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INDIAN STATISTICAL INSTITUTE

Statistical Methods III

Mid Semester Examination

B II, Semester I, 2011-12

Date: 07.09.2011 Time: 3 hours

[Total points 110. Maximum you can score is 100]

1. Suppose Z_1, Z_2 form a random sample of size 2 from a normal distribution with mean 0 and variance 1, while X_1 and X_2 form another independent random sample from a normal distribution with mean 1 and variance 1. Let $\bar{Z}$ and $\bar{X}$ be the sample means of the Z and the X observations respectively. Find the distribution of the following quantities.

   (a) $\bar{Z} + \bar{X}$

   (b) $\dfrac{(Z_1 + Z_2)}{\sqrt{[(X_2 - X_1)^2 + (Z_2 - Z_1)^2]/2}}$

   (c) $[(Z_1 + Z_2)^2 + (X_1 + X_2)^2 + (X_1 - X_2)^2]/2$

   (d) $\dfrac{(X_2 + X_1 - 2)^2}{(X_2 - X_1)^2}$

   [3 + 5 + 5 + 5 = 18 points]

2. (a) The national transportation safety board wants to examine the safety of compact cars, midsize cars and full-size cars. It collects a sample of 3 for each car type. Using the data provided below, test whether the mean pressure applied to the driver's head during a crash test is equal for each of the three type of cars. Write down the model and perform an appropriate test at level $\alpha = 0.05$.

	Compact cars	Midsize cars	Full-size cars
	643	469	484
	655	427	457
	702	525	402
Sample mean	666.67	473.67	447.33
Sample sd	31.18	49.17	41.68

   The sample standard deviations are calculated with denominator $n - 1$.

   (b) Derive the likelihood ratio test for the usual One Way Analysis of Variance model hypothesis of equality of means. Show that under the standard assumptions this test is equivalent to the usual Analysis of Variance F test. [12 + 10 = 22 points]

3. (a) Let $X_i \sim N(i\theta, 1)$, $i = 1, \ldots, 5$, where the X_is are independent. Find the Cramer-Rao lower bound to the variance of an unbiased estimator of θ. Also find a 95% confidence interval for θ, given $X_1 = 10, X_2 = 18, X_3 = 33, X_4 = 44$ and $X_5 = 40$.

(b) Let X_i, $i = 1, 2, \ldots, n$, be exponentially distributed with mean $i\theta$, where X_is are independent. Find the maximum likelihood estimator of θ. Is it an efficient estimator? [12 + 12 = 24 points]

4. (a) We wish to test the effect of the drug Prozac on the well-being of depressed individuals. A test was given to 9 depressed individuals before and after the administration of the drug to obtain a "well-being" score that could range from 0 to 20. Higher scores indicate greater well-being. The scores are given in the table below.

Patient No.	1	2	3	4	5	6	7	8	9
Pre-Prozac score	3	5	0	7	4	3	2	1	4
Post-Prozac score	5	6	1	7	10	9	7	11	8

Does the administration of Prozac improve the "well-being" of depressed individuals? Perform an appropriate test of hypothesis at level $\alpha = 0.05$. State your assumptions explicitly.

(b) We want to know if there is a gender bias in the income structures of males and females. The annual incomes of 17 independently sampled males, and 15 independently sampled females in the United States of America in 1993 generated the following data (income reported in US$).

	Sample 1 (Female)	Sample 2 (Male)
Sample Mean	12279.4667	15266.4706
Sample sd	4144.6323	3947.7352
Sample Size (n)	15	17

The sample standard deviation (sd) is calculated with denominator $n - 1$.
Test whether there is enough evidence in the above data to claim that the mean for males is greater than the mean for females in the populations represented by the above samples. State your assumptions explicitly, and use $\alpha = 0.1$.

[12 + 12 = 24 points]

5. Suppose that X is a discrete random variable having probability mass function $f_\theta($ We are interested in testing the hypothesis $H_0 : \theta = 0$ versus $H_1 : \theta = 1$. The probability distribution of the random variable for these two values of θ are given in the following table. Derive the most powerful test of level $\alpha = 0.05$ for testing the above hypotheses.

x	0	1	2	3	4	5
$f_0(x)$	0.05	0.05	0.10	0.10	0.20	0.50
$f_1(x)$	0.10	0.15	0.25	0.15	0.25	0.10

[12 points]

6. Let $X \sim Bernoulli(\theta)$, and suppose that our interest is in determining the Bayes estimator of θ. Suppose a random sample of size n shows x success' and $(n - x)$ failures.

   Suppose that the loss function is the squared error loss and the prior has a beta distribution with parameters a and b. Show that the Bayes estimator of θ for this problem is a simple linear combination of the sample proportion of success and the prior mean. [10 points]

INDIAN STATISTICAL INSTITUTE

Mid Semestral Examination : (2011-2012)

B. Stat. II Year

Physics II

Date: 09.09.2011 Maximum Marks 40 Duration $2\frac{1}{2}$ hour

Group A

Answer any three questions

1 (a) Consider a system of particles, all are in rest, and there is no external force on the system. Let the system explodes and the constituent particles of the system accelerate. Explain how the conservation of momentum still holds true.

(b) Discuss how the Coriolis force arises in a rotating frame of reference? Under what condition this force vanishes?

(c) Let in the Northern Hemisphere, a particle of mass m is falling freely from a highet h at a geographical latitude 30^o. show that there will be a deflection of the particle in the Eastward direction given by $\omega\sqrt{\frac{2h^3}{3g}}$, ω being the rotational velocity of Earth about its own axis.

$3+3+4$

2(a) Consider the central force $\vec{F}(\vec{r}) = F(r)\frac{\vec{r}}{r}$. Show that $\vec{\nabla} \times \vec{F} = 0$.

(b) Consider a central force $-\frac{K\vec{r}}{r^{n+1}}$ acting on a particle, k being a positive constant. Show that under the action of the central force, the total mechanical energy is conserved.

(c) Show that under the action of any central force, the motion of a particle is confined to a fixed plane.

$4+4+2$

3(a) A particle of mass m is moving under a repulsive inverse square law of force, where the force per unit mass is $\frac{k\vec{r}}{r^3}$, k being a positive constant. Find the nature of the orbit.

INDIAN STATISTICAL INSTITUTE

Mid Semestral Examination : (2011-2012)

B. Stat. II Year

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$4+4+2$

3(a) A particle of mass m is moving under a repulsive inverse square law of force, where the force per unit mass is $\frac{k\vec{r}}{r^3}$, k being a positive constant. Find the nature of the orbit.

(b) A particle moves in a circle under the action of a central force directed to a fixed point on the circumference of the circle. Find the law of the force.

(c) If a particle of mass m moves in a elliptic path under the action of a central force $\frac{-k\vec{r}}{r^3}$, the speed v of the particle is given by

$$v^2 = \frac{k}{m}\left(\frac{2}{r} - \frac{1}{a}\right)$$

a being the semi major axis of the ellipse. $4 + 2 + 4$

4(a) In an inertial frame a child was born and died after 1 hour at same place. Consider another inertial frame having relative velocity $\frac{4c}{5}$. Calculate the life span of the child with respect to an observer in the later frame.

(b) With respect to an inertial observer, two particles from far ends are moving towards each other. The rate of increase of distance between the particles is $\frac{7c}{6}$ where the velocity of one of the particles is $\frac{2c}{3}$ with respect to the inertial observer. Find the relative velocity between the particles.

(c) In an inertial frame, two events have the space time coordinate $\{x_1, y, z, t_1\}$ and $\{x_2, y, z, t_2\}$ respectively. Let $x_2 - x_1 = 3c(t_2 - t_1)$. Consider another inertial frame which moves along x-axis with velocity u w.r.t. the first one. Find the value of u for which the the events are simultaneous in the later frame.
(In all the problems in questions (4), c represents the velocity of light in vacuum) $2 + 4 + 4$

Group B

Answer any one question

1 The vector field $\vec{F}$ is defined as:

$$\vec{F} = 2xz\hat{i} + 2yz^2\hat{j} + (x^2 + 2y^2z - 1)\hat{k}$$

$\hat{i}$, $\hat{j}$, $\hat{k}$ being unit vectors along three orthogonal directions.

(a) Calculate $\vec{\nabla} \times \vec{F}$

(b) What can be said about the line integral $\int_c \vec{F}.d\vec{l}$ for any close curve c?

(c) Argue that one can construct a scalar function ϕ such that $\vec{F} = \vec{\nabla}\phi$.

4 + 3 + 3

2 (a) Evaluate $\int\int_S \vec{A}.d\vec{S}$, where $\vec{A} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

(b) Find electric field due to a infinite line charge with charge density ρ.

(c) Prove the identity:

$$\vec{\nabla}.(\vec{A} \times \vec{B}) = \vec{B}.(\vec{\nabla} \times \vec{A}) - \vec{A}.(\vec{\nabla} \times \vec{B})$$

4 + 3 + 3

Indian Statistical Institute

Mid-Semester Examination: 2011

Course Name: B. Stat II, **Subject name:** Biology I

Date: 09.09.11 *Maximum Marks:* 40; *Duration: 2.5 hrs*

All questions carry equal marks, answer any five

1. a. During physical activity we may feel muscle pain: what could be the reason in terms of glucose metabolism? [2]

   b. How Alanine [CH_3-CH(NH_2)-COOH] and Aspartic acid [COOH-CH_2-CH(NH_2)-COOH] are metabolized to generate ATP. [4]

   c. What happens when lemon is added to warm milk and why? [2]

2. Explain primary, secondary, tertiary and quaternary structures of hemoglobin. Lactate dehydrogenase exists in five different isoenzyme forms, how can each isoenzyme and each subunit of each isoenzyme be distinguished from each other? [4+4]

3. Distinguish between the following:

   a. Monoclonal and polyclonal antibodies [4]

   b. Phenylketonuria and alkaptonuria [4]

4. a. Why aerobic catabolism of glucose is more efficient than anaerobic metabolism of glucose in production of ATP? [4]

   b. Write down two NADH generating reactions in TCA or Krebs cycle. [4]

5 Why do the changes in pH and temperature affect the activity of an enzyme? What will be the net electrical charge of a protein below and above of isoecletric pH of that protein and why? [4+4]

6. Mention the distinguishing steps in the metabolism of these two fatty acids: **[$CH_3(CH_2)_8CH{=}CH(CH_2)_6COOH$] and [$CH_3(CH_2)_7CH{=}CH(CH_2)_7COOH$].**
In the metabolism of long chain saturated fatty acids, sometimes only acetyl~CoA is generated but in some cases propionyl~CoA and acetyl~CoA are generated: explain. (6+2)

Mid-Semester Examination 2011

9.9.11

Course – B Stat II Year (2011-2012)

Subject – Microeconomics

Maximum Marks 40 Duration 2.5 Hrs

Answer all the following questions

1. Consider a commodity bundle $x' \in$ consumption set X. Define the upper contour set Cx' and lower contour set Lx'. State the continuity axiom.
   What is the lexicographic preference rule? Explain with an example that it does not satisfy the continuity axiom.
   Show that under continuity axiom, Lx' and Cx' are closed and share a common boundary.

   (6+6+10=22)

2. a) Suppose a consumer's utility function is given by $U = \left(x_1^2 + x_2^2\right)^{\frac{1}{2}} - 30$; $p_1 = 3, p_2 = 2$ and budget M = 50. (All the symbols have usual meanings). What is the equilibrium consumption Basket?
   b) Suppose the utility function of a consumer is given by $\min\{2x_1 + x_2, 2x_2 + x_1\}$. The two commodities are bought and sold in the market at prices Rs.2 and Rs.3 respectively. To start with, the consumer has 15 units of commodity 1 only. What will be the equilibrium consumption basket of the consumer?
   c) Suppose the consumption set consists of two element commodity vectors. The prices p_1, p_2 and budget M are given. Initially, the consumer is in equilibrium. Show the change in equilibrium in a diagram following an increase in p_1. Consider two cases, viz, Case 1 and Case2. In Case 1, after the increase in p_1, the government adjusts the consumer's budget so that the consumer can just enjoy the previous utility level at minimum cost at the new set of prices. In Case 2, consumer's budget is adjusted by the government so that the consumer can just purchase the initial consumption basket at new prices. Compare the costs incurred by the government in the two situations.

