorticity is the curl of the velocity vector, and is the local spin of a fluid element). The scientific study of turbulence may be said to have begun with the experimental observations of the British engineer Osborne Reynolds more than a century ago. The first sentence of his classic paper of 1883 reads, "The results of this investigation have both a practical and philosophical aspect," and reflects already that combination of great technological relevance and deep science that is responsible for the enduring appeal of the subject to engineers, mathematicians and physicists for over a century. To this day turbulence remains a problem not solved as science. What is known about it comes directly or indirectly from experimentation and testing, coupled with some clever analysis of the data and some inspired hypothesis-making ; prediction from first principles has eluded the best and the bravest minds. Feynman called it the greatest puzzle in classical physics ; Lamb said in 1932, "......When I die and go to Heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am really rather optimistic". Aerospace technologists like to think of their progress as hampered by having to cross various "barriers"—from the sonic through the thermal to the gravity barrier, but find that it is the "turbulence barrier" that has proved impenetrable.

What is the problem ? How is it that a phenomenon in severely classical physics, whose governing laws were written down by the French engineer Navier as long ago as 1822, has defied solution ?

The problem has many different aspects and we may briefly consider each of them.

First of all we must note that those governing equations, named after Navier and Stokes, are partial differential equations that are nonlinear. A measure of this nonlinearity, and incidentally also of the ratio of inertial to viscous forces in the fluid, is given by the parameter known as the Reynolds number (proportional to flow velocity times a length scale divided by the viscosity). At sufficiently high Reynolds numbers all flows tend to be turbulent ; it follows correspondingly that they are highly non-linear as well.

Reynolds himself proceeded by decomposing the flow into mean and fluctuating components, and offered a procedure that would yield equations for moments of different orders. However, because of the non-linearity, the equations for the evolution of a moment of any order always contain moments of higher order, so the system cannot be closed. Ad hoc truncations have generally been unsuccessful. For example, the so-called quasi-normal theory, in which all cumulants above some given order greater than two are assumed to vanish, leads to absurd results such as negative spectra. (Quite generally such cumulant truncations appear to violate the basic positive definiteness of the probability distribution function itself.) To overcome this problem one has to make further ad hoc assumptions whose validity is very difficult to assess in physical terms and whose consequences are consequently difficult to accept. One school of thought that has emerged in recent years has questioned the usefulness of the Reynolds decomposition itself. I believe the mean of a fluctuating quantity is always going to be of some interest, but if male is $+1$ and female -1, it does seem that to say that the average gender of the world is neutral misses the point altogether. (Of course the fact that the mean gender in India is a small positive number is probably of great sociological significance.) Furthermore, if the fluctuations actually *drive* the mean (as they often do in turbulent flow—recall the nonlinearity), the situation is rather like the tail wagging the dog, and the Reynolds decomposition would not be fruitful.

Turbulent motions have sometimes been defined as those where the velocity (or rather its curl) takes random values which are not determined by the 'macroscopic' data of the flow. This of course immediately raises a fundamental philosophical question, namely how the basically deterministic Navier-Stokes equations could be expected to yield solutions that are apparently random—unless there was stochastic forcing on the system. Recent development in the theory of dynamical chaos may be considered to have solved this philosophical problem : for it can now be demonstrated that there are nonlinear dynamical systems whose solutions, because of sensitive dependence on initial conditions, exhibit a weak causality and hence are apparently random. Einstein's objections (in a different context) that God does not play dice can now be countered firmly : He need not, it is enough that He is nonlinear. In fact He even could, because dice may be a spectacularly simple realization of a complex nonlinear dynamical system that exhibits chaos.

But is there a more quantitative connection between dynamical chaos and turbulent flows ? In the first flush of the exciting studies that appeared in the 1970's, extending a line of thought pioneered by the meteorologist Lorenz more than a decade earlier, it looked to some as if the answers were very close. A decade or two later, however, the situation is not so clear, in particular in open flows ("ducts", boundary layers, etc.). None of the routes to chaos till now identified seems directly relevant, e.g., to transition from laminar to turbulent flow on an aircraft wing as we know it. We have recently shown that it is possible to devise a low-dimensional dynamical system in which the onset of chaos shows the right kind of dependence on external disturbance and Reynolds number : some of the gross characteristics of open-flow systems are mimicked well by this system. Such work encourages the view that unforced, nearly inviscid flow is like a homoclinic orbit, which will break into chaos at the smallest perturbation. However, to make quantitative predictions for any real flow based on these (or other similar) models has proved very difficult. For example transition on an aircraft wing occurs in isolated islands of chaos in a laminar sea, but these islands move downstream with the flow as they grow in size and cover the whole surface with turbulence. (It incidentally turns out that this physical picture shows transition to be intermittent, and we showed many years ago that there results a statistical law of the Weibull type for the intermittency distribution). No connection between these pictures of transition and dynamical chaos has yet been established.

