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PARTICLE PRODUCTION IN RHIC, THE p_T-SPECTRA AND A NEW APPROACH

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The RHIC measurements on the transverse momentum spectra of the main category of secondary particles produced in the Au + Au collisions were reported in the recent past. The combination of the phenomenological approaches we adopted provide a satisfactory, alternative framework for understanding and explaining the latest data-spurts on charged pions, kaons and the protons–antiprotons. The comparison of the performances by the contesting models on a select kind of secondary (π^+) which belongs to the most abundant variety has also been made. The impact and implications of all these have also been spelt out in the end.

Keywords: Particle and resonance production; relativistic heavy ion collision; inclusive cross section.

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1. Introduction

The studies on the behavior of particle production at the relativistic heavy ion collider (RHIC) constitute an area of great topical interest and are of paramount importance to the high energy physicists.[1–3] In the recent past Broniowski and Florkowski (BF)[4] advanced an approach for understanding the nature of the transverse momentum spectra of the various secondaries produced in the relativistic Au + Au collisions at RHIC. The approach by them was based on a "variant" of the thermal model which they themselves chose to name as "thermal model with expansion." True, the model served the data quite well. We will strive to achieve here the same end with the help of a combination of some new models which warrant to be seriously explored for understanding the deeper significance of the nature of relations used and for finding out the reasons for their remarkable phenomenological successes in interpreting the latest data on Au + Au collision at RHIC. By all indications, this fair agreement does not appear to be just a coincidence; rather a hidden harmony seems to be at work, though the exact mechanisms for such behavioral manifestation cannot right now be ascertained. The present study will concentrate on production of the charged π-mesons, charged K-mesons and the proton–antiprotons which comprise more than 95% of the produced secondaries in the Au + Au interactions at RHIC. The study presented here can certainly put a valid question mark to the conventional wisdom: the thermal model offers the last and decisive say on explaining the nature of the nuclear collisions even at very high energies. Secondly, the study presented here in some detail has had another striking revelation that the breaking of the Feynman scaling does not occur, even if it does, in any significant measure. The reason for this qualitative statement would be dwelt upon later toward the end. This introduces a new meaning and import; and adds a dimension to our perusal of this problem. In what follows we give first an outline of the approaches that have been utilized here. In the next section we briefly sketch the procedural steps and deliver the output of the results obtained by us. The last section is devoted to the summary and concluding remarks.

2. The Basic Approaches: Tracing the Outlines

Following the work of Peitzmann,[5] we propose here a generalized empirical relationship between the inclusive cross-section for pion production in nucleon(N)–nucleon(N) collision and that for nucleus(A)–nucleus(A) collision as given below:

$$E\frac{d^3\sigma}{dp^3}\,(A+B\to Q+X)\sim(A\cdot B)^{\epsilon(y,p_T)}E\frac{d^3\sigma}{dp^3}(P+P\to Q+X)\,, \tag{1}$$

where Q stands for $\pi^{\pm}$, π^0, $K^{\pm}$, P and $\bar{P}$. The function $\epsilon(y,p_T)$ could be expressed in the factorization form, $\epsilon(y,p_T)=f(y)g(p_t)$.

While investigating the specific nature of dependence of the two variables (y and p_T) either one of these is assumed to remain constant. In other words, more particularly, if and when the p_t-dependence is studied by experimental groups, the rapidity factor is treated to be constant and vice-versa. So, the formula turns into

$$E\frac{d^3\sigma}{dp^3}\,(A+B\to Q+X)\sim(A\cdot B)^{f(p_T)}\,E\frac{d^3\sigma}{dp^3}\,(P+P\to Q+X)\,. \tag{2}$$

The main bulk of work, thus, converges to the making of an appropriate choice of form for $f(p_T)$. And the necessary choices are to be made on the basis of certain premises and physical considerations which do not violate the canons of high energy particle interactions.

The expression for inclusive cross-section of pions in proton–proton scattering at high energies in Eq. (2) could be chosen to be of the form

$$E\frac{d^3\sigma}{dp^3}(P+P\to Q+X)=C_1\left(1+\frac{p_T}{p_0}\right)^{-n}\,, \tag{3}$$

Table 1. Fit results obtained on p_0 and n for PP collision.