   (8 +8+6=22)

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2011-12

B. Stat. II Yr.

Analysis III

Date: ~~20/08/2011~~ 12/09/11 Maximum Marks: 40 Duration: 3 Hours

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(1) Show that the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \frac{x^3y^4}{(x^2+y^6)^3}, & \text{if } x^2+y^2 \neq 0, \\ 0, & \text{if } x^2+y^2 = 0. \end{cases}$$

approaches 0 as (x,y) approaches $(0,0)$ along any straight line through $(0,0)$ but is not continuous at $(0,0)$. [4]

(2) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} (x-y)^2 \sin \frac{1}{x-y}, & \text{if } x \neq y, \\ 0, & \text{if } x = y. \end{cases}$$

(a) Show that both the partial derivatives $D_1 f$ and $D_2 f$ exist at each point of $\mathbb{R}^2$. Are they continuous?

(b) Show that f is differentiable at $(0,0)$. [4+6]

(3) Suppose that u, v, and f are functions from $\mathbb{R}$ to $\mathbb{R}$. Assume that u and v are differentable and f is continuous. Show that

$$\frac{d}{dx} \int_{u(x)}^{v(x)} f(t)\, dt = f\big(v(x)\big)v'(x) - f\big(u(x)\big)u'(x).$$

[5]

(4) Let $f = (f_1, f_2, \ldots, f_n) : \mathbb{R}^n \to \mathbb{R}^n$ be a differentiable function such that $f(0) = 0$. Assume that $\sum_{j=1}^{n} \sum_{k=1}^{n} |D_k f_j(0)|^2 = c < 1$. Let A denote the derivative matrix of f at 0.

(a) Show that $\|Ax\| \leq \sqrt{c}\|x\|$ for all $x \in \mathbb{R}^n$.

(b) Show that there exists $r > 0$ such that $f\Big(B(0,r)\Big) \subseteq B(0,r)$. [3+4]

(5) Find all the local maxima, local minima, and saddle points of the function

$$f(x,y) = 2x^3 + 2y^3 - 9x^2 + 3y^2 - 12y.$$

[6]

(6) (a) Let $M_{2\times 2}$ be the space of all real 2×2 matrices. For $r > 0$, let $S_r = \{A = (a_{ij})_{i,j=1}^2 \in M_{2\times 2} : |a_{ij}| \leq r\}$. Define $F : M_{2\times 2} \to M_{2\times 2}$ by $F(X) = X^2 + X$. Show that the range of F contains S_r for some $r > 0$.

(b) Show that there exists an open set U of $\mathbb{R}^5$ containing $(1, \frac{1}{2}, -1, -2, 1)$ and an open set V of $\mathbb{R}^3$ containing $(1, \frac{1}{2}, -1)$ such that for every $(x,y,z) \in V$ there exists a unique $(u,v) \in \mathbb{R}^2$ with $(x,y,z,u,v) \in U$ and satisfying the equations

$$x^2 + 4y^2 + z^2 - 2u^2 + v^2 = -4 \quad \text{and} \quad (x+z)^2 + u - v = -3.$$

Define $G : V \to \mathbb{R}^2$ by $G(x,y,z) = (u,v)$, where (x,y,z) and (u,v), are related as above. Find the derivative matrix of G at $(1, \frac{1}{2}, -1)$. [5+5]

INDIAN STATISTICAL INSTITUTE

Periodical Examination

B.STAT - II Year (Semester - I)

C & Data Structure

Date 14.9.2011   Maximum Marks : 60   Duration : 3 Hours

Note : You may answer any part of any question, but maximum you can score is 60.

1. Write C code to merge two singly linked lists A and B. The elements of both the lists are stored in the following structure. `INFO` and `LINK` have their usual meaning, and both the lists are sorted with respect to their `INFO` field. Your output should be a linked list C, which is also sorted with respect to the `INFO` field.

```
typedef struct node *nodeptr
typedef struct {
char INFO[30];
nodeptr LINK;
} node;
```

[15]

2. Show that the number of binary trees, with nodes labeled $\{1, 2, \ldots, n\}$ and produces $1, 2, \ldots, n$ as the output of the preorder traversal, is $\frac{1}{n+1}\binom{2n}{n}$. [10]

3. Consider the following representation of a matrix whose elements are 0 or 1.

   Store the index-pair (i, j) of each 1 element of the matrix in an array, where the elements in the array are sorted lexicographically.

   Write an algorithm (following the syntax of C language) for multiplying the matrices A and B. You need not have to store the resultant matrix. After computing an element of the product matrix, print it along with its indices. Don't use more that 10 locations apart from the space required for storing A and B. [15]

4. Consider an array P containing a set of n points in $\mathbb{R}^2$, where $P = \{(x_i, y_i), i = 1, 2, \ldots, n\}$, and $n > 1000$. Assume that each x_i, y_i can be stored in a location defined in C using *float*. We need to compute the convex hull of the point set using at most 20 *integer* or *float* locations apart from the array P.

   (a) Write an algorithm for solving this problem.

   (b) Derive the time complexity of your algorithm.

   [15+5=20]

5. Let $\mathcal{T}$ be a binary tree whose nodes are of the following structure. The `INFO` fields of the nodes in $\mathcal{T}$ contain distinct key values.

```
typedef struct node *nodeptr
typedef struct {
int INFO;
int MAX;
nodeptr LLINK;
nodeptr RLINK;
} node;
```

Write an algorithm following the syntax of C language whose input will be the address of the *root* node of $\mathcal{T}$, and will fill the `MAX` field of each node v with the maximum value of the `INFO` fields of all the nodes in the subtree rooted at v. [15]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2011-12

B. Stat. II Yr. Analysis III

Date: 14/11/2011 Maximum Marks: 60 Duration: 3 Hours

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(1) (a) Give an example of a function defined on a rectangle $R = [a, b] \times [c, d]$ such that the iterated integral $\int_c^d \left(\int_a^b f(x, y)\, dx \right) dy$ exists, but f is not integrable on R.

(b) Let C be a smooth simple closed curve in the first quadrant $\{(x, y) \in \mathbb{R}^2 : x > 0, y > 0\}$. Show that

$$\oint_C \ln x \sin y\, dy = \oint_C \frac{\cos y}{x}\, dx.$$

(c) Calculate $\iint_R \frac{\sin x}{x}\, dxdy$, where R is the triangle in the xy-plane bounded by the x-axis, the line $x = y$ and the line $x = 1$.

(d) Let S be a parametric surface described by $z = f(x, y)$, where (x, y) varies over a plane region T, the projection of S in the xy-plane. Let $\vec{F}$ be a vector field on S. Show that

$$\iint_S \vec{F} \cdot \hat{n}\, dS = \iint_T (-Pf_x - Qf_y + R)\, dxdy,$$

where P, Q, R are components of $\vec{F}$. [3× 4=12]

(2) Let $L(\mathbb{R}^n, \mathbb{R}^m)$ be the set of linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^m$. Let $S \subset \mathbb{R}^n$ and $f : S \to \mathbb{R}^m$ be a function. Suppose that the map $f' : S \to L(\mathbb{R}^n, \mathbb{R}^m)$ is continuous. Prove that $D_j f_i$ is continuous for all i and j. [8]

(3) Evaluate the double integral $\iint_R (2x^2 - xy - y^2)\, dxdy$, where the region R is bounded by the lines $y = -2x + 4, y = -2x + 7, y = x - 2, y = x + 1$. [8]

(4) Let R be a region in $\mathbb{R}^2$ bounded by the polygon P with vertices

$$(x_1, y_1), (x_2, y_2), \ldots (x_n, y_n).$$

Find a formula for the area of R by using Green's theorem. [7]

(5) Let V be the region bounded by the sphere $x^2 + y^2 + z^2 = 25$ and the plane $z = 3$. Let S be the boundary of V. Evaluate $\iint_S \vec{F} \cdot \hat{n}\, dS$, where $\vec{F}(x, y, z) = (xz, yz, 1)$. [10]

(6) Let f and g be scalar fields with continuous first and second partial derivatives in a region D bounded by a smooth surface S. Show that

$$\iint_S (f\nabla g - g\nabla f) \cdot \hat{n}\, dS = \iiint_D (f\nabla^2 g - g\nabla^2 f)\, dxdydz.$$

[7]

(7) Show that $\oint_C y^2 dx + xy dy + xz dz = 0$, where C is the intersection of the cylinder $x^2 + y^2 = 2y$ and the plane $y = z$. [10]

INDIAN STATISTICAL INSTITUTE

B Stat II, 1st Semester, 2011-12

Statistical Methoeds III

Semestral Examination

Date: 16.11.2011 Time: 3 hours

Total points 100. Answer all questions

1. Let $X_1, \ldots, X_n$ be a random sample from a distribution having the probability density function $f_\theta(x) = e^{-(x-\theta)}, x \geq \theta$.

   Find the method of moments estimator and the maximum likelihood estimator of θ. Are either of them unbiased? Which one would you prefer, and why. [20]

2. Suppose that the random variable of interest has a Uniform $(0, \theta)$ distribution.

   (a) Given a random sample $X_1, \ldots, X_n$ from the random variable of interest, find the most powerful test for the hypothesis $H_0 : \theta = \theta_0$ against $H_0 : \theta = \theta_1$, $\theta_1 > \theta_0$, at level α. Is the test unique?

   (b) Given a random sample $X_1, \ldots, X_n$ from the random variable of interest, find the most powerful test for the hypothesis $H_0 : \theta = \theta_0$ against $H_0 : \theta = \theta_1$, $\theta_1 < \theta_0$, at level α. Again check if this test is unique.

   (c) Given a random sample $X_1, \ldots, X_n$ from the random variable of interest, does there exist a UMP test for $H_0 : \theta \neq \theta_0$?

   [10+10+5=25]

3. The driver of a diesel operated automobile decided to test the quality of three types of diesel fuels sold in the area based on mpg (miles per gallon). The data (miles per gallon provided by randomly selected cars of the same type under the indicated fuel) are given below.

Brand A:	38.7	39.2	40.1	38.9	
Brand B:	41.9	42.3	41.3		
Brand C:	40.8	41.2	39.5	38.9	40.3

(a) Write down the standard assumptions of the analysis of variance test.

(b) Perform a test of the null hypothesis that the three means are equal using the above data and take $\alpha = 0.05$.

[3+12=15]

4. Suppose $\{a_t\}$ are i.i.d. random variables with mean 0 and variance σ^2. Define the series given by $Y_t = \mu + a_t + \frac{3}{4}a_{t-1}$ where μ is a constant. Find the mean, autocovariance function, and autocorrelation function of the series Y_t. Also determine whether Y_t is a stationary process.

Also consider the series defined by $X_t = \mu + e_t + \frac{4}{3}e_{t-1}$, where the e_t are i.i.d. with mean 0 and variance σ_e^2. Find the autocovariance and autocorrelation function of this series and compare with that of the previous one. [20]

5. (a) Discuss the different methods of analyzing the trend component of a time series.

(b) Given the following series, find the trend values by an appropriate method. [5+15=20]

Year	Quarter	Series
1	I	28
	II	33
	III	37
	IV	32
2	I	32
	II	37
	III	41
	IV	46
3	I	36
	II	41
	III	45
	IV	40
4	I	40
	II	33
	III	37
	IV	32

Indian Statistical Institute

First Semester Examination 2011-2012

Course Name: B Stat II Year

Subject Name: Economics I

Date 18 /11/2011 Maximum marks: 60 Duration 2.5 Hours

Answer all questions

1. (a) The cost function faced by a competitive firm is given by

$$C(q) = TVC(q) + F$$

where F is a constant and $TVC(q)$ (total variable cost) is upward rising and strictly convex in q with $TVC(0) = 0$. Now consider two cases, namely, Case 1 and Case 2. In Case 1, $C(0) = 0$ and in Case 2, $C(0) = F$. Draw the AVC, AC and MC schedules in a diagram and explain. Hence derive a competitive firm's supply curves in the above mentioned two cases.

