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EFFECT OF INHIBITORS ON ROW-INTERCROPPING SYSTEM

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ABSTRACT

In this paper we have studied the effect of inhibitors on row-intercropping system by considering a two component species competition model proposed by Lotka (1920). We have verified our results by our experimental data. It has been observed that the inhibitors play an important role to shape the dynamical behaviour of the system.

Keywords: Inhibitors, intercropping, persistence, global stability.

1. Introduction

Intercropping is defined as the growing of two or more common crops on the same area of land at the same time. The crops are not necessarily sown at exactly the same time and their harvest times may be quite different, but they are usually "simultaneous" for a significant part of growing periods. The advantage of the system is that there are a number of possible interactions between crops that can increase the overall crop productivity; the commonest of these is the complementary use of resource by different component crops. The term "component crops" is used here to refer to either of the individual crops making up the intercropping situation. This system has a strong attraction to the ecologists, agronomists, physiologists, economists and other scientists too. Interest on intercropping as well as multiple cropping among scientists has increased markedly which is accompained by the striking increase in number of published papers, comprehensive reviews and books on this topic. The detailed reviews by Willey [17,18] remain among the best available summary to date. The symposia in morogoro [8] were focused on these systems in semiarid zones, as was the symposium in ICRISAT (1981). Four important books by Beet [1], Steiner [5], Gomez and Gomez [4] and Francis [2] brought together in an organised manner the most relevant research done in Asia and Africa on multiple cropping as well as intercropping systems.

Ecological research on the interaction between two populations emphasizes both the relationship between populations at the same trophic level (for example, competition between two populations) and relationship between populations at different trophic levels (for example, predator/prey/host/parasite relationships). The different types of interactions between and among populations have been classified by many authors. E. P. Odum [12] identified nine types of interactions including neutralism (no interaction) and commensalism (one population benefits and other is not affected). He also identified two types of competition, direct inhibition and indirect inhibition through competition for the same resource. Beet [1] reviewed both the agricultural and ecological literature on plant competition. Smith [14] and May [10] were also devoted to this subject.

The study of competition between two component species is, of course, one of the primary interests of research on intercropping system. In this paper we have considered a two component species competition model proposed by Lotka [9] and verified our mathematical results by our experimental field data. It has been observed that the inhibitory effect on the row-intercropping system plays an vital role to shape the dynamical behaviour of the system.

2. The Mathematical Model

The two component species competition model can be written as (see [9,12,13]):

$$\begin{aligned} \frac{dp_1}{dt} &= r_1 p_1 \left(\frac{k_1 - p_1 - ap_2}{k_1}\right), \\ \frac{dp_2}{dt} &= r_2 p_2 \left(\frac{k_2 - p_2 - bp_1}{k_2}\right), \end{aligned} \tag{1}$$

where p_1 and p_2 are the growth factors of two populations, a and b are the inhibitory effects of the competing populations, k_1 and k_2 are envioronmentally defined limits (carrying capacity) and r_1 and r_2 are the intrinsic growth rates respectively.

The system (1) has four equilibria namely $E_0(0,0)$, $E_1(k_1,0)$, $E_2(0,k_2)$ and the interior equilibrium $E^*(p_1,p_2)$ where

$$p_1^* = \frac{ak_2 - k_1}{ab-1} \quad \text{and} \quad p_2^* = \frac{bk_1 - k_2}{ab-1}.$$

The criteria for the existence of (p_1^*, p_2^*) is

(i) $ab < 1$ and $\frac{k_1}{k_2} < \min\left(a, \frac{1}{b}\right)$ or

(ii) $ab > 1$ and $\frac{1}{b} < \frac{k_1}{k_2} < a$.

It is easy to show that the zero equilibrium E_0 is locally and globally unstable implying that both the population do not become extinct. Similarly, the axial equilibria E_1 and E_2 are locally asymptotically stable if $a < \frac{k_1}{k_2}$ and $b < \frac{k_2}{k_1}$ respectively.

We shall first examine the persistence of the system.

3. Persistence of the System

We shall use the method of "average Liapunov function" [3,5]. This method was first applied by Hutson and Vickers [6] in ecological problem.