Assuming that the equations do somehow permit onset of chaos without invoking stochastic forcing, the question arises whether we can construct a statistical mechanics for turbulent flow, somewhat along the lines that have been successful for other physical systems. Early attempts in this direction took inspiration from the kinetic theory of gases and sought an analogue of the mean free path in what is known as the mixing length. Unfortunately the analogy is not valid, as in turbulent flow eddy interaction and separation distances are of the same order, making the statistical mechanical situation closer to that in a liquid than in a gas. Eddies experience not the rather sharp encounters of the balls on a billiard table, but the continuous jostling of people in a crowd.

There is however one special case where some very elegant statistical mechanics can be done. This is turbulence in two dimensions : that is, there are only two velocity components with a vorticity vector perpendicular to the plane of the flow. Most real life turbulence is three-dimensional, so the two-dimensional kind sounds unreal to most students of the subject. (I must make it clear here that even in cases where the mean flow may be two-dimensional the fluctuating flow will in general not be ; for example, in the highly idealised case of turbulence in a box there may be no mean flow at all, but all the three velocity components will still be present). Nevertheless, turbulence in two dimensions is not without interest ; for example, it has been shown that large scale motions in the atmosphere (e.g. the way that the isobars at some specific level with respect to ground dance around the globe) corresponds roughly to two-dimensional turbulence. In this case the full statistical mechanics can be worked out, for the system is hamiltonian. However, the hamiltonian is so peculiar that the resulting statistical thermodynamics involves negative and infinite temperatures, corresponding to the tendency of two-dimensional vortices to aggregate.

A statistical mechanics that can handle more realistic three-dimensional vortices does not exist yet. We know that it takes only a few vortices for the resulting motion to be chaotic. The long range of vortex interaction makes us look for analogies in the statistical mechanics of gravitating or charged particles. But vortices have a way of contorting themselves into wierd structures, and while we have been able to devise reasonable methods of handling them in two dimensions, we cannot go far in three yet.

We could look for expansions of the solutions of the Navier-Stokes equations in terms of a coupling parameter. The difficulty here is that in turbulent flow the coupling parameter (basically the Reynolds number) is *large*, so such expansions can at best handle low Reynolds turbulence. One may make selective sums by diagram techniques, but there is no sure guide to what diagrams to select, and the results one gets are correspondingly ambiguous.

While we are looking at the statistics of turbulence there are two other aspects that should be mentioned. First of all to those who are brought up thinking of the Gaussian distribution as "normal", it is surprising how often it is not observed in turbulence. In fact, one more frequently encounters exponentials, by which I mean distributions with tails like exp $(-|x|)$ rather than exp $(-x^2)$ in the random variable. Examples of exponential behaviour occur in the distribution of velocity derivatives and vorticity, in the intensity of gusts in the atmosphere, in the intervals between successive zero crossings of any turbulent velocity component, etc. On the other hand quantities like dissipitation appear to obey (as suggested by Kolmogorov in 1962) the log-normal distribution, which has strange properties, such as for example that a knowledge of all the moments is not adequate to define the distribution.

An even more basic question is what the best method of *describing* turbulent flow is. The classical approach is the use of generalised harmonic analysis, following G. I. Taylor and Norbert Wiener. This involves quantities like the spectra and the correlations. This approach has led to some notable results ; in particular Taylor succeeded in showing the relation between eddy diffusivity and the correlation. However, it has in recent decades become clear that the spectral approach has not been successful in solving the problem, and the feeling has grown that discarding phase information is too limiting.

An alternative method of description of turbulent flow that I and my colleagues have been attempting for several years now is through the identification of 'events' : the recent discovery of strong coherent or ordered motions in many turbulent shear flows by Roshko and others encourages such a picture. We may think of the classical description of turbulent fluid motion as belonging to a harmonic world : a world of (random) waves, eddy viscosities, Gaussian distributions etc. On the other hand, a more natural description, i.e. one that makes more immediate sense to observers of atmospheric motion (which is really turbulent flow on a gigantic scale), is the language of events : gales, cyclones, squalls etc ("weather is a series of incidents in the working of a vast natural engine" said Napier Shaw). In contrast with the classical view, we find that this 'episodic' view can embrace non-local transport, fat- or long-tailed distributions and violent fluctuations. For example, eddy transport of momentum on scales of the order of 10^3 km in the atmosphere corresponds to a diffusion coefficient of something like 10^{11} times the thermal conductivity of copper ! The net momentum flux that enables this huge transport to take place is the outcome of violent fluctuations : deviations exceeding hundred times the mean occurred more frequently than twice a minute in atmospheric data that we have recently analysed. The atmosphere therefore has really to exert itself to eke out a mean, but this mean is enough to make air a superconductor of momentum !

In many ways the episodic description of turbulent motion is still in its infancy, and there are a wide variety of open questions. We may, for example, raise the fundamental question whether turbulent motion can be described wholly in terms of such events or whether such events need to be distinguished from a background of noisy waves, or whether the background (if it exists) is really the debris of the events (recalling in this connection the argument of Jeffreys that cyclones are not to be thought of as perturbations on the background general circulation but as essential to maintaining it). We may ask more generally what the relation is between episodic and harmonic descriptions, and speculate whether something I have called a stochastic corrective process, in which sharp jumps or events punctuate and counteract an otherwise decaying background, could provide a connection.

