FARM SIZE AND PRODUCTIVITY*
IMPLICATIONS OF CHOICE UNDER UNCERTAINTY

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SUMMARY. One of the recurring themes in the recent literature on Indian Agriculture is the alleged decline in farm productivity (yield per hectare) as farm size increases. A number of explanations of this phenomenon based on differences in the quantity and quality of inputs as between large and small farms, have been offered in the literature. Some of these explanations depend on imperfections in input markets. An alternative explanation is offered in this paper that attributes the decline in productivity to the optimal response (in terms of inputs used) of a farmer to a situation of uncertainty relating to yield per hectare due to vagaries of weather. It is shown that even in the absence of imperfections in input markets and of differences in quality of land due to differing irrigation facilities, it may still be *optimal* for a small farmer to use more inputs per hectare (and hence obtain higher expected yield) than a large farmer, provided all farmers have the same utility function for income that exhibits non-increasing absolute and non-decreasing relative risk aversion as income increases. Some remarks on uncertainty and the value of information are also offered.

1. Introduction

One of the recurring themes in the recent literature on Indian Agriculture is the alleged decline in farm productivity (yield per hectare) as farm size increases. Some of the more important contributions are listed at the end of this paper. Though Rudra (1968a; 1968b) has raised some serious doubts about the factual validity of this decline, as Saini (1969) has pointed out Rudra's discussion is not conclusive. It may not be altogether inappropriate to examine this issue once again.

Bhagwati and Chakravarti (1969) provide a penetrating discussion of the explanations of this phenomenon found in the literature and offer some of their own. For the present discussion, the following two explanations are of some relevance :

(a) the proportion of irrigated (and hence superior) land decreases as the size of holding increases, and

(b) labour and other inputs (per hectare) decrease as farm size increases.

It may be plausible to assume that the distribution of irrigation facilities cannot to changed, at least in the short run. However, the use of current inputs such as labour is indeed under the farmer's control, and one needs to explain why current inputs are used more intensively in small farms. Sen (1964b) offers an explanation in terms of a model where the smaller farms are family farms while the large farms are capitalist farms. In an extreme version of this explanation, the former are assumed to use only family labour and no member of a family with a small farm

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has any opportunity of working outside the family farm as a wage labourer. The capitalist farms are assumed to depend only on wage labour. He shows that if the real cost of labour on the small family farms is lower than the real wage the capitalist farmer has to pay, then family farms will use more labour (and other inputs complementary to labour) per hectare. Bhagwati and Chakravarti (1969) question the empirical validity of the labour market imperfection assumed by Sen.

We wish to offer an alternative explanation to this phenomenon of larger current input per hectare on small farms that is based on the optimal response of a farmer to a situation of uncertainty relating to yield per hectare due to the vagaries of weather. Such uncertainty is indeed an important aspect of agriculture in India. We show that even if there is no difference between large and small farms regarding the proportion of irrigated area to total area, and access to the labour market at a constant wage rate, it may nevertheless be *optimal* for a small farmer to use more labour per hectare than a large farmer. We assume that farmers maximize the expected value of the utility of their income, the utility function being the same for all farmers. This common utility function is assumed to satisfy the following two requirements :[1] (i) *absolute* risk aversion does not increase, and (ii) *relative* risk aversion does not decrease, as income increases. Arrow (1965) uses these two conditions to show that in the context of portfolio decisions, the first requirement implies that risky investment is not an inferior good and the second implies that non-risky investment, such as investment in cash balances, is a luxury good or more precisely, not a necessity. We shall comment later on the analogy between Arrow's portfolio model and our model.

Section 2 describes the model. In Section 3 the main results are derived. Section 4 offers some remarks on uncertainty and the value of information. In Section 5 some extensions of the model are suggested.