Theorem 1. The system (1) is persistent if

(i) $a < \dfrac{k_1}{k_2}$ and

(ii) $b < \dfrac{k_2}{k_1}$

Proof. We consider the average Liapunov function of the form

$$\alpha(p_1, p_2) = p_1^{\alpha_1} p_2^{\alpha_2}$$

for each $\alpha_i (i = 1, 2)$ which is assumed positive. In the interior of bounded region, we have

$$\begin{aligned}\frac{\dot{\alpha}}{\alpha} &= \varepsilon(p_1, p_2)\\ &= \alpha_1 \left(\frac{r_1 k_1 - r_1 p_1 - a r_1 p_2}{k_1} \right) + \alpha_2 \left(\frac{r_2 k_2 - r_2 p_2 - a r_2 p_1}{k_2} \right) .\end{aligned}$$

To establish the persistence of the system (1), it is required to prove $\varepsilon(p_1, p_2) > 0$. That is one has to satisfy the following conditions to the boundary equilibria E_0, E_1, E_2.

$$E_0 : \alpha_1 r_1 + \alpha_2 r_2 > 0 , \tag{2}$$

$$E_1 : \alpha_1 \left(r_1 - \frac{a r_1 k_2}{k_1} \right) > 0 , \tag{3}$$

$$E_2 : \alpha_2 \left(r_2 - \frac{b r_2 k_1}{k_2} \right) . \tag{4}$$

It is evident that condition (2) is always satisfied and conditions (3) and (4) are satisfied if the conditions as stated in Theorem 1 hold.

Since we are interested to examine the inhibitory effect on the system, we shall emphasize on the interior equilibrium (p_1^*, p_2^*).

4. Local Stability Analysis (of the Interior Equilibrium)

The characteristic equation of (1) is

$$\lambda^2 + \sigma_1 \lambda + \sigma_2 = 0 \tag{5}$$

where

$$\sigma_1 = \frac{r_1 p_1^*}{k_1} + \frac{r_2 p_2^*}{k_2}$$

and

$$\sigma_2 = \frac{r_1 r_2 p_1^* p_2^*}{k_1 k_2} (1 - ab) \,.$$

To examine the inhibitory effect on the system we shall now discuss the following cases:

Case 1. When $a = 0$ and $b = 0$, then $p_1^* = k_1, p_2^* = k_2$ and $\sigma_2 = r_1 r_2 p_1^* p_2^* > 0$. Hence the system is stable in nature without any parametric restriction and the growth of each population will be maximum being equal to the corresponding environmentally defined limit k_i after which $\frac{dp_i}{dt}$ will be negative indicating the wilting of the plants.

Case 2. When $a = 0$ and $b > 0$, then $p_1^* = k_1$ and $p_2^* = k_2 - bk_1$ and $\sigma_2 = r_1 r_2 (1 - b\frac{k_1}{k_2})$. Hence the system is stable if $\frac{k_1}{k_2} < \frac{1}{b}$ and the growth of the first population is maximal (k_1) whereas that of the second population is less than its maximal value (k_2).

Case 3. When $a > 0$ and $b = 0$, then $p_1^* = k_1 - ak_2, p_2^* = k_2$ and $\sigma_2 = r_1 r_2 (1 - a\frac{k_2}{k_1})$. Hence the system is stable if $\frac{k_1}{k_2} > a$ and the final growth status of each population is just the reverse of Case 2.

Case 4. When $a > 0$ and $b > 0$, then $\sigma_2 = \frac{r_1 r_2 p_1^* p_2^*}{k_1 k_2}(1 - ab)$. Hence the system is stable if $ab < 1$ and $\frac{k_1}{k_2} < \min(a, \frac{1}{b})$ implying that the final growth of both the populations remain under their corresponding carrying capacities k_i.

5. Global Stability Analysis (of the Interior Equilibrium)

Theorem 2. Local asymptotic stability of the interior equilibrium E^* implies its global asymptotic stability.

Proof. Using the positivity of $p_1(t)$ and $p_2(t)$ for $t \in [0, \infty]$, we define the Lyapunov function

$$V(p_1, p_2) = \left(p_1 - p_1^* - p_1^* \log \frac{p_1}{p_1^*}\right) + \left(p_2 - p_2^* - p_2 \log \frac{p_2}{p_2^*}\right) \,. \tag{6}$$

It is clear that the function is nonnegative for all p_1 and p_2 and the function vanishes only if $p_1 = p_1^*$ and $p_2 = p_2^*$.

