*Convocation Address by Professor Paul Erdös, Mathematical Institute, Hungarian Academy of Sciences, Budapest, Hungary.*

Mr. President, Director and Ladies and Gentlemen :

On this occasion I shall certainly speak a certain amount about myself. Since I am now over 70, I am not sure I shall have many more opportunities to do so and also I am not sure how long my mind and memory will remain intact. Fortunately, so far it has been intact.

I first came to India about 20 years ago ; I was in Chandigarh in 1965. It is my habit to learn two or three words in each language—I am not particularly interested in learning languages though my memory is good but I wanted to learn two most important words which are "old" and "stupid". I found out to my pleasure that Hindi is the best language in the world, because the two words sound almost the same, namely—buddha and buddhu. You will excuse my poor pronunciation. Probably if you pronounce them correctly that they would probably not sound so similar.

Now let me talk about myself and how I became a mathematician and what my life style is. Let me start with the question I am often asked" What will you do when you retire ?" I always answer, "I can't retire since I have been born retired." That is not as wrong as it sounds because I actually never had an ordinary job. I travelled the world practically all my life. When I am asked "When will you settle down ?" I have a very good answer, "There is plenty of time to settle down in the grave".

Now let me tell you how I became a mathematician. I had the great advantage that my parents were both mathematicians. My mother was one of the first ladies to graduate from a Hungarian University. It became possible for women to enter Hungarian University in the year 1896 and she actually entered the University in 1900. Incidentally a curious thing is that my mother travelled with me many times and her first trip by an airplane was only in 1964 when she was 84 years old. We travelled more or less all around the globe. She went to Australia, Canada, the USA, England and many other countries. When I first came to India she was also to accompany me but on the advice of her doctor she decided not to do so.

Now let me tell you how I was very interested in child prodigies, for I was one myself. Perhaps, I should tell you how it became clear that I had the ability in mathematics. I was born in 1913 and my father was drafted during World War I, served in the Hungarian army ; and my mother taught in a High School. So I was often left alone with the maid of the German governess. I, of course, often wanted to know when my mother would be home. I studied the calendar a great deal so that I knew months ahead when there would be a holiday and my mother would be at home.

By the time I was 4 years old I could multiply 3-digit number mentally. I always surprised people by asking them how old they were and telling them how many seconds they have lived. Perhaps my greater discovery was when I was 4 years old I told my mother that if we take away 250 from 100 we get 150 below zero. Actually, if I remember right, I told that in German. My interest in number theory started when I was about 10 years old when my father told me the proof, attributed to Euclid, of the fact that there are infinitely many primes and when he explained to me that there are arbitrarily long gaps between the prime numbers. To show that there are arbitrarily long gaps between the primes, all you have to notice is that if we take the numbers $n! + 2$, $n! + 3$, upto $n! + n$, these numbers are all composite, and hence then are n-1 consecutive composite numbers. Since we can do that for every $n$, we can get arbitrarily long gaps between prime numbers. The exact study of the gaps between the prime numbers is one of the most difficult problems in number theory. I have the habit of offering money for problems and the largest amount of money I have to pay so far was $1000 to my colleague Szemeredi who solved an old problem of Turan and myself. But I have a problem for which also there is a prize of $10,000, on the difference of consecutive primes. But I do not expect that I will have to pay this. Since I do not have very much money, a friend of mine once asked what would happen if all my problems were suddenly solved. After a moment's thought I said of course I would not be able to pay. But that does not seem likely. Consider what would happen to the strongest bank in the world, if all the creditors suddenly asked for their money back ? Clearly, the bank would be immediately ruined and I think it is much more likely that this will happen than that all my problems will suddenly be solved. So I think I am safe for the time being. I will not get ruined certainly not by all my problems getting solved.