Hadron	p_0	n	$\frac{\chi^2}{\text{ndf}}$
π^+	1.6 ± 0.2	13 ± 1	0.677/17
π^-	1.6 ± 0.1	14 ± 1	0.647/16
π^0	1.4 ± 0.2	16 ± 3	21.078/20
K^+	1.6 ± 0.2	11 ± 1	18.318/12
K^-	1.5 ± 0.2	10 ± 1	22.681/12
P	2.7 ± 0.5	15 ± 2	44.675/15
$\bar{P}$	2.8 ± 0.5	16 ± 4	67.869/15

where C_1 is the normalization constant, and p_0, n are interaction-dependent chosen phenomenological parameters, of which the values of p_0 and n are given in Table 1. The above form was initiated first by Hagedorn[6] for particle production in nucleon–nucleon collisions. Let us first impart the transverse momentum dependence of the first factor on the right hand side of Eq. (2) a convenient phenomenological form. Putting empirically in Eq. (2), $f(p_T) = (1+\alpha p_T - \beta p_T^2)$, the proposed final working formula for the nucleus–nucleus collisions could be expressed as follows:

$$
\begin{aligned}
E\frac{d^3\sigma}{dp^3}(A+B\to Q+X) &\sim (A\cdot B)^{(1+\alpha p_T-\beta p_T^2)} E\frac{d^3\sigma}{dp^3}(P+P\to Q+X) \\
&= C_2(A\cdot B)^{(1+\alpha p_T-\beta p_T^2)}\left(1+\frac{p_T}{p_0}\right)^{-n}, \qquad (4)
\end{aligned}
$$

where C_2 is the normalization constant; and α, β are two fit parameters which are presented in Table 2. This expression (4) represents a specifically proposed form of polynomial nature of A-dependence for nuclear reactions and this will hereafter be referred to as De–Bhattacharyya parametrization (DBP).

And the choice of this form is not altogether a coincidence. In dealing with the EMC effect in the lepton–nucleus collisions, one of the authors here (SB),[7] made

Table 2. Fit values of α and β for different hadrons produced in Au – Au collisions.

Hadron	α	β	χ^2/ndf
π^+	0.17 ± 0.01	0.04 ± 0.01	25.916/19
π^-	0.20 ± 0.01	0.04 ± 0.01	9.790/19
π^0 (min. bias)	0.22 ± 0.02	0.03 ± 0.01	0.346/6
π^0 (central)	0.25 ± 0.06	0.03 ± 0.01	18.852/12
π^0 (peripheral)	0.24 ± 0.02	0.03 ± 0.01	13.475/9
K^+	0.19 ± 0.01	0.05 ± 0.01	5.781/11
K^-	0.18 ± 0.03	0.06 ± 0.02	9.381/9
P	0.24 ± 0.01	0.051 ± 0.003	13.028/24
$\bar{P}$	0.30 ± 0.02	0.07 ± 0.01	24.491/23

use of a polynomial form of A-dependence with the variable x_F (Feynman scaling variable). This gives us a clue to make a similar choice with both p_T and $y(\eta)$ variable(s) in each case separately. In recent times, De–Bhattacharyya parametrization is being extensively applied to interpret the measured data on the various aspects[8] of the particle–nucleus and nucleus–nucleus interactions at high energies. In the recent past Hwa *et al.*[9] also made use of this sort of relationship in a somewhat different context. The underlying physics implications of this parametrization stem mainly from the expression (4) which could be identified as a clear mechanism for switch-over of the results obtained for nucleon–nucleon (P + P) collision to those for nucleus–nucleus interactions at high energies in a direct and straightforward manner. The polynomial exponent of the product term on AB takes care of the totality of the nuclear effects.