Calculating the rate of change of V along the solution of (1), we have

$$\begin{aligned} \frac{dV}{dt} &= (p_1 - p_1^*)\left(r_1 - \frac{p_1 r_1}{k_1} - \frac{a r_1 p_2}{k_1}\right) + (p_2 - p_2^*)\left(r_2 - \frac{p_2 r_2}{k_2} - \frac{b r_2 p_1}{k_2}\right) \\ &= -\frac{r_1}{k_1}(p_1 - p_1^*)^2 - \left(\frac{a r_1}{k_1} + \frac{b r_2}{k_2}\right)(p_1 - p_1^*)(p_2 - p_2^*) - \frac{r_2}{k_2}(p_2 - p_2^*)^2 \,. \end{aligned} \tag{7}$$

The above equation can be written as $-X^TAX$, where $X = (p_1 - p_1^*), (p_2 - p_2^*)$ and

$$A = \begin{bmatrix} \frac{r_1}{k_1} & \frac{1}{2}\left(\frac{ar_1}{k_1} + \frac{br_2}{k_2}\right) \\ \frac{1}{2}\left(\frac{ar_1}{k_1} + \frac{br_2}{k_2}\right) & \frac{r_2}{k_2} \end{bmatrix}.$$

From (7), it is clear that $\frac{dV}{dt} < 0$, if the matrix A is positive definite, which is possible, if,

$$\frac{4r_1r_2}{k_1k_2} > \left(\frac{ar_1}{k_1} + \frac{br_2}{k_2}\right)^2 \tag{8}$$

A sufficient condition for satisfying (8) is $ab < 1$ (9)
Hence the theorem.

6. Experiment

Paddy–Legume intercropping studies were undertaken in upland plots of agricultural farm of the Indian Statistical Institute at Giridih, situated in the eastern plateau of India. Paddy (*Oxyza sativa*) cv. culture-1, was intercropped with soyabean (*Glycine max.*) cv. alankar, greengram (*Phaseolus aureus.* Roxb.) cv. T-44, blackgram (*Phaseolus mungo* Roxb.) cv. T-9 and pigeonpea (*Cajanus cajan* (L) Millsp.) cv. T-21 during rainy seasons of 1989–1992.

Plots selected were mid-upland with sandy-loam texture having pH 5.9–6.2, organic carbon 0.31–0.42 %, total nitrogen 0.38 %, available P 15–18 kg per hectare and available K 95–105 kg per hectare. The field capacity and permanent wilting point of the soil were 29 % and 7.2 %, respectively. Values given above are obtained from the surface soil samples at the initiation of the trial. Treatments included control with sole crop paddy, soyabean, blackgram, greengram and pigeonpea at normal seed rates and spacing recommended for the plateau region [11]. For intercropped plots, after every 2 rows of rice 1 row of legume was sown in replacement series [16]. Thus the seed rate of paddy in intercropping combination was 62 % of sole crop, whereas for soyabean, blackgram, greengram and pigeonpea it was 70, 52. 36 and 95 % of sole seed rate respectively. Fertilizers applied for sole and intercropped plots were farm yard manure @ 10 tones/ha, N @ 20 kg/ha, P @ 18 kg/ha and K @ 17 kg/ha at sowing time. Additional dose of 20 kg N/ha was top dressed after a month along paddy lines only. Sole cropped legumes, however received only the same dose of N, P and K at sowing time. Treatments were laid out in randomized block design with three replications. The plot sizes varying 24 m^2 in sole to 36 m^2 in intercropped plots to accomodate appropriate plant population. The fixed plot technique was maintained. The crops were grown rainfed with only 1 weeding, done 1 month after sowing. Control treatments with sole crops were maintained in each case in identical conditions with respect to land conditions, use of fertilizers, irrigation, control of herbivores and parasites, weed control etc.

After the harvest of paddy and all legumes, a succeeding crop of "TM 17" Indian mustard was sown without disturbing the lay out. Four legumes taken as companion species for intercropping studies with paddy are of contrasting growth habits with pigeonpea, soyabean as long duration and greengram and blackgram as short duration crops.

Actual yield loss or gain (AYL) in percent was calculated as follows:

$$AYL_a(\%) = \frac{K - (LM)/N}{(LM)/N}$$

where K = Intercropped yield of species "a", L = sole crop yield of species "a", M = sown proportion of species "a" in intercropped. and N = sown proportion of species "a" in sole i.e. 100. Similarly the AYL_b (%) for component crop of "b" was calculated.