Incidentally, problems in number theory in a way are particularly difficult. It is a joke which I sometimes play with mathematicians. I ask them, "Can you tell me

a problem which is not in number theory and which is more than 100 years old" ? It is actually not very easy to do so, whereas number theory is full of unsolved problems which even a baby can understand but the wisest man cannot solve, for example, just to mention two such problems. Is it true that every even number is the sum of two primes ? Is it true that there are infinitely many pairs of prime numbers differing by 2 ? I have published many unsolved problems in many subjects. For example, my collaborator Hajnal and I published about 17 years ago some problems in Set Theory. We published about 80 such problems and more than half of them have been solved since. This wouldn't be the case in number theory, that is, the time which passes until half the problems have been settled, is very long, in fact, it may be longer than the life time of the universe. So the problems in number theory are very difficult.

Actually my first paper was on Number Theory. This was on a theorem of Chebychev which states that for every $n$, there is always a prime between $n$ and $2n$. This was actually the first time I heard of Ramanujan because I found out later that Ramanujan had also found a proof for this theorem which is comparable to mine in simplicity. During my career, there where other instances of my work being connected with his ; for example the birth of probabilistic number theory is connected with the work of Hardy and Ramanujan.

Now let us come to the question of what is needed to be a good mathematician. There is a saying attributed to Edison that genius is 99% perspiration and 1% inspiration. Now this is certainly not true in mathematics. You often don't have to work very hard in mathematics but what you need to have "your brain open". By that I mean you have to be receptive to new ideas when they come along and must be ready to interrupt any thing else you are doing and concentrate on the idea which you either get from your own mind or somebody else's. I applied this method successfully in proving an old theorem along with Katz. This was in March 1938, a quite long time ago and this sort of thing also happened since. I work very easily with others. I have more than 20 collaborators at present. There is a joke in this context that I actually have a joint paper with a rail-road conductor of Santa Fe. However, this is not true.

Now let me illustrate this idea of having your mind open with a story. It seems to be a true story, though I have not examined the original documents. There were two men, one whose mind was open and another whose mind was not open. Now there was Röntgen—the German physicist who discovered X-ray in the U. S., they are known as Röntgen rays. Röntgen in 1891 noticed by accident that if you leave a photographic plate near a Crook's tube then the photographic plate gets darkened. Now from this Röntgen somehow noticed that this must be an important phenomenon and for a few weeks he did not do anything else but work on this subject and these few weeks of course changed the world. After he finished work on X-rays, Röntgen rays were discovered for which a few years later he received the first Noble Prize in physics. These rays have immediate applications in medicine. Röntgen noticed very soon that one would take pictures of the internal organs an enormously important application. Very soon their curative effects for cancer where discovered, its dangers were realized

very soon and that one must be very careful in working with these rays. Also next year radio activity was discovered by Bequerel and then Curies came along, and a few years later the atom bomb was discovered and now we are not yet sure that you will survive this discovery. Anyway the world was never the same after Röntgen discovered X-rays.

Now let me tell the story of a man whose mind was not open. Crook's who incidentally was a great physicist himself discovered the cathode rays and the Crook's tube also noticed that the photographic plate gets darkened if you leave it near a Crook's tube, but he only deduced that you should not leave a photographic plate near a Crook's tube. We put up a morning in his laboratory for colleagues not to do so. As a joke, I sometimes call it the biggest mistake in the history of science I could tell from my own career of some instances when my mind was not open ; for example, Hajnal and I missed the discovery of the fact that the inacessible cardinals are not measurable. There are a few other examples. I missed the discovery of the extremal theory of graphs which I should have done two years before Turan did so. Every mathematician will find cases where he really should have made the discovery and he did not make it. This is true of course for every subject in the world.