Indeed, quite obviously, there is a factor unity and are two phenomenological parameters in $f(p_T)$ which need to be physically explained and/or identified. In compliance with this condition we offer the following physical explanations for the occurrence of all these factors. The term unity signifies theoretically the probability of fullest possible participation of either or both the projectile and the target which marks the very onset scenario of any real physical collision. The particle–nucleus or nucleus–nucleus collisions at high energies subsequently gives rise to an expanding blob or fireball with rising temperature. In real and concrete terms this stage indicates the growing participation of the already-expanded nuclear blob. As temperature increases at this stage, the emission of highly energetic secondaries (which are mostly peripheral nucleons or baryons) with increasing transverse momentum is perfectly possible. The coefficient α addresses this particularity of the natural event; and this is manifested in the enhancement of the nuclear contribution with the rise of the transverse momentum. Thereafter, there is a turnabout in the state of reality. After the initial fractions of seconds, the earlier-excited nuclear matter starts to cool down and there is a clear natural contraction at this stage as the system suffers a gradual fall in temperature. Finally, this leads to what one might call "freeze-out" stage, which results in extensive hadronization, especially in production of hadrons with very low transverse momentum. In other words, the production of large-p_T particles at this stage is lowered to a considerable extent. This fact is represented by the damping or attenuation term for the production of high-p_T particles. The factor β with negative values takes care of this state of physical reality. Thus the function denoted by $f(p_T)$ symbolizes the totality of the features of the expansion-contraction dynamical scenario in the after-collision stage. This interpretation is, at present, only suggestive. However, let us make some further clarifications.

The physical foundation that has here been attempted to be built up is inspired by thermodynamic pictures, whereas the quantitative calculations are based on a sort of pQCD-motivated power-law formula represented by Eq. (3). This seems to be somewhat paradoxical, because it would be hard to justify the hypothesis of local thermal equilibrium in multihadron systems produced by high energy collisions in terms of successive collision of the QCD-partons (like quarks and gluons) excited or created in the course of the overall process. Except exclusively for central heavy ion collisions, a typical parton can only undergo very few interactions before the final-state hadrons "freeze out," i.e. escape as free particles or resonances. The fact is the hadronic system, before the freeze-out starts, expands a great deal — both longitudinally and transversally — while these very few interactions take place.[10] But the number of parton interactions is just one of the several other relevant factors for the formation of local equilibrium. Of equal importance is the parton distribution produced early in the collision process. This early distribution is supposed to be a superposition of collective flow and highly randomized internal motions in each space cell which helps the system to achieve a situation close to the equilibrium leading to the appropriate values of collective variables including concerned and/or almost concerned quantities. The parameter α in expression (4) is a measure of the ratio of the net binary collision number to the total permissible number among the constituent partons in the pre-freeze out expanding stage identified to be a sort of explosive "detonation"[10] stage. This is approximated by a superposition of collective flow and thermalized internal motion, which is a function of hadronic temperature manifested in the behavior of the average transverse momentum. The post freeze-out hadron production scenario is taken care of by the soft interaction which is proportional[2,11] to the number of participant nucleons, N_{part}, according to almost any variety of wounded nuclear model. The factor β, we conjecture, offers a sort of the ratio of the actual participating nucleons to the total number of maximum allowable (participating) nucleons.