One approach that has become feasible in the last two decades is the computation of exact numerical solutions of the full unsteady three-dimensional Navier-Stokes equations. Right from the early days of electronic computers turbulence has been near the top of the computing agenda : in 1949 von Neumann spoke of breaking the "deadlock" of turbulence by computation. Such computations are very demanding because of the vast range of time and space scales that need resolution. The largest scales are comparable to the scale of variation of the mean flow, and the smallest are those where viscous action is strong, and are named after Kolmogorov, who first characterised them in terms of dissipation of turbulent kinetic energy to heat. The ratio of these two scales increases with Reynolds number like $Re^{3/4}$.

The kind of computing power needed to do "direct numerical simulation" of turbulence, as it is called, has not been available in India till recently. But with the commissioning of the Flosolver Mk-3 at National Aeronautical Laboratory, this power is now on hand, and the first such solution has been recently reported by us. The great advantage of DNS is that it provides access to any flow variable, including all those that are virtually impossible to measure at present ; e.g. the three-dimensional chaotic vorticity field that (as we have seen) is the defining characteristic of real-life turbulence. It is therefore possible to use DNS to obtain insights that are not available from experiment. Of course DNS has its limitations as well : it can now handle only modest Reynolds numbers ; and it suffers (somewhat surprisingly) from difficulties associated with prescribing boundary conditions on the surface of the (necessarily finite) computational domain within which the equations are solved, creating a situation that is strangely similar to the difficulties that wind tunnel engineers have in accounting for what they call "wall interference" !

As computers grow in power the Reynolds numbers at which DNS can be carried out will also increase, but are unlikely to match the values encountered in aerospace technology (where they are of order 10^7–10^8) at least for another decade. But, quite apart from this consideration, there is a further paradoxical difficulty with DNS, namely that, for the vast majority of applications, it provides *too much* information. So to obtain an immediate answer to an engineering problem direct numerical simulations will turn out not only to be numerically and financially expensive, but in fact to generate far more information than is necessary : DNS may be too 'rich'.

Of course engineers cannot wait for a full understanding of the problem before they start putting a fluid to work for them, so they fall back on a lot of experimental work with some clever arguments involving group theory, or on constructing ad hoc statistical-mathematical models (some rather sophisticated), or on chasing vortices on the computer. Some ingenious tinkering with the flow by those steeped in experimental lore has over the years led to some surprising methods of "managing" it. For example, we know that a ribbed surface, i.e. one with streamwise grooves (usually less than a millimetre in width and depth) can actually reduce the drag of a body (so before long we might well have aircraft whose surfaces are rough to the touch and will actually consume less fuel *because* of the roughness). But these fascinating developments are not matched by any theory that can provide inspiration or even guidance.

I once tried to summarise "engineering" experience in the form of five "working rules" (not being able to muster the courage to call them "principles"), but at the conference where this was done somebody claimed to have evidence against one of them ! Although the issue is not settled to everybody's satisfaction, it seemed prudent to add a Sixth Rule saying "None of the above may apply in your particular problem".

What I seem to see now is that during the last twenty years turbulence is again moving near the centre-stage of physics and mathematics. The current surge of interest in non-linear systems, dynamical chaos and the physics of complexity could well be tackling problems at the very heart of a new physics. For long we have proceeded on the assumption that if the basic units (like fundamental particles, for example) are identified and understood, everything else would follow. This assumption has begun to be questioned, and there is now evidence that the fascinating—and even bizarre—behaviour of certain large systems depends less on the precise nature of the interacting units than on the fact that a certain type of interaction is taking place. In studying such complex systems turbulence stands out as one classical example that is a rich storehouse of problems.

I am afraid this lecture has recounted a catalogue of non-solutions to our problem, rather in the *neti, neti* spirit of our ancient philosophers ! If I have given you the impression that we don't understand what is going on, that impression is certainly correct. Let me however hasten to say it does *not* mean that we do not know anything. Rather the problem is that we know a lot but cannot tie it together, although we suspect that that should be entirely feasible. Von Neumann felt that the solution of the turbulence problem may in fact have a profound impact on mathematics (including, we may add, statistics). I hope that there are young mathematicians and statisticians here who will absorb this message and start poking into this treasure-house, and who will even help us break the "turbulence barrier".

Samuel Butler said, "Life is the art of drawing sufficient conclusions from insufficient premises". That I pressume is what statistics is all about, so I expect you to be masters of that great art ! In the *Mahabharata*, Yudhisthira said, answering one of the Yaksha's tricky questions, *dhanyānām uttamam dākshyam* : Skill is the supreme treasure. I wish you possession of that treasure, and great happiness in your future professional and personal lives.

Please accept my warmest congratulations once again.