(b) Supply function of a firm is given by S(p) = 4p. Suppose the price p changes from 10 to 15. What is the change in the producer's surplus. [15+7=22]

2. (a) A monopolist successfully sells his products in two different markets at two different prices. The demand curves in the two different markets are $P = 40 - 0.5\,Q_1$ and $P = 18 - 0.25Q_2$ respectively. The monopolist's marginal cost of production is 10 units. Calculate the profit maximizing output and price in the two different markets.

(b) A monopolist faces the market demand curve P=100-5Q, while the total cost of the monopolist is TC = 300 + 20 Q. The monopolist practices perfect price discrimination. What will the monopolist's output and profit be? [11+11=22]

3. In an oligopolistic industry price is found not to respond to all changes in cost conditions. Discuss a suitable model to explain this situation. [22]

or

Consider a two-firm oligopoly industry. Suppose firm 1 producing a differentiated product faces an inverse demand function $P_1 = 100 - 2Q_1 - Q_2$ and have a cost function $C_1 = 2.5Q_1^2$.

Assume that firm 2 wishes to maintain a market share of $\frac{1}{3}$. Find the optimal price, output and profit of firm 1. Find the output of firm 2.

Indian Statistical Institute

18.11.11

Semester Examination (B. Stat-II, Biology I, Year-2011)

Answer any five; All questions carry equal marks; Full marks = 50; Time = 2.5 hours

1. (a) If mother and son both are color blind, is it likely that the son inherited the trait from his father? Show possible genotypes with a pedigree diagram. (5)

(b) How many different DNA sequences are possible with 6 codons: ATG, TAA, AGG, GGG, CGT and TAC provided there will not be five consecutive "G"s within the sequences. (5)

2. In rabbits, the dominant allele ***B*** causes black fur and the recessive allele ***b*** causes brown fur. Two alleles, dominant ***R*** and recessive ***r***, of another independently assorting gene can cause long and short fur respectively. A homozygous rabbit with long and black fur is crossed with a rabbit with short and brown fur. And the offspring is intercrossed. In the F_2 generation, what proportion of rabbits, with long and black fur, will be homozygous for both genes? [10]

3. Distinguish between DNA transcription and replication. Mention the effects of substitution, deletion, non-sense and mis-sense mutations in a gene on the activity of protein to be synthesized from the gene. What do you mean by the statement that "genetic code is degenerate and comma-less"? [3+4+3]

4. (A) In a random copolymer of $(AC)_n$ what will be the frequencies of different triplet codons if (a) A:C =1:1 and (b) A:C=5:1 in the copolymers? [5]

(B) Within a cycle of PCR, "initially the temperature of the reaction is raised to 95^0C, then temperature is lowered to 55^0C and finally temperature is raised to 72^0C": Answer with reasoning why the temperature changes are necessary for the reaction. [5]

5. (a) Why multiple "origin of replication" is needed in the replication of human whole DNA? [3]

(b) Why transcription and translation occur simultaneously in bacterial cell but not in human cell? [3]

(c) What happens when (a) pH is decreased and (b) temperature is increased in DNA and RNA solutions? [4]

P.T.O

6. A couple has four children and neither of the couple is bald. One of the two sons is bald b neither of the daughters is bald. If one of the daughters marries a non-bald man then what is t chance that their (a) son and (b) daughter will become bald at an adult age? Give your answ drawing a pedigree diagram with genotypes. (Assume: baldness is caused by one gene and one the two alleles behaves dominantly in male but recessively in female.) [10]

7. If both the parents are carriers of sickle cell anemia, a recessive disease caused by mutation one gene, then find out the chances that (a) all five children will be normal (b) four children w be normal and one will be affected by disease (c) at least three children will be affected disease and (d) the first child will be a normal girl. [2.5 x 4]

INDIAN STATISTICAL INSTITUTE

First-semestral Examination 2011-12

Course: B.Stat 2nd Year

Subject: Physics I

Maximum marks: 100 Date: 18../11/11 Duration: 3 hrs

Note: Use different answer sheets for different groups

Group A

Answer any five questions
If not stated otherwise, the symbols have their usual meaning

1. (a)Prove that: $(\vec{A} \times \vec{B}).(\vec{C} \times \vec{D}) + (\vec{B} \times \vec{C}).(\vec{A} \times \vec{D}) + (\vec{C} \times \vec{A}).(\vec{B} \times \vec{D}) = 0$

(b)If $\vec{F} = \vec{\nabla}\phi$ everywhere in the region R and ϕ is a single valued scaler function with continuous derivatives in R, then show that $\int_A^B \vec{F}.d\vec{r}$ is independent of the path C in R joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, whenever R is simply connected. [5 + 5]

2. (a)A rigid body is rotating with an angular velocity 5 radian per second about an axis, which is parallel to the vector joining the points $(0, 0, 0)$ and $(1, -2, 2)$. The axis of rotation also passes through the point $(2, -3, 1)$. Find the velocity of the particle at the point $(4, -2, -1)$.

(b)A force of 15 units acts in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ and passes through the point $(2, -2, 2)$. Find the moment of the force about the point $(1, 1, 1)$.

(c)Find the constant a and b, so that the surface $ax^2 - byz - (a+2)x = 0$ will be orthogonal to the surface $4x^2y + z^3 - 4 = 0$ at the point (1,-1,2). [4+3+3]

3. (a)State Gauss's theorem and show that for a dielectric medium the differential form of the law reads $\nabla.\vec{D} = \rho_f$.

(b)Show that electric field due to a spherical charge distribution of density $\rho = \frac{2k}{r^2}$ is given by $\vec{E} = \frac{2k}{\varepsilon_0 r}\hat{r}$.

(c)A spherical shell of inner radius r_1 and outer radius r_2 is uniformly charged with charge density ρ. Calculate the electric field at a distance r from the center of the spherical shell for (i) $r > r_2$, (ii) $r_1 \leq r \leq r_2$, (iii) $r < r_1$. [(1+2)+2+5]

4. (a)Let two dipoles with moments $\vec{p_1}$ and $\vec{p_2}$ be separated by a distance $\vec{r}$. Find the interaction energy between the two dipoles. Also find the torque experienced by the dipole $\vec{p_2}$ due to the presence of the dipole $\vec{p_1}$.

(b)A parabola $y^2 = 4ax$ extends from $x = 0$ to $x = a$. A uniform line charge is placed along this parabola. The charge per unit length is λ. Prove that, the electrostatic potential at the focus of the parabola is $\frac{\lambda}{\pi\varepsilon_0}\ln(1+\sqrt{2})$. [5+5]

5. (a)Prove from the statement of Biot-Savart law, that $\vec{\nabla}.\vec{B} = 0$. What physical inference can be drawn from this expression?

(b)Two identical coaxial circular coils (each having radius a, and N turns of wire/length) are kept with their axis parallel to each other and with a separation of $2d$. It is given that $a < 2d$. One end of the first coil is connected to the end of the other in such a way that they effectively constitute a circuit in series. A current I is now switched on and flows through this series combination of two coils. Find the expression for the magnetic field in the region between the two coils and show that there is a region where this magnetic field assumes the least value. Radius of both coils are 'a'.It is given that $a < 2d$ Same current is flowing to both coils in series. Find where (between the line joining their centers) the resultant magnetic field will be minimum. [3+1+6]

6. (a)Maxwell's equations are given as $\nabla.\vec{E} = \frac{\rho}{\varepsilon_0}$, $\nabla.\vec{B} = 0$, $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ and $\nabla \times \vec{B} = \mu_0(\vec{J} + \varepsilon_0\frac{\partial \vec{E}}{\partial t})$. Now for source free case, prove that electric and magnetic field, each obeys the wave equation $\nabla^2 U(\vec{r},t) = \frac{1}{c^2}\frac{\partial^2 U(\vec{r},t)}{\partial t^2}$ $[U(\vec{r},t) \equiv \vec{E}(\vec{r},t), \vec{B}(\vec{r},t)]$. Where $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ is the velocity of electromagnetic wave.

(b)What is magnetic vector potential $(\vec{A})$? Find the expression of magnetic vector potential from Biot-Savart law. If $\nabla.\vec{A} = 0$, then what will be the form of Ampere's circuital law in terms of vector potential? [6+1+1+2]

Group B

Answer all questions

If not stated otherwise, the symbols have their usual meaning

1. (a)Define action in Lagrangian mechanics. What is the dimension of action?

(b)State the principle of stationary action.

(c)Derive Euler-Lagrange equation for a particle with non-zero mass 'm' using the principle of stationary action.

(d)Consider a binary star system. (i) Write down the Lagrangian for this system in terms of the polar co-ordinates of the two stars with radius vectors $\vec{r_1}$ and $\vec{r_2}$. (ii) Show that the potential energy is a homogeneous function of co-ordinates of degree -1 obeying $V(\alpha r_1, \alpha r_2) = \alpha^{-1}V(r_1, r_2)$. Where α is a real scaling parameter. [(1+1)+1+3+(2+2)]

2. (a)What is a cyclic co-ordinate?

(b)Consider a Lagrangian in 3-dimension for a ball of mass 'm' in the air as $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mgz$. What are the cyclic co-ordinates for this system? Calculate the conserved quantities for this Lagrangian.

(c)Assume the Lagrangian for certain 1-dimensional motion is given be $L = e^{\gamma t}(\frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2)$where$\gamma, m, k$ are positive constants.What is the Lagrange's equation for this system?Is there any constant of motion?How would you describe the motion?[1+(1+2)+(3+1+1+1)]

3. (a)Show that the Euler-Lagrangian equations are equivalent to Hamilton's equation under suitable (but sufficiently general) assumptions.

(b)State Noether's theorem.

(c)Consider a Lagrangian $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}k(x^2 + y^2)$, which is invariant under the change of co-ordinates $x \rightarrow x + \epsilon y$ and $y \rightarrow y - \epsilon x$, to the first order in ϵ. What is the conserved quantity for this? What is the physical interpretation for these conserved quantities? [3+1+(3+3)]

4. (a)Show that if a particle subject to a central force only, then it's angular momentum is conserved and the motion takes place in a plane.

(b)State Kepler's laws of planetary motion. Prove that the square of the time period of an orbit(T) is proportional to the cube of the length of the semi-major axis(a).

(c)A particle of mass 'm' is bound by a linear potential $U = Kr$. (i) For what energy and angular momentum will the orbit be a circle of radius r about the origin? (ii) What is the frequency of the circular motion? [(2+1)+(1+2)+(3+1)]

5. (a) Two space ships are moving in space with velocities $\vec{V_1}$ and $\vec{V_2}$ with respect to a reference frame fixed on the earth.It is given that $(V_1)_x = \frac{c}{0}, (V_1)_y = (V_1)_z = 0$ and $(V_2)_x = (V_2)_y = \frac{c}{20}; (V_x)_z = 0$ where c is the velocity of light in vacuum. Find the velocity of the second space ship relative to the first one. $(V_1)_x$ etc represent the x-component of the velocity V_1 etc.

(b) A force is applied to a particle at rest. After time T, the velocity of the particle is v. Find the work done on the particle. Obtain the non-relativistic limit of the expression for the work done. [5+5]

PROBABILITY THEORY III
B. STAT. IIND YEAR SEMESTER 1
INDIAN STATISTICAL INSTITUTE

Semestral Examination
Time: 3 Hours   Full Marks: 50
Date: November 21, 2011

This is an OPEN NOTE examination. You may use any handwritten notes that you have brought with you, but you are not allowed to share them. Also no notes in printed, photocopied, cyclostyled or any other form (including electronic storage) is allowed.