2. The model

We consider a farmer who owns H hectares of land of which a proportion a is irrigated. The farmer uses a current input,[2] labour, of which he can supply an amount $\bar{K}$ himself. It is assumed that labour is "committed" before the harvest is determined and hence is not subject to the uncertainty with weather. More precisely, the total output that the farmer obtains from his land is given by

$$Q = H[af(k_1)r_1+(1-a)f(k_2)r_2] \quad \dots \quad (1)$$

where $k_1(k_2)$ = labour input per hectare on irrigated (unirrigated) land

r_1, r_2 = random variables reflecting the influence of weather.

[1]The earlier draft assumed a constant relative risk aversion utility function as well as an additional condition to obtain the results. It is Professor Bardhan who showed that the additional condition was not needed and the results hold good with these two (more general) conditions on the utility function.

[2]It can be shown that our results hold in case of two current inputs under plausible assumptions regarding the substitutability of one current input for the other.

It will be noted that we are assuming the *same* production function, $f(k)$, for irrigated and unirrigated land, except for the influence of weather. This production function is of constant *returns* to scale in land and labour and concave.[8] Thus we have *a priori* ruled out diseconomics of scale (with land as scale factor) as an explanation of the observed decline in productivity of larger farms. The higher yield on an irrigated hectare of land as compared to an unirrigated hectare for the same labour input is incorporated in the random terms r_1 and r_2 through the following structure :

$$r_i = v_i w + m_i \qquad i = 1, 2, \ldots \qquad \ldots \quad (2)$$

where v_i and m_i are constants and w is a random variable representing weather. This weather variable w is assumed to represent all climatological factors relevant to crop yield such as rainfall, temperature, sunshine hours, etc. With a suitable composite index of weather, it is reasonable to postulate that yield is an increasing function of the index. We assume that $m_1 > m_2 > 0$, $0 < v_1 < v_2$ and $E(w) = 0$, $V(w) = \sigma^2$ where E and V represent the expected value and variance operators. We assume that the range of w is contained in the interval $\left[\frac{-m_2}{v_2}, \frac{m_1-m_2}{v_2-v_1}\right]$. With this structure it is clear that $r_1 > r_2 \geqslant 0$ for all realisations of w.

The following implications of the structure (2) are evident :

(a) For a given labour/land ratio k the yield per hectare on irrigated land, viz. $f(k)r_1$, will be greater than the yield $f(k)r_2$ on unirrigated land for all values of the weather variable w. As a consequence the mean yield on irrigated land, namely, $f(k)m_1$ exceeds the mean yield $f(k)m_2$ on unirrigated land.

(b) The assumption $v_1 < v_2$ implies that the variance in yield per hectare of irrigated land, viz. $\{v_1 f(k)\}^2$ is *less* than the variance of yield in unirrigated land, i.e. $\{v_2 f(k)\}^2$. This characterisation of mean and variance of yield in the two categories of land is in accord with the received knowledge on irrigation. Very often irrigation is treated as a land augmenting technical change. Thus it is claimed that one irrigated hectare is equivalent to so many unirrigated hectares. However, if the influence of weather enters as a *multiplicative* factor on yield (as above), this way of treating an irrigation will imply that *both* mean and variance of yield per hectare on irrigated land is higher than on unirrigated land ! Our formulation avoids this.

The farmer's net income is

$$Y = Q + q\{\overline{K} - aHk_1 - (1-a)Hk_2\} \qquad \ldots \quad (3)$$

where $\overline{K}$ is the total labour the farmer himself can supply. It is obvious from (3) that the farmer is assumed to sell his surplus labour at a wage q in terms of output if $\overline{K} > H(ak_1+(1-a)k_2)$ or buy his excess labour requirements at the same wage q if $\overline{K} < H(ak_1+(1-a)k_2)$. The farmer is assumed to maximize his expected utility

[8] Saini (1969) offers some evidence in support of the constant returns to scale production function.

$EU(Y)$ by his choice of k_1 and k_2. The utility function is assumed to be strictly concave with a positive marginal utility of income for all Y. For deriving some of our results we shall further assume : (i) the measure of absolute risk aversion, $\frac{-U''(Y)}{U'(Y)}$, is a nonincreasing function of Y, and (ii) the measure of relative risk aversion $\frac{-U''(Y)Y}{U'(Y)}$ is a nondecreasing function of Y. The first assumption implies, as Arrow points out (1965), that the willingness to engage in small bits of a fixed size does not decrease as income increases. The second assumption implies that if both the size of the bet and income are increased in the same proportion, the willingness to accept the bet does not increase.