Now let me tell you one or two jokes about the Erdös number. Graham, a good friend of mine—under an assumed name wrote a paper about the "collaboration graphs" of mathematicians. He writes about this paper, "This paper fills the much needed gap in the literature". The paper considers the problems, namely ; "does Gauss have an Erdös number,?" "Can Gauss and I be connected by a line as collaborators ?" Now this is not as easy as it sounds ; Gauss did not really write any joint papers : it was not customary those days but Gauss collaborated with a German physicist called Weber, in fact, with some justice it can be said that he really discovered the telegram though as far as I know he did not utilize it. Now Weber has joint papers with several others and you can reach the famous physicist Helmholtz and Gauss can be joined to Helmholtz. I can be joined to Einstein quite easily becuase I have several joint papers with Strauss who has joint papers with Einstein. So the only outstanding problem remains whether Einstein and Helmholtz can be joined by a line. This could certainly be decided by somebody who knows physics better than I do. But perhaps rightly nobody took the trouble to settle this outstanding problem. Another such question after Goldman's paper on the Erdös number appeared ; I wrote a paper "The fundamental problem of mathematics". The problem was "You join two people if they have a joint paper. Is this graph planar ?" and Sims noticed that the graph was not planar because it contained a Kuratowski graph. By now it has been found that it contains 5 points—any two of which can be joined by a line. Let us now stop talking about not so serious subjects and let me tell you a few more things about my child progidigies.

First of all there is a curious fact that not all subjects have child prodigies ; there are child prodigies in mathematics and child prodigies in music. Everybody knows that Motzart was a child prodigy and there are several other musicians who were child prodigies. Now I don't know Indian music well enough in fact I hardly know it, to know whether there are child prodigies in Indian music. I am almost certain there are.

In mathematics there were several child prodigies. An outstanding example was Gauss. Let me tell you another anecdote about Gauss which probably is true. Gauss himself said that he learn to count before he learnt to speak. His great ability was discovered when he was 7 or 8 years old ; he was a very poor boy ; his mother might have been illiterate and he went to a little village school, the school master wanted to have a quiet hour and he told the children to add the numbers from 1 to 100. A few seconds later Gauss came out and said 5050. The teacher first thought that Gauss wanted to be naughty and rather severely asked him ; "How do you know ?" Gauss said "Look, 1 and 100 is 101, 2 and 99 is 101, 3 and 98 is also 101 and if we continue down 50 and 51 is also 101. Therefore the sum of the integers from 1 to 100 is simply 50 times 101 which is 5050. Now the teacher realised that this was the most important event of his life and from then on he gave Gauss special instruction and by the time Guass was 20 he was one of the greatest mathematicians of the world. And I often say that every high school or elementary school teacher should know enough at least to discover a very clever boy or girl if he finds one and give him or her special encouragement. And if there are quicker than the rest do not try to hold them back but encourage them. This is not always easy and I know of many cases of teachers who actually discovered clever boys.

Let me tell you a few things about my association with child prodigies. Perhaps the most striking child prodigies I came across were Pósa, and Rusza. I came across several child prodigies cutside Hungary too but about Hungarian ones I had a little more time to learn. Let me tell you a few words about Pósa. In 1959 when I just returned from Colorado I was told about a little boy who had won a competition which was held for high school students who had just matriculated. The next day I had lunch with the little boy and while he was eating his soup I posed him this question : "Prove that if you have $n + 1$ integers less than or equal to $2n$ then two of them must be relatively prime". This is not true for $n$ integers because we can simply take the multiples of 2 ; then we have $n$ integers no two of which are relatively prime. Pósa was drinking his soup and he finished his soup and said "If you have $n + 1$ integers up to $2n$ then two of them must be consecutive and therefore they must be relatively prime". Needless to say, I was very impressed and then on I worked a great deal with Pósa ; when he was $14\frac{1}{2}$ we wrote already a substantial joint paper and we had many more. This is not an unqualified success story. Pósa is teaching now at the University of Budapest and he is a good mathematician but I still think for his ability he should have been one of the leading mathematicians of the world which he has certainly not become until now. He is very interested in teaching and is a very successful teacher but I always tease him by telling him that if he spends his time teaching elementary mathematics, it is like Bach, instead of writing his immortal music spends his time teaching music to children. Anyway, Pósa is relatively happy and likes what he is doing.