1. Let (X_1, X_2) be bivariate normal with means μ_1, μ_2, variances σ_1^2, σ_2^2 and correlation ρ. Obtain the conditional distribution of X given Y and find the conditional mean and variance of X given Y. [6]

2. Consider two sequences $\{a_n\}$ and $\{b_n\}$ with $a_n > 0$ and $b_n > 1$ for all n and $\lim_{n\to\infty} a_n = 0$ and $\lim_{n\to\infty} b_n = 1$. Define A_n to be the interval $[a_n, b_n)$. Find $\limsup_{n\to\infty} A_n$ and $\liminf_{n\to\infty} A_n$. [8]

3. Let $\{X_n\}$ be an i.i.d. sequence with finite mean μ and variance $\sigma^2 > 0$. Let $\overline{X}_n = \frac{1}{n}\sum_1^n X_i$ and $S_n^2 = \frac{1}{n}\sum_1^n (X_i - \overline{X}_n)^2$ be the sample mean and variance respectively. Show that $\sqrt{n}(\overline{X}_n - \mu)/S_n$ converges weakly and find the limiting distribution. [4]

4. Let X_n be independent Poisson random variables with mean λ_n, where $\sum \lambda_n = \infty$ and let $S_n = X_1 + \cdots + X_n$. Show that $S_n/\mathrm{E}[S_n] \xrightarrow{P} 1$ and $S_n/\mathrm{E}[S_n] \to 1$ a.s. [3+7=10]

5. For some number $0 < \alpha \leq 2$, consider the characteristic function $\exp(-|t|^\alpha)$. (You do not have to prove that it is a characteristic function as it was done in the class.) Show that the corresponding distribution function has a density function, which is differentiable. (Caution: Do not try to obtain the density function explicitly.) [12]

6. Let $\{X_n\}$ be a sequence of uncorrelated random variables, that is, for any $i \neq j$, $\mathrm{cov}(X_i, X_j) = 0$. Further assume that there exists a constant c such that for all n, $\mathrm{E}[X_n^2] \leq c$. Show that, for any $\alpha > 1/2$, we have $n^{-\alpha}\sum_{i=1}^n X_i \xrightarrow{L^2} 0$. [10]

INDIAN STATISTICAL INSTITUTE

Semester Examination

B.STAT - II Year (Semester - I)

C & Data Structure

Date : 24.11.2011 Maximum Marks : 100 Duration : 3.5 Hours

Note : You may answer any part of any question, but the maximum you can score is 100.

1. Many numerical applications use square symmetric matrices in the upper triangular form, that is, the entries lying on or above the main diagonal are stored. We want to implement a data structure to store such a matrix A using minimum space. To do this we store the entries in a one-dimensional array, and use an indexing function to map a row-column pair (i, j) to an integer offset in this array.

   Write a C code for multiplying a matrix A by the same matrix A, and store the result in a linear array with minimum amount of space. [15]

2. A polynomial $f(x)$ is stored in computer memory using a linear linked list. State the variable declarations in C code to represent the polynomial $f(x)$.

   Write a computer program in C language for computing a polynomial $h(x)$ by multiplying two polynomials $f(x)$ and $g(x)$, where all the polynomials f, g and h are represented as above. [15]

2. T is a binary search tree. Its each node consists of three fields – (info, left, right). The left and right fields point to its left and right children. Let r be an internal node of T.

   (a) Write the pseudo-code following the syntax of the C programming language for the left- and right-rotation at node r.

   (b) Suppose T has $2^k - 1$ nodes for some $k > 0$, and it is a linear chain, where each node is connected with its adjacent node using its left link. Use left- and right-rotations at appropriate nodes to convert T to a perfectly balanced complete binary tree.

   [8+8+14=30]

3. You are given an unweighted directed graph $G = (V, E)$. The objective is to generate a graph $G' = (V, E')$ where $(u, v) \in E'$ if $u, v \in V$ and there is a directed path in G from u to v. Write an algorithm for this problem, and derive the time complexity of your algorithm. [12+4=16]

4.(a) Consider an insertion of the key $x = 227$ into the hash table shown in the figure. For each of the following probing methods, indicate the sequence of table entries that would be probed, and the final location of insertion for the key. Assume that $h(x) = 7$ for $x = 227$.

Table index	1	2	3	4	5	6	7	8	9
Key value			152	53		86		68	999

(i) Linear probing

(ii) Quadratic probing

(iii) Double hashing with $g(x) = 3$ for $x = 227$

(b) Show that for a suitable hash function that can assume any location with equal probability for each key value, the number of probes required to get a key value that already exists in the hash table is $O(\frac{1}{\lambda} \log_c \frac{1}{1-\lambda})$. Here $\lambda = \frac{n}{m}$, where n is the number of keys present in the table and m is the size of the table.

[(5 × 3)+10=25]

5.(a) Consider a weighted undirected graph $G = (V, E)$, where $|V| = n$ and $|E| = O(n)$. Suggest a technique for storing the graph in computer memory. Write the variable declarations in C for storing the graph.

(b) Write an algorithm for computing the minimum spanning tree of the aforesaid graph G, where a node $r \in V$ is specified as the root. Derive the time complexity of your algorithm.

(c) After computing the tree, we need to store the tree so that every node is reachable from the root r of the tree. Suggest an algorithm for the post-processing of the result obtained in part (b) of this question (if anything is necessary).

[10+(8+4)+10=32]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2011-12

B. Stat. II Yr.

Analysis III (Backpaper)

Date: 22/12/2011 Maximum Marks: 100 Duration: 3 Hours

All questions carry equal marks.

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(1) Let f be defined on an open set S in $\mathbb{R}^n$. We say that f is homogeneous of degree p over S if $f(rx) = r^p f(x)$ for every real $r > 0$ and for every x in S. If such a function f is differentiable at x, then show that $x \cdot \nabla f(x) = pf(x)$. Also prove that if $x \cdot \nabla f(x) = pf(x)$ for all x in an open set S, then f must be homogeneous of degree p over S.

(2) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} x^{4/3} \sin \frac{y}{x}, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Determine all the points at which f is differentiable.

(3) Show that the system of equations

$$3x + y - z + u^2 = 0$$

$$x - y + 2z + u = 0$$

$$2x + 2y - 3z + 2u = 0$$

can be solved in terms of x, in terms of y, in terms of z, but not in terms of u.

(4) Use the transformation $x + y = u, y = uv$ to evaluate the integral

$$\iint_R [xy(1-x-y)]^{1/2} dxdy,$$

where R is the triangle with sides $x = 0, y = 0, x + y = 1$.

(5) (a) Let C be the line segment from $(1,0)$ to $(0,-1)$ followed by the line segment from $(0,-1)$ to $(-1,0)$. Find the line integral of the vector field $F(x,y) = (x+y)\hat{\imath} - (x^2+y^2)\hat{\jmath}$ along C.

(b) Suppose that f is differentiable and positive on $[a,b]$. Let C be the path $\hat{r}(t) = t\hat{\imath} + f(t)\hat{\jmath}$, $t \in [a,b]$, and $\vec{F}(x,y) = y\hat{\imath}$. Is there any relation between the line integral of $\vec{F}$ along C and the area of the region bounded by the t-axis, the graph of f, and the lines $t = a$ and $t = b$? Justify your answer.

(6) Show that $\vec{F}(x,y,z) = (e^x \cos y + yz)\hat{\imath} + (xz - e^x \sin y)\hat{\jmath} + (xy + z)\hat{k}$ is the gradient of a scalar field φ. Find a function φ such that $\vec{F} = \nabla\varphi$.

(7) Let S be the surface cut from the cylinder $y^2 + z^2 = 1, z \geq 0$ by the planes $x = 0, x = 1$, and let $\vec{F} = (0, yz, z^2)$. Evaluate $\iint_S \vec{F} \cdot \hat{n}\, dS$.

(8) Find the area of the portion of the sphere $x^2 + y^2 + z^2 = 4$ between the planes $z = -1$ and $z = \sqrt{3}$.

(9) Let $\vec{F} = y\hat{\imath} + xz\hat{\jmath} + x^2\hat{k}$, and C be the boundary of the triangle cut from the plane $x + y + z = 1$ by the first octant, counterclockwise when viewed from above. Use the surface integral in Stokes' theorem to evaluate the line integral of $\vec{F}$ along C.

(10) Let D be the region inside the solid cylinder $x^2 + y^2 \leq 4$ between the planes $z = 0$ and the paraboloid $z = x^2 + y^2$. Let $\vec{F} = y\hat{\imath} + xy\hat{\jmath} - z\hat{k}$. Evaluate $\iint_S \vec{F} \cdot \hat{n}\, dS$.

PROBABILITY THEORY III
B. STAT. IIND YEAR SEMESTER 1
INDIAN STATISTICAL INSTITUTE

Backpaper Examination

Time: 3 Hours Full Marks: 100
Date: December 26, 2011

This is an OPEN NOTE examination. You may use any handwritten notes that you have brought with you, but you are not allowed to share them. Also no notes in printed, photocopied, cyclostyled or any other form (including electronic storage) is allowed.

1. Let U be uniform on $(0,1)$ and given U, the conditional distribution of X be binomial with parameters (n, U). Show that X is uniformly distributed over the set $\{0, 1, \ldots, n\}$. [15]

2. Given two sets B and C, define

$$A_n = \begin{cases} B, & \text{when } n \text{ is odd,} \\ C, & \text{when } n \text{ is even.} \end{cases}$$

Find $\limsup_{n\to\infty} A_n$ and $\liminf_{n\to\infty} A_n$. [12]

3. Let $X_1, \cdots, X_n$ be i.i.d. random variables with distribution function F and characteristic function $\exp(-|t|^\alpha)$, where $0 < \alpha \leq 2$. Write down the distribution function of $S_n = X_1 + \cdots + X_n$ in terms of F. [8]

4. Show that $X_n \xrightarrow{P} 0$ if and only if $\mathrm{E}[|X_n|/(1 + |X_n|)] \to 0$. [20]

5. Using Central Limit Theorem or otherwise, prove that

$$\lim_{n\to\infty} \frac{1}{3^{3n}} \sum_{k=0}^{n} \binom{3n}{k} 2^{3n-k} = \frac{1}{2}.$$

[17]

6. Let ϕ and Φ denote the standard normal density and distribution functions. Prove that

$$\lim_{x\to\infty} \frac{1 - \Phi(x)}{\phi(x)/x} = 1.$$

If $\{X_n\}$ is an i.i.d. sequence of standard normal random variables, show that

$$\mathrm{P}\left[\limsup_{n\to\infty} \frac{|X_n|}{\sqrt{\log n}} = \sqrt{2}\right] = 1.$$

[10+18=28]

INDIAN STATISTICAL INSTITUTE

Back Paper Examination

B.STAT - II Year (Semester - I)

C & Data Structure

Date : 29.12.11 | Maximum Marks : 100 | Duration : 3 Hours

1. A polynomial $f(x)$ is stored in computer memory using a linear linked list. State the variable declarations in C code to represent the polynomial $f(x)$.

   Write a computer program in C language for computing a polynomial $h(x)$ by adding two polynomials $f(x)$ and $g(x)$, where all the polynomials f, g and h are represented using linear linked lists. [15]

2. Consider an array P containing a set of n points in $\mathbb{R}^2$, where $P = \{p_i = (x_i, y_i), i = 1, 2, \ldots, n\}$, and $n > 1000$. Assume that each $p_i = (x_i, y_i)$ can be stored in a location defined in C using *float*. Write an algorithm for computing the convex hull of the point set. Derive the time complexity of your algorithm. [15+5=20]

3. Let $\mathcal{T}$ be a binary tree whose nodes are of the following structure. The `INFO` fields of the nodes in $\mathcal{T}$ contain distinct key values.

```
typedef struct node *nodeptr
typedef struct {
int INFO;
nodeptr LLINK;
nodeptr RLINK;
} node;
```

   Write an algorithm following the syntax of C language to find the maximum value stored in the `INFO` fields of all the nodes in the tree. [10]

4. A weighted directed graph $G = (V, E)$ is stored in the form of adjacency matrix.

   (a) Write an algorithm for computing the shortest path for every pair of vertices in the graph.

   (b) Derive the time complexity of your algorithm.

   (c) State a method of storing the shortest paths so that after the computation, if a user gives a pair of vertices $u, v \in V$, then the list of vertices on the shortest path from u to v can be printed efficiently. [15+5+10=30]

5.(a) Consider an insertion of the key $x = 227$ into the hash table shown in the figure. For each of the following probing methods, indicate the sequence of table entries that would be probed, and the final location of insertion for the key. Assume that h(x) = 7 for $x = 227$.

table index	1	2	3	4	5	6	7	8	9
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(i) Linear probing

(ii) Quadratic probing

(iii) Double hashing with $g(x) = 3$ for $x = 227$

(b) Show that for a suitable hash function that can assume any location with equal probability for each key value, the number of probes required to search for a key value that does not exist in the hash table is $O(\frac{1}{1-\lambda})$. Here $\lambda = \frac{n}{m}$, where n is the number of keys present in the table and m is the size of the table.