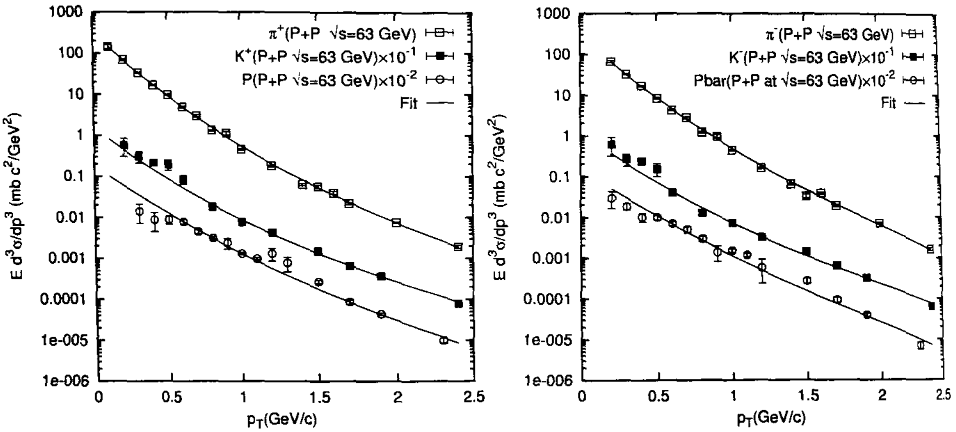
3. The Procedure and the Final Results

The first step towards progress in the present work consists of fitting the inclusive transverse momentum cross-section of secondary hadrons produced in P + P collisions at highest available energies ($\sqrt{s} = 63$ GeV) with the help of the proposed form given in Eq. (3). The values of p_0 and n for different secondary hadrons produced in P + P collisions at $\sqrt{s} = 63$ GeV are specifically noted in Table 1.

Hereafter, in studying the nature of p_T-spectra of all secondary hadrons produced in Au + Au collisions at $\sqrt{s_{\text{NN}}} = 130$ GeV the values of p_0 and n for corresponding hadrons are picked up from Table 1. While analysing nucleus-dependence with expression (4) and trying with the fit parameters α and β, we have inserted these values of p_0 and n for P + P cross-section term occurring in the expression (4) given above. The values of α, β and χ^2/ndf for various hadron production in Au + Au collisions are depicted in tabular form (Table 2).

The solid curves in Figs. 1 and 2 depict the theoretical description based on expression (3) for production of various secondaries of the data on P+P interactions at an energy which was so far the highest available one for P + P reactions at CERN SPS.

The reasons for presenting these graphical plots are two fold: (a) Firstly, they give a check-up for the efficiency of the basic model for P + P reactions. In this



(a) (b)

Fig. 1. Plot of $E\frac{d^3\sigma}{dp^3}$ vs. p_T for secondary $\pi^{\pm}$, $K^{\pm}$, P and $\bar{P}$ produced in P + P collisions at $\sqrt{s} = 63$ GeV in the rapidity region $y = 0$. Various experimental data points are taken from Ref. 12. The solid curves give the theoretical fits on the basis of Eq. (3).

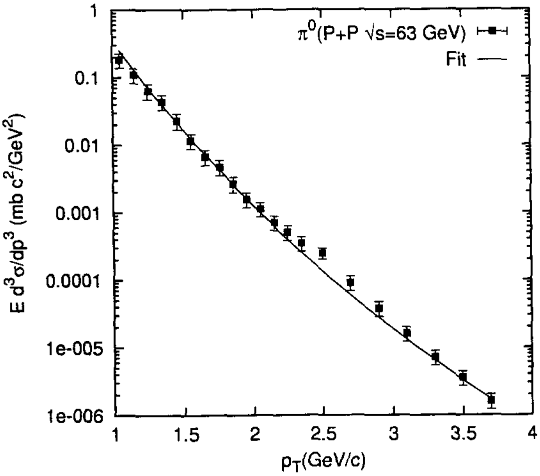
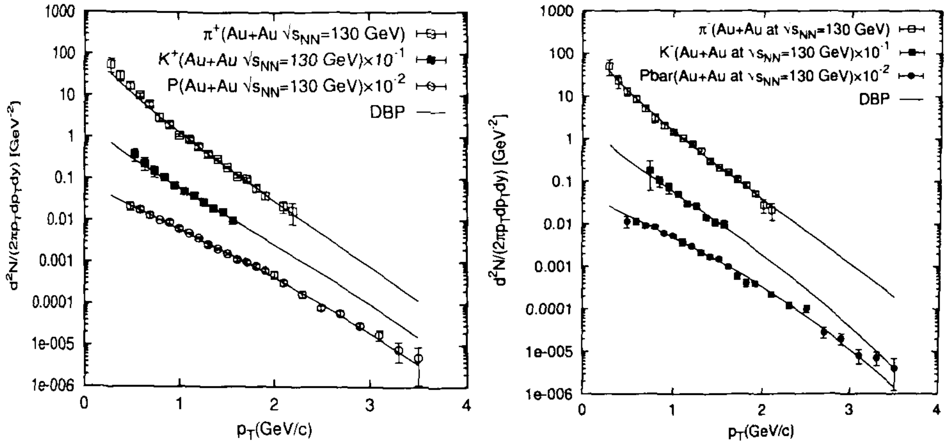


