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Book Reviews

)ecoupling
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Doeblin, in the late thirties, introduced the Coupling Technique – a method that went unnoticed till the seventies. Since then it developed as a powerful tool with wide ranging applications to the asymptotics of regenerative processes, interacting particle systems, point processes, Poisson approximations etc. Here is a modern day definition of coupling: A bivariate process $Z_n = (Z_n^1, Z_n^2)$ is a coupling of two univariate processes (X_n) and (Y_n) if $(X_n) \overset{Law}{\sim} (Z_n^1)$ and $(Y_n) \overset{Law}{\sim} (Z_n^2)$. Ideally the (X_n) process is being investigated; (Y_n) process is known. For a coupling to be useful we need good coupling inequalities, for instance, estimates of $\sup_x |P(Z_n^1 \leq x) - P(Z_n^2 \leq x)|$. An important by-product of this technique is estimates of rates of convergence. Incidentally the term "Coupling" itself was coined by F. Spitzer.

Suppose $(X_n, \mathcal{H}_n)$ is an adapted sequence on a space $(\Omega, \mathcal{H}, P)$. Let Q_0 be the distribution of X_0 and for $n \geq 1$ let $Q_n(w, \cdot)$ be the conditional distribution of X_n given $\mathcal{H}_{n-1}$. For each w, let $P(w, \cdot)$ be the product probability $\Pi_{n=0}^{\infty} Q_n(w, \cdot)$ on $\mathbf{R}^{\infty}$. On $\Omega \times \mathbf{R}^{\infty} = \tilde{\Omega}$ let $\tilde{P}$ be the probability having first marginal P and the conditionals being $P(w, \cdot)$. Denote $\tilde{X}_n(w, \underset{\sim}{x}) = X_n(w)$ and $Y_n(w, \underset{\sim}{x}) = x_n$, the n-th coordinate of $\underset{\sim}{x}$. Let $\tilde{\mathcal{H}}_n = \sigma(\tilde{X}_i, \tilde{Y}_i \ i \leq n)$. Then both $(\tilde{X}_n, \tilde{\mathcal{H}}_n)$ and $(\tilde{Y}_n, \tilde{\mathcal{H}}_n)$ are replica of $(X_n, \mathcal{H}_n)$. The added advantage is that given w, $(\tilde{Y}_n)_{n \geq 0}$ are conditionally independent. The sequence $(\tilde{Y}_n)$ is called a *decoupling* of the sequence (X_n).

The origins of the technique of *decoupling* are in the works of Burkholder, McConnell, Jacod and others in the eighties. Decoupling is the breaking of a dependence structure, and has evolved as a powerful technique to prove limit theorems for dependent sequences by using the analogous results for independent sequences. The authors have contributed to this area significantly and their book is a masterly state of the art exposition of the theory and applications of decoupling.

Chapter 1 presents a host of inequalities for moments and tail probabilities o sums of independent random variables. The authors not only give all the important inequalities but also in many cases provide recent and improved proofs of sharpe results. Notable among the techniques, used heavily in the next chapter, is that o L function bounds.

Chapter 2 deals with randomly stopped processes with independent increments This chapter discusses the famous Wald's identities, the Burkholder-Gundy inequal ities for L_p norms of Brownian motion and the inequalities of Klass and others. A decoupling interpretation is provided for these inequalities. An interesting result is (in some sense) the weakest possible conditions under which the famous Wald's first identity holds.

Chapters 3, 4 and 5 illustrate, applications to U-statistics and U-process, the technique of decoupling. A decoupling of U-statistics, say, $\binom{n}{m}^{-1} \sum h(X_{i_1}, \ldots, X_{i_m}$ based on a symmetric kernel h means the following : consider independent copies of your X-process say, $(X_i^j)_{i \geq 1} j = 1, 2, \ldots, m$ and $\binom{n}{m}^{-1} \sum h(X_{i_1}^1, \ldots, X_{i_m}^m)$ "This produces enough independence so that this average conditioned on all but one of the independent X-sequences becomes a sum of independent random variables". The "Three gems" - SLLN, CLT, LIL are developed in a systematic way for U-statistics as well as U-processes (an indexed family of U-statistics). The necessity of the integrability of the Kernel for SLLN, the necessity of the square integrability of the Kernel for CLT are well treated. The combined power of randomization (introducing Rademacher Variables) and decoupling (introducing independent copies of your process) is really revealing. Among the notable applications to statistics is the proof of strong consistency and asymptotic normality of the simplicial median.

Chapter 6 treats the general concept of decoupling and discusses several inequalities including Burkholder-Davis-Gundy inequalities.

Chapter 7 presents the principle of conditioning which roughly says that the convergence properties of decoupled sums are shared by the original sums. Applications include proofs of the three series theorem for dependent variables and the martingale central limit theorem by using the corresponding results for independent sequences. Another notable application is a central limit theorem for a sequence of dependent two by two tables where standard limit theorems do not seem to be applicable. There are also powerful decoupling inequalities for the L_p $(p > 1)$ norms of a sum of arbitrary random variables.

Chapter 8 deals with randomly stopped U-statistics and establishes Wald's equation, moment bounds and moment convergence in Anscombe's random central limit theorem. The last section of this chapter proves exponential inequalities for martingales normed by its conditional variances. The general principle which emerges is that when extending results for sums of independent random variables to martingales and to the ratio of martingale over its conditional variance, one should replace the variance by the conditional variance and the exponential of a function of the variance by the expectation of the exponential of the same function of the conditional variance.

The book is very well written with ample motivation and discussion being provided for all topics. It is a pleasure to read, except for the abundant typographic errors. We hope these will be corrected in the next edition. There is enough interesting material for anyone who works in statistics and probability, irrespective of whether he/she works in decoupling. Read it.

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