[(5 × 3)+10=25]

INDIAN STATISTICAL INSTITUTE

B. Stat II: 2011 – 2012

Mid-semester Examination

Economic Statistics and Official Statistics

Duration: 3 hours | Maximum marks: 100 | 20 February 2012

Economic Statistics and Official Statistics

(Answer question no. 1 and any two from the rest of the questions)

1. Suppose the urban population is just one-fourth of the rural population in a country. The share of income of top 50% people in the urban population is four times that of bottom 50% people. Also in the urban sector there are 10% people with income Rs. 100 or less. The average monthly income of the bottom 10% population of rural India is Rs. 30. The Lorenz Ratio of income of rural India is 0.28. Assume that income follows lognormal distribution separately for rural and urban sectors. Calculate the percentage of people with income more than Rs.145/- for the country as a whole. [28]

2. (i) Define two parameter lognormal distribution. (ii) Derive its moment generating function. (iii) Find the skewness and kurtosis coefficients (γ_1 and γ_2, say) of this distribution. (iv) Prove that both these coefficients are positive. (v) State the moment distribution property of lognormal distribution and hence derive the Lorenz Curve and the Lorenz Ratio of it. (vi) State and prove the properties of the Lorenz Curve of lognoramal distribution. [2+5+5+5+9+10=36]

3. (i) Define Dalton's measure of inequality. (ii) Describe how Atkinson modified it using "Equally Distributed Equivalent Income" approach. (iii) Demonstrate the advantage of this measure over the Dalton's measure. (iv) Give an outline of the steps towards arriving at the final form of Atkinson's measure of inequality. (v) Describe how one can interpret the unknown parameter of this measure by finding the values of this measure taking different values of the unknown parameter. (vi) Examine this measure in the light of Pigou-Dalton principle of transfer property. [4+3+4+5+8+12=36]

4. Write short notes on any three of the following:

   (i) Treatment of qualitative explanatory variables in economics.

   (ii) First order statistics of n i.i.d. random samples vs. P^{th} quantile of Pareto distribution where $P = 1/n$.

   (iii) Law of Proportionate Effect.

   (iv) Three parameter lognormal distribution.

   (v) Properties of Lorenz Curve. [12×3=36]

Indian Statistical Institute
Mid-Semester Examination 2012
Course: B-Stat II Year 2012
Subject: Economics II (Macroeconomics)

Duration: 150 minutes
Date: 22.2.2012

Answer all questions **Maximum Marks40**

1. In an economy, in the private sector there are only two firms, Firm 1 and Firm 2. The public sector consists only of government administration and defence. Receipts and payments of the two firms and the government administration and defence in a given period are recorded in the table given below. You have to compute from the table the GDP of the country using all the three methods. There did not occur any involuntary change in inventory in any of the firms in the given period.

Receipts	Payments
Firm 1	
Sales of Rs.500 to households Sales of Rs.300 to Firm 2 Sales of Rs.200 to government administration and defence Rs.100 in subsidy	Purchase of goods of Rs.200 from Firm 2 half of which it used to add to its capital stock Rs.100 in wages and salaries to households Rs.100 in rent to households Rs.100 in interest to households
Firm 2	
Sales of Rs.1000 to households Sales of Rs.200 to Firm 1	Purchase of goods of Rs.300 from Firm 1 which it used in production in the given period Rs.200 in wages and salaries to households Rs.100 in indirect taxes
Government Administration and Defence	
Sales: nil Rs.100 in indirect taxes	Purchase of goods of Rs.200 from Firm 1 Rs.300 in wages and salaries to households Rs.100 in interest to households Rs.100 in subsidy

[26]

2. (a) Suppose in a closed economy aggregate saving = Rs.2000, C = Rs.500, G = Rs.300, government saving = Rs.100 and business saving = business transfer = 0. Compute personal disposable income and from personal disposable income derive NDP using the given data. **[8]**

(b) Suppose a firm appoints an audit firm to audit its accounts. Which category of expenditure will its payment to the audit firm belong to? Explain. **[4]**

P.T.O.

(c) Suppose an individual gets Rs.100 in interest on his bank deposits and Rs.200 in interest from his holding of government bond. Do they constitute a part of GDP? Explain.

[4]

(d) Suppose a coal mine produces coal of Rs.1000 without using any intermediate input and sells it to an electricity generating company, which uses it entirely as an intermediate input. Will the output of the coalmine be included in the country's GDP? Explain. **[4]**

INDIAN STATISTICAL INSTITUTE

Mid- Semestral Examination: 2011 – 12

B. Stat II Year

Biology II

Date 22.02.12 Maximum Marks 30 Duration Three hours .
(Attempt any three questions)
(Number of copies of the question paper required 15)

1. Name different meteorological variables that are related to crop production. Write down the names of apparatus along with the units used to measure those variables.

5+5

2. Define Moisture Availability Index. Draw a suitable rice calendar with the following data.

2+8

Week No.	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*
Rainfall (mm) at 0.5 Prob.	*0*	*5*	*15*	*10*	*29*	*38*	*28*	*35*	*18*	*49*	*98*	*142*	*95*	*80*	*15*	*32*	*22*	*5*	*0*
PET (mm)	*45*	*42*	*33*	*37*	*33*	*24*	*25*	*20*	*22*	*20*	*19*	*17*	*19*	*20*	*25*	*28*	*31*	*33*	*34*

3. Briefly discuss about 5+5
   i) Drought.
   ii) Weather forecasting.

4. Write short notes on any four of the following: 2.5 x 4
   a) Monsoon onset
   b) Microclimate
   c) Estimation of Evapotranspiration
   d) Cessation of Monsoon
   e) Weather and climate
   f) Cup anemometer

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2011-12
B.Stat Second year
Physics II
Date 22.02.12 Maximum Marks 30 Duration 1hr 30 min
All questions carry equal marks.

Group A

Answer any three questions

1. Show that a free electron at rest can not absorb a photon.

2. Using Bohr model determine the energy values of a particle moving under the influence of a potential $V(r) = kr^2/2$.

3. Using the uncertainty principle estimate the size of the hydrogen atom and the ground state energy. Given $m_e = 9.31 \times 10^{-31} kg$ and $e = 1.6 \times 10^{-19} C$.

4. Hydrogen atom in its ground state is excited by monochromatic radiation of wavelength 970.6Å. How many different wavelengths are possible in the resulting emission spectrum? Find the longest among these wavelengths. $(h = 6.62 \times 10^{-34}\ m^2 kg/s,\ \ c = 3 \times 10^8 m/s)$.

5. Evaluate the commutator $[x, p_x^n]$ and hence find $[x, sinp_x]$.

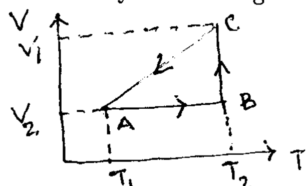
Group B

Answer any three questions

1. A gas molecule at the Earth's surface has rms speed for that gas at exactly 0^oC. If it goes up without colliding with any other molecule, determine how high it will rise. Given mass of a molecule of the gas as $4.65 \times 10^{-26} kg$ and Boltzmann constant $k_B = 1.38 \times 10^{-23} J/K$.

2. At what pressure will the mean free path be 50cm for spherical molecules of radius of $3 \times 10^{-10} cm$? Assume ideal gas at 20^oC.

3. The following figure shows the process ABCA performed on an ideal gas. Find the net heat given to the system during the process.



4. A sample of gas ($\gamma = 1.5$) is taken through an adaibatic process in which the the volume is compressed from 1600 cm^3 to 400 cm^3. If the initial pressure is 150kPa, find (a) the final pressure (b) how much work is done by the gas during the process.

5. A Carnot engine working between 0^oC and 100^oC takes $746J$ of heat from high temperature reservoir in one cycle. Calculate the (a) Work done by the engine (b) the heat it rejects and (c) it's efficiency.

DEMOGRAPHY

Date: 24 February, 2012 Duration: 1½ hours

<u>Answer as many as you can</u>. Maximum marks (50)

1. Discuss a suitable method for adjustment of infant mortality rate (IMR) (7)

(b) Explain how do you calculate Standardized Death Rate by direct and indirect methods of standardization. (4 + 4)

2. What are the usual sources of data on vital events? Explain. (5)

3. What are Morbidity Incidence Rate and Morbidity Prevalence Rate? Explain. (4+4)

4. What are the necessary assumptions for constructing a life table. Explain how to construct a complete life table on the basis of observed age-specific death rates. (4+6)

5. (a) What is the average age at death of those who die between age x and x+n ? (5)

(b) Find the probability (determine from life table) that two females aged 15 years will both die within 10 years. (3)

6. In a stationary population we consider the working population to be all persons aged 20 and 65.

(a) what is the number of persons of working age? (3)

(b) How many deaths will occur each year of persons of working age? (3)

© What is the average death rate for the particular age group? (3)

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2011-12

Course Name : B.Stat II

Subject Name: SQC & OR

Date: 24/02/2012 Maximum Marks: 50 Duration: $1\frac{1}{2}$ hrs

Note, if any: Answer all questions. marks for each question are given in [ ]

1. (a) A tailor has 90 sq.m of cotton material and 120 sq.m of woolen material. A suit requires 1 sq.m of cotton and 3 sq.m of woolen material and a dress requires 2 sq.m of each. A suit sells for Rs. 600 and a dress sales for Rs. 450. Formulate the LPP to maximize the profit.

   (b) Solve the above problem of maximization. [6 + 8]

2. Show that any convex combination of 'k' different optimum solutions to an LPP is again an optimum solution to the problem. [8]

3. Using simplex method, solve the following system of linear equations: $x_1 + x_2 = 1,\ 2x_1 + x_2 = 3.$ [14]

4. Using simplex method, find the inverse of the following matrix :

$$A = \begin{pmatrix} 4 & 2 \\ 1 & 5 \end{pmatrix}$$ [14]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2011-12

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FEBRUARY 27, 2012

INDIAN STATISTICAL INSTITUTE

B.STAT SECOND YEAR, SECOND SEMESTER, 2011-12

MID-SEMESTRAL EXAMINATION

TIME : 2 HOURS — STATISTICAL METHODS - IV — FULL MARKS : 80

1. Suppose that (X_1, X_2) follows a bivariate normal distribution with $\mathrm{E}(X_i) = \mu_i$, $\mathrm{Var}(X_i) = \sigma_i^2$ for $i = 1, 2$, and $\mathrm{Corr}(X_1, X_2) = \rho$.

   (a) Find the conditional distribution of X_2 given $X_1 = x_1$ and show that the regression line of X_2 on X_1 is linear. [3+1]

   (b) If this bivariate normal distribution remains invariant under rotations, show that $\mu_1 = \mu_2 = 0$, $\sigma_1^2 = \sigma_2^2$ and $\rho = 0$. [2+2+2]

   (c) If $\mu_1 = \mu_2 = 0$, show that the correlation coefficient between X_1^2 and X_2^2 is ρ^2. [5]

   (d) Consider a circle $\mathcal{C}$ and a square $\mathcal{S}$ having the same area and origin at the center. If the distribution of (X_1, X_2) is invariant under rotations, then show that $P\{(X_1, X_2) \in \mathcal{C}\}$ is bigger than $P\{(X_1, X_2) \in \mathcal{S}\}$. [5]

2. Consider a simple linear regression model with an intercept. Let $\widehat{\beta}_1$ be the least squared estimator of the regression coefficient β computed based on a set of m observations (x_{11}, Y_{11}), $(x_{12}, Y_{12}), \ldots, (x_{1m}, Y_{1m})$ and $\widehat{\beta}_2$ be that computed from another set of n observations (x_{21}, Y_{21}), $(x_{22}, Y_{22}), \ldots, (x_{2n}, Y_{2n})$. Now consider a new unbiased estimator $\widehat{\beta}_0 = \alpha_1\widehat{\beta}_1 + \alpha_2\widehat{\beta}_2$.