Fig. 2. The inclusive spectra for production of neutral pions in P + P collisions at $\sqrt{s} = 63$ GeV in the rapidity region $y = 2.0 - 2.25$. The filled squares depict the experimental data points,[13] while the solid curve represents the theoretical fits on the basis of Eq. (3).

particular case it is Hagedorn's model[6] with some specifically chosen parameters. (b) Secondly, they help us to estimate the parameter values which would be utilized for also the Au + Au reactions at $\sqrt{s_{\rm NN}} = 130$ GeV with a specific polynomial pattern of A-dependence of nuclear reactions. The concept of Feynman types of scaling comes into play when we plan to use the same parameter values for two different energies viz. $\sqrt{s} = 63$ GeV and $\sqrt{s_{\rm NN}} = 130$ GeV. The calculations were



(a) (b)

Fig. 3. Nature of p_T-dependence of $E\frac{d^3N}{dp^3}$ for secondary $\pi^{\pm}$, $K^{\pm}$, P and $\bar{P}$ produced in Au+Au collisions at $\sqrt{s_{\rm NN}} = 130$ GeV for minimum bias events. Different experimental data points are taken from Ref. 4. The solid curves provide the theoretical fits on the basis of Eq. (4).

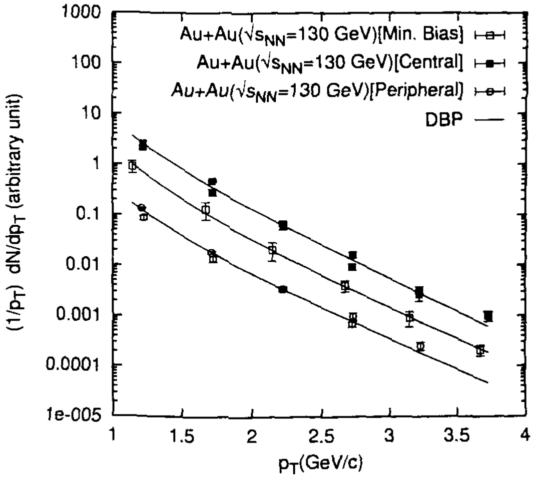


Fig. 4. Invariant spectra for neutral pions produced in Au + Au reaction at $\sqrt{s_{\rm NN}} = 130$ GeV (RHIC). The various experimental points are from Refs. 14 and 15. The solid curves give the theoretical fits on the basis of Eq. (4).

done with the help of the parameter values shown in Table 1. The solid curves in Figs. 3 and 4, drawn on the basis of Eq. (4), demonstrate the cases of particle production phenomena for Au + Au reactions at RHIC. The parameter values for calculations are supplied in the adjoining Table 2.

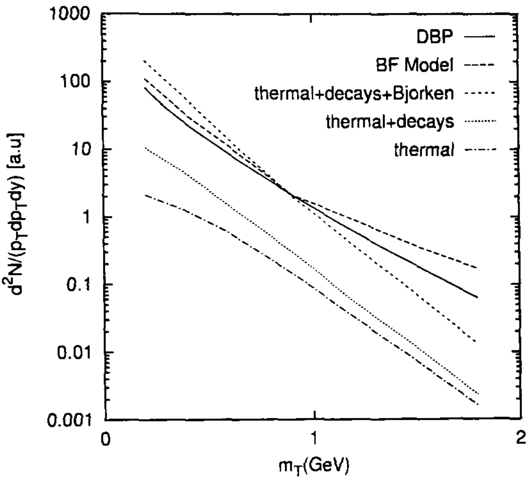


Fig. 5. Comparison of various models *vis-a-vis* π^+ production in Au + Au collisions at RHIC ($\sqrt{s_{NN}} = 130$ GeV).

Table 3. Comparison of the particle ratios: measurements versus present (DBP-based) calculations ($0.15 < p_T < 1$).

