Large-amplitude solitary waves in finite temperature dusty plasma

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Large-amplitude solitary waves in dusty plasma are investigated, taking into account the dusty particle temperature. It is shown that finite dusty temperature restricts the region for the existence of solitary waves.

Recently a lot of interest[1–8] (for more references see Ref. 8) is being shown in the study of dusty plasmas. Dusty plasmas occur in nature in various forms. Some experimental studies[9–12] of dusty plasmas have been made recently. Very recently Mamun *et al.*[13] showed that a dusty plasma with inertial dust fluid and Boltzmann-distributed ions admits only negative solitary potentials associated with nonlinear dust acoustic waves. In this Brief Communication we study large-amplitude solitary waves in a dusty plasma, taking into account the temperature of dust particles that may not be negligible. We consider a two-fluid model of dusty plasmas with Boltzmann-distributed ions and extremely massive negatively charged inertial dusty grains. The one-dimensional equations governing the dynamics of dusty plasma are

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial n}(n_d u_d)=0, \tag{1}$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial n_d}{\partial x}+\frac{\sigma}{n_d}\frac{\partial p_d}{\partial x}=\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial n^2}=n_d-e^{-\phi}, \tag{3}$$

where n_d is the dusty particle density normalized by the unperturbed dust number density n_{0d}, u_d is the dusty particle velocity normalized by the dust acoustic speed $(c_d)=(T_i/m_d)^{1/2}$, and ϕ is the electrostatic wave potential normalized by T_i/e_i, T_i being the ion temperature. The time and length are normalized by the dust plasma period,

$$w_d^{-1}=(m_d/4\pi n_0 Z_d^2 e^2)^{1/2},$$

Z_d being the dust charge, and the Debye length

$$\lambda_D=(T_i/4\pi Z_d n_{0d} e^2)^{1/2},$$

respectively.

Also, $\sigma=T_d/T_i$, T_d being the dust particle temperature. We also take the equation of state as

$$p=n^\gamma p_0. \tag{4}$$

To find the Sagdeev potential we make all the dependent variables depend on a single variable $\xi=x-Vt$, V being the soliton velocity, normalized by c_d. The equations (1)–(3) written in terms of ξ yield the following set of ordinary differential equations:

$$-V\frac{dn_d}{d\xi}+\frac{d}{d\xi}(n_d u_d)=0, \tag{5}$$

$$-V\frac{dn_d}{d\xi}+u_d\frac{dn_d}{d\xi}+\gamma\sigma n_d^{\gamma-2}\frac{dn_d}{d\xi}=\frac{d\phi}{d\xi}, \tag{6}$$

$$\frac{d^2\phi}{d\xi^2}=n_d-e^{-\phi}, \tag{7}$$

where we have used the relation (4) and taken $p_0=1$. From (5) we obtain

$$n_d=V/(V-u_d), \tag{8}$$

(6) gives

$$\phi=-Vu_d+u_d^2/2+\frac{\gamma\sigma}{\gamma-1}\left(\frac{V}{V-u}\right)^{\gamma-1}-\frac{\gamma\sigma}{\gamma-1}, \tag{9}$$

for $\gamma\neq 1$ and

$$\phi=-Vu_d+(u_d^2/2)+\ln[V/(V-u_d)], \tag{10}$$

for $\gamma=1$.

In general, (9) is an implicit equation for u_d in terms of ϕ. However for $\gamma=3$ the equation can be solved, and we obtain

$$u_d=V-(1/\sqrt{2})[V^2+2\phi+3\sigma+\sqrt{(v^2+2\phi+3\sigma)^2-12\sigma V^2}]^{1/2}. \tag{11}$$

The Sagdeev's pseudopotential is defined by

$$\frac{d^2\phi}{d\xi^2}=-\frac{\partial\psi}{\partial\phi}. \tag{12}$$

It is to be noted that to derive (8)–(11) we have used the following boundary conditions: $u_d\to 0$, $n_d\to 1$, $\phi\to 0$ when $\xi\to\infty$. Here ψ can be obtained from the set of Eqs. (5)–(7) and is given by

$$\psi(\phi)=1+\sigma+Vu_d-e^{-\phi}-\sigma[V/(V-u_d)]^\gamma, \tag{13}$$

where we have used the boundary conditions $\phi\to 0$, $\psi(\phi)\to 0$. For the existence of soliton solutions the following conditions must be satisfied:

$$\left.\frac{d^2\psi}{d\phi^2}\right|<0; \tag{14}$$

this gives

$$[1/(V-\gamma\sigma)-1<0] \tag{15}$$

and

$$\psi(\phi_m)=0 \tag{16}$$

and

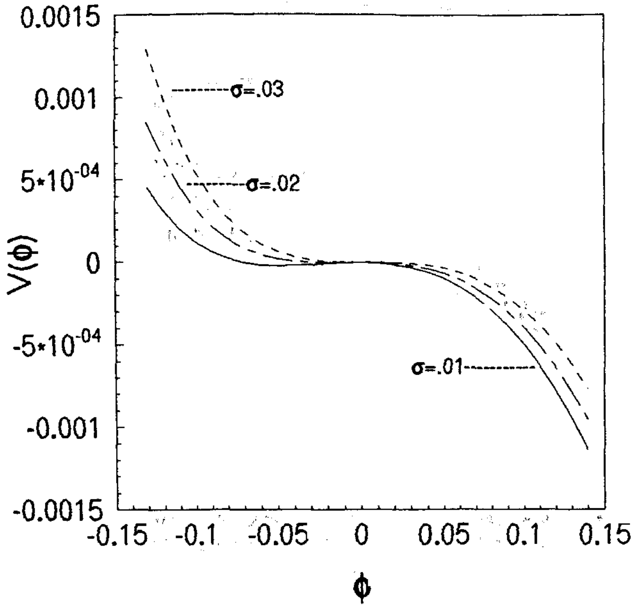

FIG. 1. Plot of the pseudopotential $V(\phi)$ against ϕ for various values of σ viz. $\sigma=0.01$, 0.02, and 0.03. The solid line is for $\sigma=0.01$, the middle broken line is for $\sigma=0.02$, and the top broken line is for $\sigma=0.03$. Here V is 1.04 and $\gamma=3$ for all the cases.

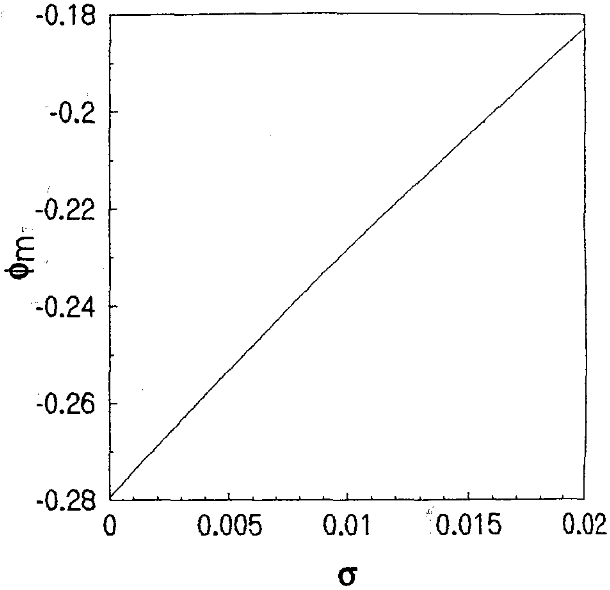

FIG. 2. Plot of ϕ_m, the soliton amplitude against ... re also $\gamma=3$.

$$\left.\frac{\partial \psi}{\partial \phi}\right|_{\phi=\phi_m} >0(<0), \tag{17}$$

for a compressive (rarefactive) soliton, where ϕ_m is the amplitude of the soliton. In Fig. 1 the pseudopotential $V(\phi)=\psi$ is plotted against ϕ for various values of σ, viz., $\sigma=0.01$, 0.02, and 0.03 when V, the soliton velocity, is taken to be 1.04.

It can be seen that for $\sigma\geqslant 0.02$ the soliton solution does not exist because though $V(\phi)$ vanishes for a positive value of ϕ (for $\sigma>0.02$) it remains positive between $\phi=0$ and $\phi=\phi_c$ where $V(\phi_c)=0$, $\phi_c\neq 0$. The dependence of soliton amplitude ϕ_m on σ is shown in Fig. 2, where ϕ_m is plotted against σ for $V=1.1$. It is seen that the amplitude decreases with the increase of σ.

Finally, we give the small-amplitude expansion of $V(\phi)$ and the corresponding analytical form of the soliton solution.

If one neglects term of $O(\phi^5)$, then $V(\phi)$ is found to be

$$V(\phi)=A_1(\phi^2/2)-A_2(\phi^3/6)-A_3(\phi^4/24), \tag{18}$$

where

$$A_1=1/(V^2-\gamma\sigma)-1, \tag{19}$$

$$A_2=[(3V^2+\gamma\sigma)/(V^2-\gamma\sigma)^3]-1, \tag{20}$$

$$A_3=\frac{\sigma V^2(\gamma-1)}{(V^2-\gamma\sigma)^4}+\frac{3V^2+\sigma\gamma(\gamma-2)}{(V^2-\gamma\sigma)^4}\times\left(\gamma-\frac{3(1+\gamma)V^2}{V^2-\gamma\sigma}\right). \tag{21}$$

The soliton solution of the differential equation (12) is given by

$$\phi=\frac{2a_1}{a_2\mp\sqrt{a_2^2-4a_1a_3}}\left[2\cosh^2\left(\frac{\xi}{\partial}\right)-1\right], \tag{22}$$

where $a_1=A_1/2$, $a_2=A_2/6$, $a_3=A_3/24$, and $\partial=2/\sqrt{-A_1}$. It can be noted from the condition for existence of the soliton that A_1 is negative throughout the solution region. Therefore $A_2<0$ gives compressive and $A_2>0$ gives rarefactive solitary waves. However this holds only for small-amplitude solitary waves.

To conclude, we have shown that finite dust temperature restricts the region of existence of solutions for solitary wave solutions. For example, even for $\sigma=0.0001$, soliton solutions would not exist for $V=1.5$, whereas for $\sigma=0$ soliton solutions exist for $V<1.58$. Also, we found that compressive soliton solutions do not exist for any value of σ.

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