   (a) Choose the constants α_1 and α_2 such that the variance of $\widehat{\beta}_0$ is minimum. Compute this minimum variance. [4+2]

   (b) If $\bar{x}_1$ and $\bar{x}_2$ are the mean of the X-values in the first and the second set, respectively, show that $\widehat{\beta}_0$ is the best linear unbiased estimator if and only if $\bar{x}_1 = \bar{x}_2$. [3+5]

   (c) Describe how will you construct a 95% confidence interval for β based on $m+n$ observations. If this confidence interval contains the origin, will you accept the null hypothesis $H_0 : \beta = 0$ while testing it against the alternative $H_1 : \beta \neq 0$ at 5% level ? Justify your answer. [3+3]

[P.T.O.]

3. The scores of B.Stat second year students in two different subjects are given below.

Scores in subject A (X)	Scores in subject B (Y)	Scores in subject A (X)	Scores in subject B (Y)	Scores in subject A (X)	Scores in subject B (Y)
71	76	69	67	79	72
67	82	78	83	83	82
76	86	70	70	78	85
78	85	88	94	86	95
69	87	70	73	70	83
84	87	74	84	78	76
88	91	79	83	81	85
73	70	72	73	64	69
75	80	68	63	66	77
79	79	71	78	77	80

(a) Using appropriate model assumptions, test whether the overall performance of the students is significantly better in subject B compared to subject A. [5]

(b) Test whether there is significant correlation between these two sets of scores. [3]

(c) Find a 95% confidence interval for the ratio of the variance of X to the variance of Y. [8]

(d) For a student, who scored 70 in subject A, find a 95% prediction interval for his score in subject B. [4]

4. Consider a linear regression model $Y = \alpha_1 + \beta_1 x + \epsilon$, where x is non-stochastic and $\epsilon \sim N(0, \sigma^2)$. Suppose that we have m independent observations from this model.

(a) Suppose that we want to test whether the values of x have any significant effect on the values of Y. Compute the likelihood ratio test statistic for this problem and use that to perform an exact test . [5+3]

(b) Describe how will you test whether the regression line passes through the point $(1, 2)$. [4]

(c) Consider another regression model $Z = \alpha_2 + \beta_2 w + \delta$, where w is non-stochastic and $\delta \sim N(0, \tau^2)$. Suppose that we have n independent observations from the second model.

(i) Describe how you will test $H_0 : \sigma^2 = 4\tau^2$ against $H_1 : \sigma^2 = 4\tau^2$. [3]

(ii) Assuming $\sigma^2 = 4\tau^2$, how will you test whether the two regression lines are parallel ? [5]

INDIAN STATISTICAL INSTITUTE

B. STAT. SECOND YEAR

Elements of Algebraic Structures

Date: 2·3·2012 Mid.Semestral Examination Time : 3 hrs.

This paper carries 70 marks.The maximum you can score is 60.

1. Let H be a subgroup of finite index of a group G. Show that there exist only finitely many distinct groups of the form gHg^{-1} where $g \in G$. [10]

2. If H is a subgroup of a group G , show that $\cap\{gHg^{-1} : g \in G\}$ is a normal subgroup. [10]

3. Let p be a prime and G a group satisfying $(ab)^p = a^p b^p$ for all $a, b \in G$. Let $N = \{g \in G : g^{p^m} = e$ for some m (m depends on g) $\}$, where e is the identity element. Show that N is a normal subgroup of G. [10]

4. Let G be a group and H the subgroup generated by all elements of the form $xyx^{-1}y^{-1}$. Show that H is normal and G/H abelian. [10]

5. Let S_n be the permutation group on n elements. Show that the alternating group $A_n \subseteq S_n$ is generated by the set of 3-cycles. [10]

6. Let σ be a k-cycle in S_n. Show that σ^2 is a cycle if and only if k is odd. [10]

7. How many non isomorphic abelian groups are there of order 36? Justify your answer. [10]

INDIAN STATISTICAL INSTITUTE
B. STAT. SECOND YEAR
Elements of Algebraic Structures

Date: 23.4.12 Semestral Examination Time : 3 hrs.

This paper carries 80 marks.The maximum you can score is 70.

1. Let p be a prime and S_p the permutation group on p elements. How many elements of order p does S_p have? Justify your answer. [10]

2. Let $(R, +, .)$ be a ring with 1. Define $+', *$ on R by $a +' b = a + b + 1, a * b = a.b + a + b$. Show that $(R, +', *)$ is a ring isomorphic to $(R, +, .)$ [10]

3. Let Q be the field of rationals. Find a basis for the extension $Q(\sqrt{2}, \sqrt{3})$ over Q. Justify your answer. [15]

4. Let F be a finite field of characteristic p.

   (a) Show that for any $a \in F$ there is $b \in F$ satisfying $b^p = a$. [13]

   (b)Show that no irreducible polynomial over F has a multiple root. [12]

5. How many irreducible polynomials of degree 4 are there over the field of integers mod 2 ? Justify your answer. [10]

6. Let R be the ring of numbers of the form $a + bi$: a, b being integers. Which integers m are prime elements of R ? Are there prime elements of R which are not integers ? Justify your answer. [10]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: (2011 – 2012)

B. Stat II Year

Biology II

Date 25.04.12 Maximum Marks 50 Duration Three hours.

(Attempt any five questions)

(Number of copies of the question paper required 15)

1. Write in brief about different types of rice. Briefly describe the cultural practices associated with rainfed lowland rice cultivation. Critically highlight the variation in yield of winter (*Aman*) rice and summer (*Boro*) rice. 2+5+3

2. What are the differences between Manures and Fertilizers? Calculate the quantity of VC, Urea, Single super phosphate and Muriate of potash required for 1 ha rice crop to meet the nutrient requirement of 120kg N, 60 kg P_2O_5 and 60 kg K_2O per hectare. 25% of required N should be given through VC.

   3+7

3. Describe the suitable agrotechniques for rice nursery bed preparation. Estimate the expected yield of rice grain in t/ha from the following data:

   i) Spacing – 20 x 20 cm ii) Average no. of tillers/hill –60 iii) Average no. of effective tillers/hill –52 iv) Average no. of grains/panicle –160 v) Average no. of unfilled grains/panicle – 22 vi) Test weight -24 g. 4+6

4. Write short notes on any five of the following: 2 x 5
   a) Moisture availability index
   b) Soil pH
   c) Reproductive stages in rice
   d) Bulk density of soil
   e) Transgenic rice
   f) Capillary water
   g) Field capacity

5. Write in brief about any two of the following: 2X5
   a) System of Rice Intensification.
   b) Soil texture.
   c) Intercropping and Mixed cropping

6. Let x be the density of plant biomass and y denotes the density of the herbivore at time t. In the absence of herbivore, the plant grows in a logistic manner with an intrinsic rate of increase r and environmental carrying capacity k. The herbivore consumes the plant following Holling type II functional response. The death of the herbivore is density dependent.

   a) With the above assumption, write down the basic model of plant-herbivore in terms of ordinary differential equations.
   b) Find out the biologically feasible equilibria.
   c) Find out the conditions for which both the plant and the herbivore will co-exist.

   3+2+5

Indian Statistical Institute
Second Semester Examination 2011-12
Course Name: BStat Second Year
Subject: Economics II (Macroeconomics)

Date 26.04.12 **Maximum Marks: 60** **Duration: 2.5 Hours**

Answer any three of the following questions

1. In the central budget of this fiscal, the Government of India has raised taxes and reduced subsidy. Do you consider such policies appropriate for India which is slowing down at ~~the~~ present? Answer this question using the IS-LM model. **[22]**

2. If the government raises its consumption expenditure (G) and does not want investment, which you assume to be a function of interest rate alone, to be crowded out, what other policy will it have to undertake? Explain your answer. **[22]**

3. **a.** Explain the concept of "paradox of thrift" in the simple Keynesian model.
b. Consider an initial equilibrium situation in the simple Keynesian model where investment is an increasing function of Y (GDP). Now suppose the saving and investment functions shift upward by 4 and 7 units respectively. Following these changes, saving in the new equilibrium is found to have increased by 16 units. Marginal propensity to invest with respect to Y is 0.3. Compute the value of the autonomous expenditure multiplier. **[8+14=22]**

4. **a.** What is high-powered money? In what different forms is high-powered money held in the economy?
b. Consider an economy where there is no currency holding and CRR is 25 per cent. Banks are fully loaned up. Describe the process of change in money supply following an increase in central bank's loan to the government by Rs.100. **[8+14=22]**

Indian Statistical Institute
Second Semestral Examination : 2011-12
B.Stat II
Physics II

Date : 27.04.12 Maximum Marks :70 Duration : 3 Hours

Group A

Answer $Q1$ and any two questions from the rest

1. (i) Write down Einstein's photoelectric equation explaining clearly the notations used. [1]

(ii) The operator $\left(\frac{d}{dx}+x\right)$ has an eigenvalue α. Find the corresponding normalised eigenfunction if $-\infty < x < \infty$. [2]

(iii) Find the average life of a radioactive sample whose half-life is 60 days. [2]

2. (a) Show that for large quantum number n, the mechanical orbital frequency is equal to the frequency of the photon which is emitted between adjacent levels. [8]

(b). If v_p and v_g denote the phase and the group velocity respectively, show that $v_g = v_p - \lambda\frac{dv_p}{d\lambda}$ where λ is the wavelength. [7]

3. (a) Suppose ψ_0 and ψ_1 are normalized ground and the first excited state wave functions of the linear harmonic oscillator. Consider the wave function $\psi = [a\psi_0 + b\psi_1]$ where a and b are real numbers. Determine the values of a and b which maximize and minimize $<\psi|x|\psi>$. [8]

(b). Show that $\int_{-\infty}^{\infty} j(x,t)dx = \frac{<p_x>}{m}$ where $j(x,t)$ denotes probability current density. [7]

4. (a) Let $\phi_1(x)$ and $\phi_2(x)$ denote normalized eigenfunctions of an infinite square well potential of width a. Now consider the state $\phi(x) = [\alpha\phi_1(x) + \frac{3}{5}\phi_2(x)]$. Find the constant α such that $\phi(x)$ is normalized.
Also find $\phi(x,t)$. [4+4]

(b). An electron is moving with kinetic energy $5eV$ from the left and strikes a potential barrier of the form

$$\begin{aligned} V(x) &= 0, \quad x<0 \\ &= 10\,eV, \quad x>0 \end{aligned}$$

Obtain the solutions of the Schrödinger equations for the regions $x<0$ and

$x > 0$. Sketch the wave function for $x > 0$. [3+3+1]

5. (a) A free neutron decays into a proton, electron and antineutrino i.e, $n \rightarrow p + e^- + \bar{\nu}$. If $M_n = 1.00898u, M_p = 1.00759u$ and $M_e = .00055u$, find the kinetic energy shared by the electron and the anitneutrino. [6]

(b) Explain qualitatively the main features of insulators, conductors and semiconductors. What are n and p type semiconductors? [6+3]

Group B

Answer $Q6$ and any two questions from the rest

6. (i) Represent an isobaric and an isochoric process on a p-V diagram. [1]

(ii) Represent Carnot's cycle on a T-S diagram and show that the area represents available energy. [2]

(iii) Write down the Jacobian of T and S with respect to x and y and expand it in determinant form. [2]

7. (a) Assuming Maxwellian distribution of velocity, show that the following $v_P :< v >: \sqrt{< v^2 >} :: \sqrt{2} : \sqrt{8/\pi} : \sqrt{3}$ holds, where the symbols have their usual meaning. [10]

(b). Calculate the mean free path and the collision frequency of air molecules at 0^oC and 1atm pressure, given that collision cross section (σ)=2×10^{-8}cm and density of molecules (n)=3×10^{19} molecules/c.c and average velocity =10^5 cm/s. [5]

8. (a) One mole of an ideal gas ($\gamma = 5/3$) at 27°C is adiabatically compressed in a reversible process from an initial pressure of 2atm to a final pressure of 100atm. Calculate the resulting temperature difference. [4]