	π^-/π^+	K^-/K^+	$\bar{P}/P$
PHOBOS	$1.00 \pm 0.01 \pm 0.02$	$0.91 \pm 0.07 \pm 0.06$	$0.60 \pm 0.04 \pm 0.06$
DBP	1.11	0.96	0.82

The various curves drawn in Fig. 5 present a comparative picture of the performances by the various model on the particle production scenario, in particular, with regard to production of positive pion. For the sake of the clarity of the picture we did not insert the data points on even the Au + Au reaction. Our curve could be considered as a mean of the BF model and TDB (thermal + decays + Bjorken) model.

All parametrizations here have been done (i) for minimum bias events; (ii) for the whole available range of $p_T (0 \leq p_T \leq 3.5$ GeV/c); (iii) with the assumption that there is an observance of the spirit of the Feynman scaling (FS) in the nuclear collisions at RHIC energies.

Next, we present comparison of the charged particle ratios obtained by DBP-based calculations with those measured by the PHOBOS collaboration.[3] The particle ratios are calculated in the range $0.15 < p_T < 1$ (Table 3) and are diagrammatically depicted in Fig. 6 against actual data. Besides, a comparison of DBP with other standard version of models, like String Fusion Model (SFM), has also been made in Table 4 where NF, F and FR stand, respectively, for the SFM without string fusion or rescattering, the SFM with string fusion and the SFM with string fusion and rescattering.[3]

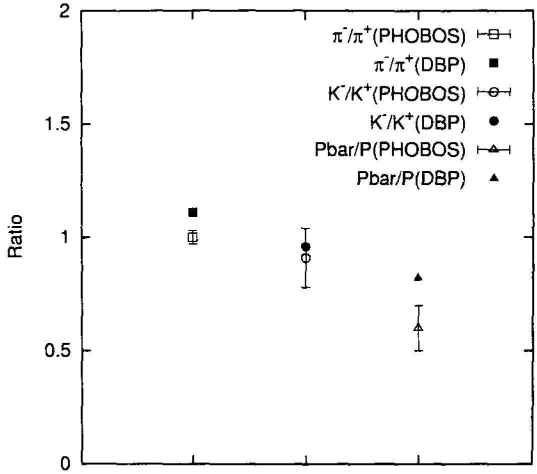


Fig. 6. Comparison of the charge-ratios for the various secondaries produced in Au+Au collisions at RHIC: measured data versus DBP-based calculations.

Table 4. Calculations on the particle ratios: comparison between DBP and the String Fusion Model (SFM).

	π^-/π^+	K^-/K^+	$\bar{P}/P$
DBP	1.11	0.96	0.82
SFM (NF)	1.02	0.92	0.81
SFM (F)	1.02	0.97	0.85
SFM (FR)	1.01	0.96	0.80

A fair agreement in both the cases is quite spectacular. So, our parametrization is seen to work quite well even when such comparisons are made either with measurements or with some specific model(s) as well.