The first order conditions for $EU(Y)$ to attain its maximum are (assuming that differentiation under the expectation operator is permitted) :

$$E\{U'(Y)aH(f_1^1r_1-q)\} = 0 \quad \dots \quad (4)$$

$$E\{U'(Y)(1-a)H(f_1^2r_2-q)\} = 0 \quad \dots \quad (5)$$

where $f_1^1r_1$ represents the marginal product of labour on irrigated land and $f_1^2r_2$ represents the marginal product of labour on unirrigated land. Cancelling out the terms aH and $(1-a)H$ which do not depend on the random variables, we can rewrite the above conditions as

$$\frac{q}{f_1^1} = \frac{EU'r_1}{EU'} \equiv r_1^* \equiv v_1w^*+m_1 \quad \dots \quad (6)$$

$$\frac{q}{f_1^2} = \frac{EU'r_2}{EU'} \equiv r_2^* \equiv v_2w^*+m_2 \quad \dots \quad (7)$$

where

$$w^* = \frac{EU'w}{EU'}. \quad \dots \quad (8)$$

By our assumption $r_1 > r_2$ for all values of the weather variable w. Hence if we set k_1, k_2 at their respective optimal values k_1^*, k_2^* and evaluate $r_1^* \left(= \frac{EU'r_1}{EU'}\right)$ and $r_2^* \left(= \frac{EU'r_2}{EU'}\right)$, we will find $r_1^* > r_2^*$. This implies that $f_1^1 = f_1(k_1^*) < f_1^2 = f_1(k_2^*)$, i.e., the marginal product[4] of labour is lower in irrigated land when labour input is used at its respective optimal levels in each type of land. From the concavity of f, it follows that $k_1^* > k_2^*$ which implies that the optimal labour input per hectare of irrigated land is higher than that on unirrigated land.

[4]The marginal products (for a given relisation of w) are really $f_1^1r_1$, $f_1^2r_2$. For brevity we have called f_1^1, f_1^2, etc., as marginal products rather than more precisely as marginal products when the weather factors r_i are kept constant at unity.

Let us now explore the relationship between r_i^* and Er_i for $i = 1, 2$. Using our characterisation of r_1 and r_2 we get

$$Y(w) \, (\gtreqless) \, Y(w^*) \text{ according as } w \, (\gtreqless) \, w^*.$$

Now Y is an increasing function of both r_1 and r_2 which are in turn increasing functions of w. Hence Y is an increasing function of w. Hence :

$$Y(w) \, (\gtreqless) \, Y(w^*) \text{ according as } w \, (\gtreqless) \, w^*.$$

This together with the fact U is strictly concave (U' is a decreasing function of Y) implies :

$$U'(Y(w))(w-w^*) < U'(Y(w^*))(w-w^*). \qquad \ldots \quad (9)$$

Thus

$$EU'(Y(w))(w-w^*) < U'(Y(w^*))E(w-w^*). \qquad \ldots \quad (10)$$

But from (4) and (6) we know $EU'(Y(w))(w-w^*) = 0$. Hence we get $E(w-w^*) > 0$ or $w^* < Ew$, implying also that $r_i^* < Er_i$. We can interpret these results as follows. It can be seen from (6) and (7) that a risk-neutral farmer (i.e. one for whom $U'(Y)$ is a positive constant) will (in an optimal situation) equate the expected marginal product of labour input to its price, i.e., he will allocate resources so that $q = f_1^i Er_i$. But a risk averting farmer so allocates his resources that q equals $f_1^i r_i^*$. Since $r_i^* < Er_i$ this means that $f_1^i (i = 1, 2)$ for the risk-neutral farmer has to be smaller than the corresponding f_1^i for the risk-neutral farmer. This would immediately imply that the risk-neutral farmer will use more of labour input per hectare than the risk averting one.

