International Journal of Modern Physics A
Vol. 21, No. 30 (2006) 6229–6247

HIGH ENERGY PROTON–BERYLLIUM COLLISIONS: IS NATURE DICTATED BY POWER LAW?

S. K. BISWAS
West Kodalia Adarsha Siksha Sadan, New Barrackpore, Kolkata-700131, India
sunil_biswas2004@yahoo.com

BHASKAR DE
Institute of Mathematical Sciences, CIT Campus, Taramani, Chennai-600113, India
bhaskar@imsc.res.in

P. GUPTAROY
Department of Physics, Raghunathpur College, Raghunathpur-723133, Purulia, India
gpradeepta@rediffmail.com

A. BHATTACHARYA
Department of Physics, Jadavpur University, Kolkata-700032, India
aparajita_bh@yahoo.co.in

S. BHATTACHARYYA*
Physics and Applied Mathematics Unit (PAMU),
Indian Statistical Institute, Kolkata-700108, India
bsubrata@www.isical.ac.in

Received 28 May 2005

In the present study on proton–Beryllium collisions at two distinctly different energies obtained by one FERMILAB Collaboration, we attempt to focus on the unsettled controversy between the exponential models versus power law models, both of which are found to be in wide applications. The study concludes that none of them could be abandoned finally. And the resolution of the debate, the authors argue, might rest, not on the acceptance of just any one of them, but on a suitable combination of both of them, acting in the different domains of the transverse momentum values.

Keywords: Hadron–nucleus collisions; inclusive production; scaling phenomena; power laws.

PACS numbers: 12.38.Mh, 13.60.Hb, 13.85.Ni

*Corresponding author.

1. Introduction

In the recent past Fermilab E706 Collaboration, Apanasevich *et al.*,[1] reported a set of interesting experimental results on production of neutral mesons in proton–berrylium collision at 530 and 800 GeV/c and compared data with two existing phenomenological models. Besides, Stewart *et al.*[2] also reported earlier some measurements on high p_T particles produced in proton–lead, proton–copper and proton–carbon interaction at 800 GeV/c. All these would attract our attention and interest in the present work.

Our objective here is to use these recent data sets to test and to assess the present status of some of the prevalent and accepted views in the domain of particle physics literature. At the very start the exponential nature[3] was almost taken for granted. This reproduced data on the "soft" (small-p_T) production of particles. But with the advent of the oncoming stream of high-p_T ("hard") data the exponential law somewhat receded in the background and yielded place to the power law behavior[4–9] which now constitutes the dominant trend in the literature of particle physics phenomenology.

Furthermore, very recently, d'Enterria[10] suggested a novel combination of exponential and power law with five parameters and claimed very good agreement with RHIC data at superhard region of transverse momenta. So the controversy boils down to the point whether nature observes exponential law, or power law or just a combination[10–12] of both. In other words, the question is whether it is a choice between exponential versus power law or exponential and power law combined together.

Before we proceed, further, some comments are in order here. In both particle–nucleus ($pA/\pi A$) and nucleus–nucleus collisions at high energies and at large transverse momenta, the nature of mass number ($A_{\mathrm{m.n.}}$) dependence constitutes a puzzling question. The measured values of mass number ($A_{\mathrm{m.n.}}$)-dependence subscribe quite well to a parametrization expressed in the form, $Ed^3\sigma/dp^3 \propto A^{\alpha}_{\mathrm{m.n.}}$ where the exponent α exhibits some peculiar changes which are uptill now left, at best, to only some educated guesses. No comprehensive theoretical–physical models on these puzzling traits of the exponent are yet available. In the present work, we have consciously sidetracked this already much-discussed issue by attempting to interpret the available data on p_T-spectra mainly with the generalized structure of a power law as was done by WA80 Collaboration.[6]

The organization of the paper is as follows. In the next section (Sec. 2) we present the most generalized basic working formula for the present work, with some highlights on the details of the debate and the background. The results have been depicted in Sec. 3 in tabular forms and in graphical plots. In Sec. 4 we have attempted to emphasize the overwhelming success of power laws in the various spheres of particle/ultrahigh energy physics wherein we confront almost the unique choice of the power law(s). In Sec. 5 we have tried to draw some conclusions, of course under certain conditions and constraints.

2. The Background in Some Detail and the Working Formula

In general, the simple rules of statistical mechanics govern, it is generally taken for granted, the single particle energy distribution in the local rest frame which is represented by an exponential form as was originally suggested by Boltzmann:

$$\frac{dn}{dp^*} \sim \exp\left(\frac{-E^*}{T}\right) \tag{1}$$

with $E^* = \sqrt{m^2 + p^{*2}} = m_T \cosh(y^*)$ where m_T denotes the transverse mass given by $m_T = \sqrt{m^2 + p_T^2}$ and y^* is the total hadron rapidity and T is a common (for all particles) temperature parameter which is normally extracted by a comparison with the experimental data. Based on such approach the transverse mass spectra is expressed in the form

$$\frac{dN}{dm_T^2} \sim \exp\left(\frac{-m_T}{T}\right) \tag{2}$$

and the mean hadron multiplicity (for $m \gg T$) is given by the form

$$N(m) \sim \exp\left(\frac{-m}{T}\right). \tag{3}$$

Gazdzicki and Gorenstein[3] maintained that the exponential distribution offer modestly good description of data on p_T spectra in the transverse momentum region $p_T \leq 2$ GeV/c. The normalization factors in the expression (1)–(3) involve physically (i) a volume parameter for production of hadron fluid elements, (ii) a degeneracy factor $g = (2j+1)$ where j is the particle spin and (iii) a chemical factor which accounts for material conservation laws in grand canonical approximation. Combining all these factors and absorbing the parameters let us put here the most generalized form for the exponential distribution