7. Results of the Experiment

Grain yield (q/ha) and actual yield loss in percentage are presented in table 1. It shows that in intercropping with four legumes the yield of paddy is inhibited by 18 % – 22 %. 34 % inhibition of yield of blackgram and 15 % inhibition of pigeonpea could be noted whereas no inhibition of soyabean and greengram could be observed. Thus it may be concluded that paddy has insignificant inhibitory effect on the yield of soyabean and greengram although the latter two legumes have significant inhibitory action on the yield of paddy.

In this paper we have considered two "component species" competition model proposed by Lotka [9] and verified the mathematical results by our experimental findings. The mathematical treatment of the model carried by us consists of local and global stability analysis of the system assuming that the yield of a plant is proportional to its growth. We have compared our mathematical observations with crop yield data as obtained from our field experiments on intercropping. Stability of the system will henceforth imply stability of the interior equilibrium only. The results and experimental observations are as the following:

In Case 1, when there is no inhibitory effect on the system, the system is stable in nature. Our experimental data (Table 1) also reveal that when the species of different crops like paddy and other legumes were sown in separate plots, produced higher amount of grains than intercropping, reflecting our mathematical results.

In Case 2, when species 1 (p_1) has an inhibitory effect on species 2 (p_2) but species 2 has no inhibitory effect on species 1, then we would have observed from our mathematical analysis that the inhibitory rate (b) has some threshold value to stabilize the system. But in our experiment we have not observed such types of phenomena.

In Case 3, when species 1 (p_1) has no inhibitory effect on species 2 (p_2) but species 2 has an inhibitory effect on species 1; then the inhibitory rate (a) has some threshold value to stabilize the system.

Table 1. Grain yield and actual yield loss of different sole and 2:1 intercrop systems.

	Grain yield (Q/ha)						Pooled actual yield	
	Paddy			Legume			loss %	
Year	1989	1990	1991	1989	1990	1991	Paddy	Legume
Paddy	18.4	26.7	20.2					
Soyabean				19.0	26.7	23.9		
Blackgram				7.4	7.5	6.2		
Greengram				5.7	9.6	6.0		
Pigeonpea				16.0	20.0	19.2		
Paddy + soyabean (2:1)	8.2	11.2	13.5	13.0	17.6	17.5	−19.5	− 1.2
Paddy + blackgram (2:1)	10.9	12.2	9.6	1.5	2.6	2.9	−18.6	−34.2
Paddy + greengram (2:1)	10.2	11.3	9.7	1.6	3.5	2.6	−22.1	− 0.3
Paddy + pigeonpea (2:1)	9.2	11.6	12.1	13.0	16.5	15.1	−18.0	−15.4
SE (±)	1.3	3.0	0.8					

In our experiment, when paddy was intercropped with greengram with 2:1 "replacement series" the yield of greengram was not affected due to the association of other species i.e. paddy, although 0.34 % yield loss of greengram has been noted which is negligible from agricultural view point. Our field experiment shows greengram in intercropped situation with 36 % of the sole greengram seed rate yielded nearly 36 % of the sole crop yield whereas paddy with 62 % of sole seed rate yielded only 49 % of the sole crop.This clearly indicate that in case of greengram the yield is at par in both sole and intercropped situation. Hence we may conclude that paddy (p_1) has no inhibitory effect on greengram (p_2), but greengram can inhibit the yield of paddy in intercrop system.

In Case 4, when species 1 (p_1) has an inhibitory effect on species 2 (p_2) and species 2 also has an inhibitory effect on species 1, the system is stable in nature if the product of the inhibitory rates a and b is under some threshold value.

When paddy was intercropped with soyabean, blackgram and pigeonpea with 2:1 "replacement series", paddy produced 19, 19, 18 % less yield with 62 % of sole seed rate, whereas soyabean, blackgram and pigeonpea yielded 1, 34 and 15 % less yield with the seed rate of 70, 52 and 95 % of sole seed rate respectively. Thus in this case too paddy as well as legumes inhibit each other. From mathematical viewpoints, we may conclude that the paddy has an inhibitory effect on soyabean but from agricultural point of view 1 % yield loss may be considered at par with their sown proportion. Hence this situation reflects the findings of Case 3 for soyabean.

Hence from the above study we may finally conclude that in intercropping system inhibitors are one of the key factors for the dynamical behaviour of the system. We have mathematically proved in this paper that the inhibition based two population competition model (1) is persistent under some parametric conditions and the interior equilibrium point is globally asymptotically stable. The final size of yield which is proportional to the size of the interior equilibrium depends upon the inhibitory rates a and b.

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