3. Farm size and productivity

We shall first evaluate the sign of $\frac{\partial k_i^*}{\partial H}$. This will enable us to answer the question whether k_i^* decreases as H increases, all other things remaining the same. We shall then evaluate the signs of $\frac{\partial k_i^*}{\partial K}$, and $\frac{\partial k_i^*}{\partial a}$. These, together with plausible assumptions about direction of change of a and $\overline{K}$ as H increases will enable us to determine the signs of $\frac{\partial k_i^*}{\partial H}$, thus answering the question whether the optimal levels of labour input per hectare decrease with farm size when changes in irrigated area and own labour input consequent on the increase in farm size are taken into account.

Eliminating w^* from (6) and (7) we get :

$$\frac{1}{v_1}\left(\frac{q}{f_1^1}-m_1\right) = \frac{1}{v_2}\left(\frac{q}{f_1^2}-m_2\right). \qquad \ldots \quad (11)$$

Differentiating partially both sides of (11) yields

$$-\frac{1}{v_1}\frac{q}{(f_1^1)^2}f_{11}^1\frac{\partial k_1^*}{\partial -}=-\frac{1}{v_2}\frac{q}{(f_1^2)^2}f_{11}^2\frac{\partial k_2^*}{\partial -} \quad \ldots \quad (12)$$

where f_{11}^i denote the second derivative of f^i with respect to k_i and $\frac{\partial k_i^*}{\partial -}$ denote $\frac{\partial k_i^*}{\partial H}$, $\frac{\partial k_i^*}{\partial a}$ and $\frac{\partial k_i^*}{\partial K}$ as required. It is easily seen from (12) that $\frac{\partial k_1^*}{\partial -}$ and $\frac{\partial k_2^*}{\partial -}$ are of the same sign and hence it is enough to evaluate one of the two. For convenience let us rewrite (12) as

$$\frac{\partial k_2^*}{\partial -}=g\frac{\partial k_1^*}{\partial -} \text{ where } g=\frac{v_2(f_1^2)^2f_{11}^1}{v_1(f_1^1)^2f_{11}^2}>0.$$

Differentiating (4) after cancelling aH and using (12) we get

$$\frac{\partial k_1^*}{\partial -}=\frac{N}{D} \quad \ldots \quad (13)$$

where

$$N=-EU''(f_1^1r_1-q)\frac{\partial Y}{\partial -}=-f_1^1v_1EU''(w-w^*)\frac{\partial Y}{\partial -} \quad \ldots \quad (14)$$

and

$$D=f_{11}^1EU'r_1+aHEU''(f^1r_1-q)^2+(1-a)HgEU''(f_1^1r_1-q)(f_1^2r_2-q)$$

$$=f_{11}^1EU'r_1+(f_1^1v)^2aHEU''(w-w^*)^2+(1-a)Hgf_1^1f_1^2v_1v_2EU''(w-w^*)^2.$$

Since $f_{11}^1<0$ (by concavity of f), $EU'r_1>0$ (given $U'>0$, $r_1\geqslant 0$), and $U''<0$ (by concavity of U) it is clear that D is negative. Hence the sign of $\frac{\partial k_1^*}{\partial -}$ is the same as that of $EU''(w-w^*)\frac{\partial Y}{\partial -}$

Now

$$EU''(w-w^*)\frac{\partial Y}{\partial H}=EU''(w-w^*)\left(\frac{Y-q\overline{K}}{H}\right)$$

$$=\frac{1}{H}EU''(w-w^*)Y-q\overline{K}EU''(w-w^*) \quad \ldots \quad (15)$$

$$EU''(w-w^*)\frac{\partial Y}{\partial a}=HEU''(w-w^*)[f^1r_1-f^2r_2-qk_1^*+qk_2^*]$$