(b) Two moles of oxygen are heated from 0°C to 10°C at constant volume. If the change in internal energy is 420J, find the molar specific heat of oxygen at constant volume. [4]

(c) Find the thermal efficiency of an ideal gas engine operating in a reversible cycle consisting of two isotherms T_1 and $T_2(< T_1)$ and two isochores V_1 and $V_2(< V_1)$ (See Fig 1). Assume C_p, C_v to be constant during the process. [7]

9. (a) m gm of water at temperature T_1 is mixed isobarically and adiabatically with equal mass of water at temperature T_2. Show that change in entropy is $2mln(T_{av}/\sqrt{T_1T_2})$ where $T_{av} = (T_1 + T_2)/2$. [8]

(b) If $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$ and $K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$, then show that $C_p - C_V = T\alpha^2 V/K_T$. [7]

10. (a) Three distinguishable particles labeled A, B and C are distributed among four energy levels 0,E,$2E$ and $3E$. The total energy is $3E$. Calculate the number of possible macrostates and the microstates. [7]

(b) If U and Z denote respectively the internal energy and partition function of a Maxwell-Boltzmann gas, then show that $U = Nk_BT^2\frac{d}{dT}logZ$ where N is the number of molecules, k_B is Boltzmann constant and T denotes the temperature in Kelvin. [8]

or

If a system of N particles obeying Maxwell-Boltzmann statistics possesses three energy levels $E_1 = 0, E_2 = \epsilon, E_3 = 10\epsilon$. Find the temperature below which only the levels E_1 and E_2 are occupied. [8]

INDIAN STATISTICAL INSTITUTE

B. Stat. (Hons.) II: 2011 – 2012

Second Semester Examination

Economic Statistics and Official Statistics

Duration: 3 hours | Maximum marks: 100 | Date: 30.04.2012

Economic Statistics and Official Statistics

(Answer question no. 1 and any **four** from the rest. Marks allotted to each question are given within brackets [ ].)

1. Using the following data on the Per-capita Expenditure (PCE) compute (i) the Head Count Ratio (H), (ii) the Income Gap Ratio (I) and (iii) the Sen's Index of Poverty (P). Assume that the poverty line was Rs. 20 per 30 days in the base period and the current CPI for agricultural labourers with that base is Rs. 400.

Table 1: The size distribution of population by PCE

PCE (Rs./30 days)	Percentage of population	Average PCE (Rs./30 days)
0 – 30	0.91	24.80
30 – 40	2.48	35.79
40 – 50	5.10	45.42
50 – 60	7.98	55.23
60 – 70	9.75	65.15
70 – 85	15.35	77.35
...	...	...

[20]

2. Describe how you will estimate Engel elasticity using Specific Concentration Curve. (*You should show the derivations of the associated results.*) [20]

3. (a) Define Head Count Ratio and Income Gap Ratio and discuss their merits and demerits.
   (b) How did Amartya Sen improve these indices? Does Sen's index satisfy Transfer Axioms? Discuss.
   [10+10=20]

4. Define Cobb-Douglas (CD) production function. State and prove its properties. Describe the cost minimization and profit maximization procedures of obtaining the optimum quantities of inputs and outputs under CD production function set up.
   [2+6+12=20]

[PTO]

5. (a) Write a brief account of Sample Registration System that is followed in India.
(b) Write down the names of principal organs of United Nations.
(b) Who is the ex-officio Secretary of NSC?
(c) Write down the names of the divisions under NSSO.
(d) How was slum area defined in Census of India, 2001? [10+3+1+2+4=20]

6. Write down the five axioms of the economic-theoretic approach to Price Index Numbers. Prove that none of these is superfluous. Give examples of at least five index number formulae satisfying all these five axioms. [7.5+10+2.5=20]

7. Write short notes on any two of the following:

a. Properties of Lorenz curve of two-parameter lognormal distribution.
b. Linear Expenditure System.
c. Pareto law of income distribution.
d. Popular forms of Engel Curves. [10+10=20]

INDIAN STATISTICAL INSTITUTE

Semestral Examination: 2011-12

Course Name : B.Stat II

Subject Name: SQC & OR

Date: 3rd May, 2012 Maximum Marks: 50 Duration: 2 hrs

Answer as many as you can. Maximum you can score is only 50. Marks for each question are given in [ ]

1. The following linear program is fed to a computer for solution:
maximize $Z = 3x_1 + 2x_2$
subject to

$$\begin{aligned} -x_1 &+ 2x_2 + x_3 = 4 \\ 3x_1 &+ 2x_2 \qquad + x_4 = 14 \\ x_1 &- x_2 \qquad\qquad + x_5 = 3 \\ x_1, &\ x_2, \cdots, x_5 \geq 0 \end{aligned}$$

Using the revised simplex method, the computer has the following information corresponding to a basis B at some stage:

$$x_B = (x_3, x_4, x_1) \text{ and } B^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}.$$

a) Compute the current basic feasible solution and the simplex multipliers corresponding to the basis B.
b) Show that the present solution is not optimal. Which variable should now be introduced into the basis?
c) Having chosen a variable to enter the basis, now select a variable to leave and describe the selection rule. Is it possible that no variable meets the criterion of your rule? If this happens in some problem, what does that indicate about the original problem?
d) Using (c), find the new inverse of the basis and the new simplex multipliers.

e) Write down the new basic feasible solution. Is it optimal? Why or why not? [4 + 3 + 4 + 4 + 4 = 19]

2. A fraction nonconforming control chart with center line 0.10, UCL = 0.19 and LCL = 0.01 is used to control a process.

a) If 3 - sigma limits are used, find the sample size for the control chart.
b) Use the Poisson approximation to the binomial to find the probability of the type I error.
c) Use the Poisson approximation to the binomial to find the probability of the type II error if the process fraction defective is actually p = 0.20. [3 + 2 + 3 = 8]

3. a) The 3 - sigma control limits for an $\overline{X}$ chart is given by $\overline{\overline{X}} \pm A_2\overline{R}$. Find out the 2 - sigma control limits for the $\overline{X}$ chart in terms of A_2 and $\overline{R}$. b) Why is 'np' chart not appropriate with variable sample size? [3 + 2 = 5]

4. Control chart for $\overline{X}$ and 'R' are maintained for an important quality characteristic. The sample size is n = 7. $\overline{X}$ and 'R' are computed for each sample. After 35 samples, we have found $\sum_{i=1}^{35} \overline{X}_i = 7805$ and $\sum_{i=1}^{35} R_i = 1200$.
a) Setup $\overline{X}$ and 'R' charts using the data.
b) Assuming that both charts exhibit control, estimate the process mean and standard deviation.
c) If the quality characteristic is normally distributed and the specifications are 220 ± 35, can the process meet the specifications? Estimate the fraction non-conforming.
d) Assuming the variance to remain constant, state where the process mean should be located to minimize the fraction non-conforming. What would be the value of the fraction non-conforming under these conditions?
[3 + 3 + 4 + 5 = 15]

5. The purity of a chemical product is measured on each batch. Purity determinations for 20 successive batches are shown below. Is the process in statistical control? Justify your answer.

Batch	Purity	Batch	Purity
1	0.81	11	0.81
2	0.82	12	0.83
3	0.81	13	0.81
4	0.82	14	0.82
5	0.82	15	0.81
6	0.83	16	0.85
7	0.81	17	0.83
8	0.80	18	0.87
9	0.81	19	0.86
10	0.82	20	0.84

[13]

INDIAN STATISTICAL INSTITUTE
B.Stat. II Year Course, 2011-12
Second Semestral Examination
Subject: Demography

Date: 3 May, 2012 Duration: 1½ hours

Maximum marks :50

Answer all questions

1. Distinguish between (1) stationary and stable population,(2) birth order and parity, (3) permanent and temporary sterility, (4) natural and controlled fertility, (5) period life table and cohort life table, (6) morbidity incidence rate and morbidity prevalence rate.
[2 x 6=12]

2. Fill in the blanks.
   i) Computation of TFR is based on births of both sexes, whereas GRR is based on .... only.
   ii) Like TFR, GRR also assumes that women in reproductive age groups .... till the end of their reproductive period.
   iii) GRR in a population is 1.7 means if 100 mothers follow the current schedule of fertility, they will be replaced by......... daughters.
   iv) Fertility of replacement level corresponds to the value of NRR =......
   v) A married woman having no live births is said to be of parity .....
[5]

3. i) For a certain life table with $l_x = 20{,}900 - 80x - x^2$, what is the ultimate age in the life table?
   ii) Given $l_x = (1 - \frac{x}{105})$,
   find the probability that among two persons aged 20 years and 30 years, only one will be attaining the age of 70 years.
   iii) Show that life table death rate = life table birth rate = $\frac{1}{e_0^0}$

[3+3+2=8]

4. The following information for a certain community was obtained within a given period.

Age group	Female population' (000)	Live births	Female survival rates
15 – 19	200.6	4227	0.969
20 – 24	173.5	26099	0.967
25 – 29	161.7	32844	0.963
30 – 34	160.9	23449	0.958
35 – 39	155.7	11588	0.952
40 – 44	125.6	2071	0.942
45 – 49	87.6	122	0.928

Calculate the Gross Reproduction Rate (GRR) and the Net Reproduction Rate (NRR) from the given information.
(Assume that the sex ratio at birth is 105 males per 100 females).

[4+3]

5. i) What is logistic model of population growth? What are local stability points?

ii) Obtain one sex age structured population growth model equations by writing all three basic assumptions.

iii) Explain the method of population projection to project population after 10 years. (Assume that population by 5-year age intervals is known in the beginning).

[2+3+7+6]

MAY 08, 2012

INDIAN STATISTICAL INSTITUTE

B.STAT SECOND YEAR, SECOND SEMESTER, 2011-12

SEMESTRAL EXAMINATION

TIME : $3\frac{1}{2}$ HOURS | STATISTICAL METHODS - IV | FULL MARKS : 100

[Answer as many as you can. The maximum you can score is 100]

1. Some observations were generated from a bivariate normal distribution. Suppose that we do not know the actual values of those observations, but we know that the numbers of observations in the four quadrants are n_1,n_2, n_3 and n_4, respectively, taken in order.

   (a) Using this information, how will you perform an asymptotic test of level α for testing $H_0 : (\mu_1, \mu_2) = (0,0)$ against $H_1 : (\mu_1, \mu_2) \neq (0,0)$? [6]

   (b) If we also know that the distribution is symmetric about $(0,0)$, how will you construct an exact test of level α for testing $H_0 : \rho = 0.25$ against $H_1 : \rho > 0.25$? [6]

2. (a) Consider a binary logistic regression model, where the response has two categories. If the sample observations from these two categories are linearly separable, show that the maximum likelihood estimates of the model parameters do not exist. [6]

   (b) Suppose that we have n observations from a mixture of J multivariate normal distributions, which differ only in their locations. Now, for each observation $\mathbf{x}_i$ $(i = 1, 2, \ldots, n)$ define the response variable $y_i = j$ if $\mathbf{x}_i$ comes from the j-th $(j = 1, 2, \ldots, J)$ distribution. In this set up, can we use the logistic regression model for estimating the conditional probability $p(y = j \mid \mathbf{x})$ for different values of j given the value of $\mathbf{x}$? Justify your answer. [6]

3. Suppose that two linear regression equations are computed based on 25 sample observations from the bivariate normal distribution $N_2(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, and these equations are $x - 0.2337y - 0.7887 = 0$ and $x - 1.0301y - 0.0081 = 0$.

   (a) Find unbiased estimates for μ_1 and μ_2. [2]

   (b) Perform an exact test at 5% level for testing $H_0 : \rho = 0$ against $H_1 : \rho > 0$. [4]

   (c) Perform an exact test at 5% level for testing $H_0 : \sigma_1^2/\sigma_2^2 = 0.5$ against $H_1 : \sigma_1^2/\sigma_2^2 \neq 0.5$. [6]

4. Assume that $\mathbf{X} = (X_1, X_2, X_3)'$ follows the multivariate normal distribution with the mean vector $\boldsymbol{\mu} = (1, 2, 3)'$ and the dispersion matrix $\boldsymbol{\Sigma} = 4\mathbf{I}_3 + \mathbf{1}\mathbf{1}'$, where $\mathbf{1} = (1, 1, 1)'$.