Rusza, another child prodigy of mine started life doing analytic number theory was more successful. When he was 15, he was more successful aheads a mathematician. In fact, he should have been in India. He was invited to a conference which was

held in Ooty in my "memory" ; by "memory" I mean for my 70th birthday. I already said it when I was 60 when a conference was held in my honour that it was in my memory ; it is much more true now that I am 70. Anyway, he could not come to this conference because he is in West Germany now.

Another child prodigy who was very successful was Lovasz but in a way his mathematical ability manifested itself a little later and Lovasz wrote his first significant paper when he was an old man of 17 and Posa had already done it when he was $14\frac{1}{2}$. But by no means all mathematicians are child prodigies. Some counter examples would be Weierstrass ; his talent manifested itself relatively late. Also Hardy was not a cild prodigy. As I said, Gauss was a child prodigy. For example Cantor who is one of the deepest thinkers and who opened up infinite sets to mathematics, which according to Betrand Russel was perhaps the greatest single intellectual feat was certainly not a child prodigy. By child prodigy I mean one whose talent manifests itself by 10 or 12 years. There are exceptions ; for example, my collaborator Hajnal when he got to the University, did not work on studies. He thought he was not suitable for mathematics.

Another, outstanding child prodigy in mathematics was Pascal. His ability showed up at the time when he was 12. His parents, especially, his father, was very worried about his rapid advancement. He tried to discourage him but soon gave up. It is often said about child prodigies that it is dangerous to push them too hard. In some cases it might be justified. For example the great mathematician Norbert Weiner complained often that his father pushed him too hard ; and he almost never praised him and made him work very very hard. But Norbert Weiner overcame this difficulty and became a great man in many subjects. He always remained a little bit of a child prodigy in his behaviour, for example he was wanted people to praise him, even people who evidently did not understand his greatness. Norbert Weiner gives a few examples of child prodigies whose work petered out, on the other hand, certainly a child prodigy should be encouraged to go ahead as fast as he can.

Let me tell you a little joke about my childhood. My father and of course my mother too worked a great deal with me and they were careful not to push me and I always had to quarrel before they taught me various things. For example, I remember very well when I first heard about the absolute zero. I knew that there was something called absolute zero but did not understand it. Once, my parents had to go out and it was not convenient to take me. I was very unhappy, but my parents said that if I were a good boy, they would tell me more about the absolute zero. That is the way I learnt about the absolute zero when I was 10 or 11 years old. As I said I did not start publishing phenomenally early. My first published paper appeared when I was 18 but I kept on working fairly steadily. Now I have more than a thousand papers, more than half of them are joint papers. Despite my enormous age I am still continuing to work as much as I can. As I said, despite the fact that I travelled almost all the time I was asked how I still find time for serious work. The answer is very simple. I have got accustomed to disregard my surroundings. I can work very well in an airplane, in a train and even in a bus, if it is not too crowded and unpleasant. Certainly

in a train I can work very well ; it is not necessary for me to have paper to be able to work ; I can just think about the problems. I learnt to do this when I was a young man. In Hungary we have made many excursions ; Budapest is nice scenic town, with very beautiful surroundings and we often took long walks. In the walks, I talked about mathematics and I get accustomed to talk about mathematics without paper. As I explained before with examples, how important it is that you should keep your mind open and keep it receptive to new ideas. On the other hand there are no general principles as to how best it is to work. There are people who cannot work well with others ; for example, Mordell told me that the reason he did not have joint papers even though he likes to talk mathematics is that he can concentrate only when he is alone in his study sitting at his desk with paper and pencil. This makes it difficult for him to work jointly. Also, ability to work depends on one's life style. For example, my collaborator Turan, with whom I have more than 20 joint papers once told me after he was over 60 years old, that he had slowed down and it was harder for him then to work jointly by talking ; he had to write things down and see it in a manuscript. Nevertheless, he wrote several joint papers after that. But it was not an easy thing to do for him from then on.