$$=H\{(r_1^*f^1-r_2^*f^2)EU''(w-w^*)+(v_1f^1-v_2f^2)EU''(w-w^*)^2$$

$$+q(k_2^*-k_1^*)EU''(w-w^*)\} \quad \ldots \quad (16)$$

$$EU''(w-w^*)\frac{\partial Y}{\partial \overline{K}}=qEU''(w-w^*).$$

Let us now bring in the assumptions on the attitudes towards risk taking on the part of farmers. Our first assumption was that $\frac{-U''(Y)}{U'(Y)}$ is a non-increasing function of Y. Since Y is an increasing function of w this means that $\frac{-U''(Y)}{U'(Y)}$ is a non-increasing function of w. Hence

$$\frac{-U''(Y)(w-w^*)}{U'(Y)} \leqslant \frac{-U''\{Y(w^*)\}(w-w^*)}{U'\{Y(w^*)\}}.$$

Multiplying both sides by the positive quantity $U'(Y)$ and taking expectations we get

$$-EU''(Y)(w-w^*) \leqslant \frac{-U''\{(w^*)\}EU'(Y)(w-w^*)}{U'\{Y(w^*)\}}.$$

But (4) implies $EU'(Y)(w-w^*) = 0$. This in turn implies

$$EU''(Y)(w-w^*) \geqslant 0. \qquad \dots \quad (18)$$

Our second assumption on the utility function was that $\frac{-U''(Y)Y}{U'(Y)}$ is a non-decreasing function of Y. This, together with the fact that Y is an increasing function of w implies that $\frac{-U''(Y)Y}{U'(Y)}$ is a non-decreasing function of w. Hence

$$\frac{-U''(Y)Y(w-w^*)}{U'(Y)} \geqslant \frac{-U''\{Y(w^*)\}Y(W^*)(w-w^*)}{U'\{Y(w^*)\}}.$$

Again multiplying both sides by the positive number $U'(Y)$ and taking expectations we get

$$-EU''(Y)Y(w-w^*) \geqslant \frac{-U''\{Y(w^*)\}Y(w^*)EU'(Y)(w-w^*)}{U'\{Y(w^*)\}} = 0$$

or

$$EU''(Y)Y(w-w^*) \leqslant 0. \qquad \dots \quad (19)$$

We have already shown that $r_1^* > r_2^*$ and $k_1^* > k_2^*$. Hence $r_1^* f^1 - r_2^* f^2 = r_1^* f(k_1^*) - r_2^* f(k_2^*) > 0$. The sign of $v_1 f^1 - v_2 f^2$ will be negative (positive) according as the variance in yield per hectare of irrigated land (i.e. $(v_1 f^1 \sigma)^2$) is less (greater) than that on unirrigated land when optimal levels of labour input are used in both categories of land. It is plausible to assume that the variance in yields per hectare is less in irrigated areas. With this assumption it follows that

$$v_1 f^1 - v_2 f^2 < 0. \qquad \dots \quad (20)$$

Using (15)-(20) we conclude :

(1) A *ceteris paribus* increase in the size of holding H, will *decrease* labour input per hectare of irrigated as well as unirrigated land.

(2) A *ceteris paribus* increase in the own labour supply ($\overline{K}$) of the farmer will *increase* labour input per hectare of irrigated as well as unirrigated land;

(3) A *ceteris paribus* increase in the proportion of irrigated area will *increase* labour input per hectare on irrigated as well as unirrigated land.

If we further assume that both the own labour supply $\overline{K}$ of the farmer and the proportion a of irrigated area to total area decrease as farm size increases, we can conclude that, taking into account the variation with farm size of all relevant variables, *the optimal level of labour input (per hectare) will decrease (both on irrigated and unirrigated lands) as farm size increases resulting in higher expected yield per hectare on small farms.*

We mentioned earlier that our results are analagous to those of Arrow (1965) for a portfolio choice model involving a risky asset and a secure asset such as cash balances. The reason for this is essentially this : in our model the role of a secure asset is played by the availability of employment at a secure wage rate q and the role of a risky asset is that of cultivation. The farmer in part decides on the optimal allocation of his endowment of labour between these two activities. However, this analogy should be pushed too far since our model has no constraint corresponding to Arrow's wealth constraint.