   (a) Find the region $A \subseteq R^3$ with $P(\mathbf{X} \in A) = 0.75$, which has the smallest volume. Give justification for your choice. [6]

   (b) Find the volume of this region. [6]

5. Consider the multiple linear regression model $\mathbf{Y}_{n\times 1} = \mathbf{X}_{n\times(p+1)}\boldsymbol{\beta}_{(p+1)\times 1} + \boldsymbol{\epsilon}_{n\times 1}$, where $r(\mathbf{X}) = p+1$ and $\boldsymbol{\epsilon} \sim N_n(\mathbf{0}, \mathbf{I})$. Let $\widehat{\boldsymbol{\beta}}$ be the least squares estimator of $\boldsymbol{\beta}$, and define $\widehat{\mathbf{Y}} = \mathbf{X}\widehat{\boldsymbol{\beta}}$.

(a) Show that for any $\mathbf{m} \in R^{p+1}$, $\mathbf{m}'\widehat{\boldsymbol{\beta}}$ is the best linear unbiased estimator for $\mathbf{m}'\boldsymbol{\beta}$. [6]

(b) Let λ be the largest eigenvalue of $\mathbf{X}'\mathbf{X}$. If $\|\mathbf{m}\| = 1$, show that $\text{Var}(\mathbf{m}'\widehat{\boldsymbol{\beta}})$ cannot be smaller than $1/\lambda$. [6]

6. Assume a linear regression model $\mathbf{Y}_{n\times 1} = \mathbf{X}_{n\times p}\boldsymbol{\beta}_{p\times 1} + \boldsymbol{\epsilon}_{n\times 1}$, where $\boldsymbol{\epsilon} \sim N_n(\mathbf{0}, \sigma^2\mathbf{I})$. Consider a partition of $\mathbf{X} = (\mathbf{X}_1 : \mathbf{X}_2)$, where $\mathbf{X}_i$ is a matrix of order $n \times p_i$ $(i = 1, 2)$, and a partition of $\boldsymbol{\beta}' = (\boldsymbol{\beta}_1' : \boldsymbol{\beta}_2')$, where $\boldsymbol{\beta}_i$ is a p_i-dimensional vector $(p_1 + p_2 = p)$.

(a) Describe how you will test $H_0 : \boldsymbol{\beta}_2 = 0$ against $H_1 : \boldsymbol{\beta}_2 \neq 0$. [6]

(b) For any matrix $\mathbf{A}$, define $\mathbf{P_A} = \mathbf{A}(\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'$. If $\mathbf{P}_{\mathbf{X}_1}\mathbf{Y}$ is unbiased for $\boldsymbol{\beta}_1$, find the distribution of $\mathbf{Y}'\mathbf{P}_{\mathbf{X}_1}\mathbf{Y}/\mathbf{Y}'\mathbf{P}_{\mathbf{X}}\mathbf{Y}$. [6]

7. Scores of 15 students in mathematics (X_1) and statistics (X_2) are given below.

X_1	51	67	46	68	69	84	48	73	91	70	35	65	72	41	39
X_2	76	82	66	85	87	78	55	69	80	77	45	76	80	62	35

[$\sum X_1 = 919$, $\sum X_2 = 1053$, $\sum X_1^2 = 60297$, $\sum X_2^2 = 77119$, $\sum X_1X_2 = 67270$]

(a) If the joint distribution of (X_1, X_2) in the population is assumed to be bivariate normal, find a 95% confidence region for the mean vector. [6]

(b) Under the assumption of bivariate normality, find a confidence interval for the population correlation coefficient with approximately 95% confidence coefficient. [6]

8. Consider a linear regression model with an intercept and p covariates $X_1, X_2, \ldots, X_p$.

(a) Check whether testing $H_0 : \beta_1 = \beta_2 = \ldots = \beta_p = 0$ against $H_1 : \beta_i \neq 0$ for some $i \in \{1, 2, \ldots, p\}$ is equivalent to testing the significance of $\rho_{Y\cdot 12\ldots p}$, the multiple correlation coefficient between Y and $X_1, X_2, \ldots, X_p$. [6]

(b) Check whether testing $H_0 : \beta_p = 0$ against $H_1 : \beta_p \neq 0$ is equivalent to testing the significance of $\rho_{Yp.12\ldots(p-1)}$, the partial correlation coefficient between Y and X_p. [6]

9. Suppose that $X_{n\times p}$ is a data matrix from a p-variate normal distribution with the mean vector $\mathbf{0}$ and the dispersion matrix $\boldsymbol{\Sigma}$.

(a) Consider a partition of $\mathbf{X} = (\mathbf{X}_1 : \mathbf{X}_2)$, where $\mathbf{X}_1$ and $\mathbf{X}_2$ are matrices of orders $n \times p_1$ and $n \times p_2$, respectively $(p_1 + p_2 = p)$. Find the distribution of the matrix $\mathbf{X}_2'\mathbf{X}_2 - \mathbf{X}_2'\mathbf{X}_1(\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'\mathbf{X}_2$. [6]

(b) If $\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_k$ are $n \times n$ real symmetric matrices with $\text{rank}(\mathbf{C}_i) = r_i$ for $i = 1, 2, \ldots, k$ and $\sum_{i=1}^{k}(\mathbf{C}_i) = \mathbf{I}_n$, find the distribution of $\mathbf{X}'(\mathbf{C}_1 + \mathbf{C}_2)\mathbf{X}$. [6]

INDIAN STATISTICAL INSTITUTE

B.STAT SECOND YEAR, SECOND SEMESTER, 2011-12

SEMESTRAL (BACKPAPER) EXAMINATION

TIME : 3 HOURS | STATISTICAL METHODS - IV | FULL MARKS : 100

27·06·12

1. Consider a multiple linear regression model $\mathbf{Y}_{n\times 1} = \mathbf{X}_{n\times(p+1)}\boldsymbol{\beta}_{(p+1)\times 1} + \epsilon_{n\times 1}$, where rank($\mathbf{X}$) = $p+1$ and $\epsilon \sim N_n(\mathbf{0}, \mathbf{I})$. Let $\widehat{\boldsymbol{\beta}}$ be the least squares estimator of $\boldsymbol{\beta}$ and $\widehat{\mathbf{Y}} = \mathbf{X}\widehat{\boldsymbol{\beta}}$. Now, prove or disprove the following statements.

   (a) If Y is regressed on $\hat{Y}$, the regression line will pass through the origin, but if $\hat{Y}$ is regressed on Y, the regression line will not pass through the origin. [6]

   (b) If $Y - \widehat{Y}$ is regressed on Y, the fitted regression line will always have a positive slope, but if it is regressed on $\widehat{Y}$, it may have negative slope. [6]

   (c) For any $\boldsymbol{\alpha} \in R^{p+1}$, $\boldsymbol{\alpha}'\widehat{\boldsymbol{\beta}}$ is the best linear unbiased estimator for $\boldsymbol{\alpha}'\boldsymbol{\beta}$. [6]

   (d) If λ is the largest eigenvalue of $\mathbf{X}'\mathbf{X}$, for any $\boldsymbol{\alpha} \in R^{p+1}$, we have Var$(\boldsymbol{\alpha}'\widehat{\boldsymbol{\beta}}) \geq \|\boldsymbol{\alpha}\|^2/\lambda$. [6]

2. Suppose that a random vector $\mathbf{X}$ follows a p-variate distribution with the mean vector $\mathbf{0}$ and the dispersion matrix $\mathbf{\Sigma}$, assumed to be positive definite.

   (a) Show that for any $t > 0$, $P(\mathbf{X}'\mathbf{\Sigma}^{-1}\mathbf{X} \geq t) \leq p/t$ [4]

   (b) If $\mathbf{R}$ is the corresponding correlation matrix, show that the mean of the off-diagonal elements of $\mathbf{R}$ cannot be smaller than $-1/(p-1)$. [4]

   (c) If $\mathbf{X}$ is spherically symmetric, show that trace$(\mathbf{\Sigma}) = p|\mathbf{\Sigma}|^{1/p}$. [6]

   (d) If $\mathbf{X}$ is spherically symmetric, show that $\|\mathbf{X}\|$ and $\mathbf{X}/\|\mathbf{X}\|$ are independently distributed. [6]

   (e) In case of $p = 2$ (i.e., when $\mathbf{X} = (X_1, X_2)'$), show that $P(X_1 > 0, X_2 > 0) = 0.25 + 0.5 \sin^{-1}\rho$, where ρ is the correlation coefficient between X_1 and X_2. [6]

3. Suppose that we have 100 observations from the bivariate normal distribution $N_2(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, and the two regression lines computed based on those 100 observations are $x - 1.0301y - 0.0081 = 0$ and $x - 0.2337y - 0.7887 = 0$.

   (a) Find a confidence interval for ρ with approximately 95% confidence coefficient. [4]

   (b) Perform an exact test for $H_0 : 2\sigma_1^2 - \sigma_2^2 = 0$ against $H_1 : 2\sigma_1^2 - \sigma_2^2 > 0$ at 1% level of significance. [6]

4. Let $\mathbf{X}_{n \times p}$ be a data matrix from a p-multivariate normal distribution with the mean vector $\boldsymbol{\mu}$ and the dispersion matrix $\boldsymbol{\Sigma}$. If $\mathbf{A}_{m \times n}\mathbf{X}\mathbf{B}_{p \times q}$ is also a data matrix, show that

(a) $\mathbf{A1} = \alpha\mathbf{1}$ for some scalar α or $\mathbf{B}'\boldsymbol{\mu} = \mathbf{0}$ [3]

(b) $\mathbf{A}'\mathbf{A} = \beta\mathbf{I}_n$ for some scalar β or $\mathbf{B}'\boldsymbol{\Sigma}\mathbf{B} = \mathbf{O}$. [7]

5. Let $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$ be n independent observations from $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$.

(a) Find the maximum likelihood estimators for $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, and check whether these estimators are unbiased. [2+4+2+2]

(b) Find the distribution of $\mathbf{S} = \sum_{i=1}^{n}(\mathbf{X}_i - \bar{\mathbf{X}})(\mathbf{X}_i - \bar{\mathbf{X}})'$, where $\bar{\mathbf{X}} = \sum_{i=1}^{n} \mathbf{X}_i/n$. [4]

(c) Show that $\bar{\mathbf{X}}$ and $\mathbf{S}$ are independent. [4]

(d) Show that for any $\boldsymbol{\alpha} \in R^p$, $\boldsymbol{\alpha}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\alpha}/\boldsymbol{\alpha}'\mathbf{S}^{-1}\boldsymbol{\alpha}$ follows a χ^2 distribution. [6]

(e) Find the distribution of $\frac{n-p}{p}(\bar{\mathbf{X}} - \boldsymbol{\mu})'\mathbf{S}^{-1}(\bar{\mathbf{X}} - \boldsymbol{\mu})$ and describe how you will use it to find a 95% confidence region for $\boldsymbol{\mu}$. [6]

INDIAN STATISTICAL INSTITUTE
B. STAT. SECOND YEAR
Elements of Algebraic Structures
Date: 28.06.12 Backpaper Examination Time : 3 hrs.
Maximum Marks-100

1. Let m be a positive integer not divisible by n^2 for any integer n. Show that any abelian group of order m is cyclic. [15].

2. Let D be an integral domain and m a positive integer. If $ma = 0$ for some $a \epsilon D$, show that $ma = 0$ for all $a \varepsilon D$. [10]

3. Let R be a commutative ring. Let $I = \{x \varepsilon R : x^n = 0 \text{ for some } n\}$. Show that I is an ideal of R . [15]

4. Let $f(x)$ be a polynomial with rational coefficients . If p, q are rational numbers such that $p + q\sqrt{2}$ is a root of f show that $p - q\sqrt{2}$ is a root of f. [15]

5. Is $x^3 - 9$ irreducible over the field of integers mod 11. Justify your answer. [15]

6. Show that the splitting fields of $x^2 - 3$ and $x^2 + 1$ over the field of integers mod 7 are isomorphic but their splitting fields over the rationals are not isomorphic. [15 + 15]