I do not want to keep your attention and time infinitely long. So I will finish in a few minutes. Before I stop let me tell you a few more things about myself. In a sense I have no permanent job ; I do not even have a permanent home. I have no apartment. I just travel around from place to place. An American mathematician called Curtis at the Institute for Numerical Analysis got first the following kind of position for me. I had this position at that time at the Institute of the Numerical Analysis, I got paid either at the Institute for Numerical Analysis at Los Angeles or at the Bureau of Standards at Washington. For every day I was there I got a certain amount of money and I could leave any time I wanted to. I did not have to get a leave of absence and if I was away for more than 3 or 4 days, it was upto the Director whether he considered my trip as a mathematical trip or not and whether I should get paid or not. Since then I had many such positions. At the moment I have got a so-called permanent position at the Hungarian Academy of Sciences and at the Technion at Haifa at what I call the Curtis conditions. Clearly not many people would benefit by such a way of life. People like to travel but do not like to be constantly on the move. Also, they like to have a permanent home. So far, I have managed to keep myself free of all these. But I cannot tell what would happen if I really got old and sick. I am already old but my health is still good. I have another curious habit. I am rather uninterested in material possessions. I would say I have almost no possessions except my immediate needs. My only valued possessions are my note books which I have kept since 1933. There was a French Socialist thinker pre Marxist who said that private property was like stealing. I modified it by saying that property is a nuisance. This is clearly a way of life which is quite unsuitable for one who has a family and especially to one who has children. Even I, when I was a child, liked a certain amount of stability ; that need has slowly disappeared now.

Let me tell you a few more things. I played the following games on science. Some of you will be familiar with the Iliad which is the Ramayana or the Mahabharata

of the Greeks where the following scene often occurred. The Iliad deals with the Seige of Troy by the Greeks ; if two heros fought, the chief Greek God Herod sometimes favoured one or the other but He was not allowed to interfere. He had to wait for Fate's decision. He had a balance ; he put the fates of both of the heroes and he who weighed heavier sank down and that man was supposed to win. He was not allowed to interfere. I made the following joke out of this for myself : I had a large number of collaborators. Unfortunately many of them died now. God has a transfinite balance. He puts the mathematical value of the papers of my collaborators who died and the value of my other papers and when the value of the papers of the person who died is heavier then I have to leave the world too. So I was given an advice which I shall certainly follow, that I should get younger collaborators as much as possible.

Another joke of mine is that you do not have to believe in God but you should believe in the existence of this balance. Another joke of mine is the book : God has a transfinite book which contains all the theorems and their best proofs. If he is well disposed towards you he shows the book to you for a moment. Here, as again it is not necessary to believe in God but it is certainly necessary to believe in the existence of the book. Incidentally I never had the privilege of meeting Ramanujan because he died in 1920 when I was only 7 years old and never even met his widow who is living in Madras. But when I was here last, I saved a certain amount of money. Since rupees cannot be exchanged so I sent the money I saved to Mrs. Ramanujan. This I will do now again. Now I will tell the joke that I hope that in gratitude Ramanujan will steal the book for me for a few seconds. Since I have grown older I will need the book more than when I was younger.

Incidentally, Hardy very strongly emphasised that mathematics is a young man's game, perhaps he was unduly pessimistic but still as far as I know very few great theorems have been found by mathematicians who are older than 70. As far as I know the record is held by Weierstrass. He was born in 1815 and his approximisation theorem that every continuous function can be approximated by a polynomial was published in 1885. It was published when he was 70 years old and he probably found it only one or two years earlier. And also Ellie Cartan did important work when he was between below 70. But as far as I know, no really important work has been done by mathematicians over 70 though Hadamard published a paper when he was 90. I hope to break this record but only the future will tell whether my hope will be fulfilled ; first of all it is necessary to live that long. I thank you for listening me so carefully. Let me finish my talk wishing all the young people who got their degrees good luck and good luck for the whole Institute and if I live I will surely be back again.

Thank you,