4. Final Discussion and Conclusions

The theoretical curves drawn by the present De–Bhattacharyya parametrization (DBP) illustrate a very transparent description of the data already measured and reported on Au + Au collision at RHIC. And the description is also consistent as it is based on a successful fit to p_T data on the basis of Eq. (4). The nature of agreement obtained by DBP is satisfactory even from the quantitative angle of fits, i.e. by χ^2/ndf. The study could be concluded with the following summary statements: (i) It throws a soft question-mark to the validity of what are known as the "key-ingredients"[4] in the relativistic nuclear collisions like freeze-out, decays of resonances, longitudinal and transverse flows. All of them, admittedly, are only the model-based constructs; and so they are to be treated more as artifacts than facts related to the really measured observables. This turns into a meaningful observation, because we are capable of offering an adequate explanation to the measured data without resorting to any of these ideas, at least, in a straightforward manner. (ii) The assumption on validity of Feynman Scaling is also substantiated by its application to the RHIC data. (iii) Quite admittedly, the main focus of the work rests on the minimum bias events for any secondary. Despite this preferential bias, as a test case, we have tried to interpret the data on inclusive cross-sections for produced neutral pions in both the peripheral and central collisions with the help of the present approach. To our surprise, we discover that the parametrizations for both cases with minor shifts in the values of the two parameters reproduce the measured data quite well. So, this parametrization, by all indications, is not extremely sensitive to the centrality of the collisions. (iv) There are free parameters in almost all of the models in both high energy physics and nuclear physics. In fact, the model (BF) that served the stimulus to take up the present work is also riddled with two chosen parameters. (v) That the two adjustable parameters take care of the data on all varieties of secondaries produced in the heavy nucleus–nucleus collisions, like Au + Au interactions, from P + P reactions at the highest available energy, is a pointer to the intrinsic strength of the phenomenological flyover that has here been constructed for obtaining the final results for nucleus–nucleus reactions from those for P + P collisions at high energies. (vi) The just preceding observation is in agreement with what was indicated by us in a previous work related to proton-air collision.[16] The context is certainly different here; but the main theme that some parametrizations address physics data too well, and that too in a very unified manner, is a convincing demonstration of the strength of and support to the phenomenological approaches. (vii) In asserting some points and in preempting possible queries, let us state and apprise beforehand that the model has been applied to analyze the data on production of all the three types of particles in several other collisions as well which are so far available with a high degree of reliability. Thus, the model can explain one of the crucial properties of high energy physics, called the aspect of "universality" or "globality." (viii) The fair agreement that is obtained here must not be viewed as just a coincidence, as the parametrizations has been checked with data on the p_T-spectra of all these varieties in a large number of diverse high energy particle and nuclear reactions.[17] (ix) The fair degree of compatibility between the present calculations and the measurements and also between the various shades of the SFM and the present one is quite evident from Fig. 6 and also from the supporting tables (Tables 3 and 4). The charge-ratio-values offer us a convenient tool to cross-check the validity of the new parametrization presented here. In sum, the parametrization (DBP) is also corroborated by such exercises of comparison.

Quite logically, and as a follow up of what is so far mentioned here categorically, it would be only fair to recognize a basic flaw of the present approaches: the parameters are not and could not in concrete terms be identified quantitatively with any of the very basic physical observable; this limitation constitutes a probing point for the future studies on such question(s) and issue(s).

Note Added in the Proofs

The nature of the function in expression (4) here has subsequently been finalized t be of the form given in Eq. (8) of Ref. 17 of this paper. With suitable adjustment of the values of the arbitrary parameters like α and β and of the normalizatio constant, the basic physics presented and the conclusions arrived at are found t remain only the same.

References

1. J. Schaffner-Bielich, D. Kharzeev, L. Mclerran and R. Venugopalan, nucl-th/0202054
2. S. Y. Li and X. N. Wang, *Phys. Lett.* **B527**, 85 (2002).
3. N. Armesto, C. Pajares and D. Souza, *Phys. Lett.* **B527**, 92 (2002).
4. W. Broniowski and W. Florkowski, *Phys. Rev. Lett.* **87**, 272302 (2001), and reference therein.
5. T. Peitzmann, *Phys. Lett.* **B450**, 7 (1999).
6. R. Hagedorn, *Riv. Nuovo Cimento* **6**, 1 (1983).
7. S. Bhattacharyya, *Lett. Nuovo Cimento* **44**, 119 (1985).
8. B. De, S. Bhattacharyya and P. Guptaroy, *Int. J. Mod. Phys.* **A17**, 4615 (2002).
9. R. C. Hwa *et al.*, *Phys. Rev.* **C64**, 054611 (2001).
10. L. Van-Hove, *Z. Phys.* **C21**, 93 (1983).
11. M. M. Aggarwal *et al.* (WA98 Collab.), *Eur. Phys. J.* **C23**, 225 (2002).
12. B. Alper *et al.*, *Nucl. Phys.* **B100**, 237 (1975).
13. T. Akesson *et al.*, *Z. Phys.* **C18**, 5 (1983).
14. J. S. Bielich *et al.*, *Nucl. Phys.* **A705**, 494 (2002).
15. K. Adcox *et al.* (PHENIX Collaboration), *Phys. Rev. Lett.* **88**, 022301 (2002).
16. B. De, P. Guptaroy and S. Bhattacharyya, *Mod. Phys. Lett.* **A16**, 2387 (2001).
17. B. De, S. Bhattacharyya and P. Guptaroy, *J. Phys.* **G28**, 2963 (2002).