4. Uncertainty and value of information

In the model of Sections 2 and 3 it was assumed that the farmer had no way of predicting the value of the weather variable w in any given season. He had knowledge only of the probability density function of w. In this section we explore the value of information (which in this context means information pertaining to the weather variable w) to the farmer. In order to simplify the discussion, let us consider a model where there is only one type of land and a single purchased input, nitrogen. Let the farmer have just one unit of land. Then his net income will be $Y = f(n)r-pn$ where r is a linear function of w. In the absence of any information pertaining to weather the farmer will maximize $EU(Y)$ leading to the first order condition.

$$EU'(Y)(f'r-p) = 0. \qquad \dots \quad (21)$$

Let the value of n satisfying (21) be denoted by n^*.

*We have stated the results in a slightly stronger form than our assumptions strictly would imply. But a slight strengthening of one of our assumptions to read that absolute risk aversion decreases with income will yield the stated results.

Suppose now this farmer is offered the knowledge of another variable s which is related to r or more precisely the farmer is offered the joint distribution $g(r, s)$ of r and s where s can be observed prior to r, i.e. prior to taking a decision on n. Suppose this information is available at a fixed cost c in terms of output. The maximum c that the farmer will be willing to pay is the value of this knowledge to the farmer. To evaluate this maximum let us proceed as follows :

Given that s can be observed prior to the realisation of r, the farmer will now maximize $EU(Y)$ where $Y = f(n)r - pr - c$ and the expectation is taken over the conditional distribution of r given s. The conditional density is $g(r, s)/h(s)$ where $h(s) = \int g(r, s)\,dr$ is the marginal density of s. Hence the first order condition of maximization can be written as

$$\frac{1}{h(s)} \int U'(Y)(f'(n)r - p)g(r, s)dr = 0$$

or

$$\int U'(Y)(f'(n)r - p)g(r, s)dr = 0. \qquad \ldots \quad (22)$$

The solution n^{**} of this equation will in general be a function of s. Let us denote it by $n^{**}(s)$.

When the farmer maximises his expected utility in the absence of the knowledge of $g(r, s)$ the expectation is with respect to the *marginal* density of r. We can therefore rewrite the earlier first order condition as

$$\int U'(Y)(f'(n)r - p)j(r)dr = 0 \text{ where } j(r) = \int g(r, s)ds \qquad \ldots \quad (23)$$

and $$Y = f(n)r - pn.$$

The solution of this equation, it will be recalled, is n^*. By sticking to n^*, the farmer realises

$$\int U(f(n^*)r - pn^*)j(r)dr = \int ds \int U((n^*)r - pn^*)g(r, s)dr$$

as his expected utility. On the other hand, if he buys the information on $g(r, s)$ and uses an amount $n^{**}(s)$ of nitrogen depending on the observed value of s, he will realise the following expected utility :

$$\int h(s)ds \int U\{f(n^{**})r - pn^{**} - c)\} \frac{g(r, s)dr}{h(s)}$$

or $$\int ds \int U(f(n^{**})r - pn^{**} - c)g(r, s)dr.$$

Hence the gain in expected utility through the use of $g(r, s)$ is

$$\int\int \{U(f(n^{**})r - pn^{**} - c) - U(f(n^*)r - pn^*)\}\, g(r, s)dr\, ds. \qquad \ldots \quad (24)$$

It should be obvious that the farmer will use the information on $g(r, s)$ if it were free, i.e. if $c = 0$! For, in that case the gain in utility is certainly positive, since the farmer can ensure himself as much utility as in the case without information, by sticking to n^*, i.e., by not using the information. However, when $c > 0$, buying the information and not using it will lower income and hence expected utility. The maximum value of c, i.e., the value of information, is that value of c which makes the gain in expected utility just equal zero. There is an unique value of c which will make this gain zero, since it is easily seen that the gain is a decreasing function of c. Let us denote this value of information by c^*. We have seen that $c^* > 0$. Let us now derive an upper bound for c^*.

By definition of c^* we have

$$\int\int U(Y^{**})\, g(r, s) dr\, ds = \int\int U(Y^*)\, g(r, s) dr\, ds$$

where $\quad Y^{**} = f(n^{**})r - pn^{**} - c^*$ and $Y^* = f(n^*)r - pn^*$.

Hence $\quad \{U(Y^{**}) - U(Y^*)\}\, g(r, s) dr\, ds = 0. \qquad \ldots \quad (25)$

By the concavity of U, $\quad U(Y^{**}) - U(Y^*) \leqslant U'(Y^*)(Y^{**} - Y^*)$.

By the concavity of f, $\quad Y^{**} - Y^* \leqslant (f'(n^*)r - p)(n^{**} - n^*) - c^*$.

Since $U'(Y^*) > 0$ we have

$$U(Y^{**}) - U(Y^*) \leqslant U'(Y^*)[(f'(n^*)r - p)(n^{**} - n^*) - c^*].$$

From the fact that $g(r, s)$ is a probability density function, it follows that it is nonnegative and hence

$$0 \leqslant \int\int U'(Y^*)[(f'(n^*)r - p)(n^{**} - n^*) - c]\, g(r, s) dr\, ds$$

or

$$c^* \leqslant \frac{\int\int U'(y^*)[f'(n^*)r - p)(n^{**} - n^*)]g(r, s) dr\, ds}{EU'(Y^*)} \qquad \ldots \quad (26)$$

This upper bound has a natural interpretation. With the information, the farmer's input of nitrogen changes to $n^{**}(s)$ from n^*. Now $U'(Y^*)(f'(n^*)r - p)$ is the marginal utility of nitrogen when it is used at the level n^*. Thus $(n^{**} - n^*)U'(Y^*)(f'(n^*)r - p)$ is the upper bound on the gain (or lower bound on the loss) in utility by changing to n^{**} from n^*. It is only a bound since it ignores the fact that marginal productivity of nitrogen as well as marginal utility of income are diminishing. Thus the numerator of the above expression is the maximum gain in expected utility when information is utilized. The denominator of course is the expected marginal utility of income or the 'price' of a unit of income in terms of utility. Hence by dividing an upper bound for the expected gain in utility by the utility price of income we get an upper bound for c^* in terms of its own unit, namely, unit of income.

5. Some possible extensions

Our model is admittedly very simple. It does however provide a possible rational explanation as to why small farmers may use current inputs more intensively than large farmers even though they face the *same* competitive market for their inputs, have the *same* constant returns to scale, concave production functions. However, this model has abstracted from several aspects of reality. Some of the more important of these are : (i) not all of farmer's current input decisions are taken before he has any knowledge of the behaviour of the random factor. This is true particularly of nitrogen. A decision as to the use of nitrogen (other than a basal dose) can be taken after observing the weather and growth of the plant for part of its growth duration. It is also true of labour used in harvesting : it depends on the size of the crop and the decision on it is taken only after all the weather uncertainty is over; (ii) the model completely ignores capital as an input and the farmer is assumed to have no saving and investment opportunities; (iii) it covers only the class of owner cultivators with no leased in or leased out are in their holdings.

A more sophisticated model will have to remedy the above omissions. It can be attempted along the following lines : instead of having a single random variable describing the influence of weather on yield, one can have at least two. The first one can relate to the weather conditions at the sowing and early growth stage of the crop and the second can relate to the flowering and grain or fruit formation stage. Inputs also could be divided into two groups : those on which decisions have to be taken prior to or during the first stage and those on which decisions are taken after the first stage. The decisions on the second group of inputs will naturally have the benefit of the knowledge of the actual weather conditions at the first stage. Consumption and saving decision can be incorporated into the model by using a dynamic programming approach[7] with capital as the state variable.

The naive "information" model of Section 4 is no more than illustrative. The economics of information, its production, dissemination and use in the field of agriculture is yet to be sufficiently explored.

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