$$f(p_T) = a \exp(-bp_T). \tag{4}$$

However, Gazdzicki and Gorenstein observed rightly that for $p_T > 2$ GeV/c, the data sharply deviates from the exponential nature, for which they proposed a power law distribution of certain forms for both p_T spectra and particle multiplicity.[13] Indeed, for both p_T spectra and multiplicity such power law forms have become now the most dominant tools in dealing with the transverse momentum spectra of all hadrons. Gazdzicki and Gorenstein showed that the normalized multiplicities and ($m_T \sim p_T$) spectra of neutral mesons obey the m_T-scaling which has an approximately power law structure of the form $\sim (m_T)^n$. This scaling behavior is analogous to that expected in statistical mechanics: the parameter n plays the role of temperature and any normalization constant to be used (say c) resembles the system volume. Thus the basic modification of the statistical approach needed to reproduce the experimental results on some hadron production process in $p(\bar{p}) + p$ interaction in the large $m_T \equiv p_T$ domain is to change the shape of the distribution functions. Thus, the Boltzmann function $\exp\left(-\frac{E^*}{T}\right)$ appearing in expression (1) above had to be altered to the power law form as given by $\left(\frac{E^*}{\Lambda}\right)^n$ with some changed parameters, viz. a scale parameter Λ and an exponent n, both are assumed to be common for all hadrons. It is to be noted that we are attempting here to study the properties of two varieties of neutral mesons only, for which we can skip here the proper canonical treatment of material conservation laws needed for description of charged hadrons in small systems.

In what follows we are going to choose the power law in a much more convenient way. With a view to accommodating some observed facts, it is tempting try to fit the whole distribution for the inclusive p_T-spectra with one single expression in the form of power law as was done by G. Arnison *et al.*[4] and Hagedorn:[7]

$$E\frac{d^3\sigma}{dp^3} = \text{const}\,\frac{d(dN/dy)}{2\pi p_T\,dp_T} = A\left(\frac{q}{p_T+q}\right)^n, \tag{5}$$

where the letters and expressions have their contextual significance.

Indeed for $p_T \to 0,\, \infty$, we have

$$\left(\frac{q}{p_T+q}\right)^n \approx \left(1-\frac{n}{q}p_T\right) \approx \begin{cases} \exp\left[\frac{n}{q}p_T\right] & \text{for} \quad p_T \to 0\,, \\ \left(\frac{q}{p_T}\right)^n & \text{for} \quad p_T \to \infty\,. \end{cases} \tag{6}$$

Thus along with impressive fit, which now includes the large p_T domain, the estimate of $\langle p_T\rangle$ assumes with the help of expression (5):

$$\langle p_T\rangle = \frac{\int q/(p_T+q)^n p_T^2\,dp_T}{\int q/(p_T+q)^n p_T\,dp_T} = \frac{2q}{n-3}\,. \tag{7}$$

So, in clearer terms, let us put the final working formulae as follows with substitution of p_T (transverse momentum) as x for the exponential model[8] and power law model[9] respectively

$$f(x) = a\exp(-bx)\,, \tag{8}$$

$$f(x) = A(1+x/q)^{-n}\,. \tag{9}$$

Besides, combining the exponential and power law model, d'Enterria[10] proposed a five-parameter functional form for neutral pion production which is named here as the mixed model (MM): the form is given by

$$f(x) = B\left[\exp(\alpha x^2+\beta x) + \frac{x}{\kappa}\right]^{-\nu}. \tag{10}$$

3. Results and Discussions

The parameters of the exponential model are given in Tables 1, 3, 5 and 7. For all practical purposes, weaker fits based on the exponential model are given here for the sake of mere comparison with the power law model (PLM). In order to test the given working expressions related to power law here we have used three parameters

Table 1. Numerical values of the fit parameters of exponential equation for neutral pion (π^0) production in p–p and p–Be collisions at 530 GeV, $p_T = 1$ to 9 GeV/c.

Collisions	Rapidity ($y_{\rm cm}$)	a	b	$\frac{\chi^2}{ndf}$
p–Be	$-0.750 < y_{\rm cm} < -0.625$	0.08 ± 0.05	2.5 ± 0.1	12.235/9
p–p		0.5 ± 0.7	2.9 ± 0.3	3.747/7
p–Be	$-0.500 < y_{\rm cm} < -0.375$	2 ± 1	3.1 ± 0.2	12.051/9
p–p		0.2 ± 0.1	2.6 ± 0.2	2.850/7
p–Be	$-0.250 < y_{\rm cm} < -0.125$	2 ± 1	3.1 ± 0.2	5.449/10
p–p		0.5 ± 0.4	2.7 ± 0.2	2.695/7
p–Be	$0.000 < y_{\rm cm} < 0.125$	2 ± 1	3.1 ± 0.2	85.452/10
p–p		1.0 ± 0.8	2.9 ± 0.2	40.096/8
p–Be	$0.250 < y_{\rm cm} < 0.375$	2 ± 1	3.1 ± 0.2	84.101/10
p–p		0.8 ± 0.6	2.9 ± 0.2	28.469/7
p–Be	$0.500 < y_{\rm cm} < 0.625$	1.3 ± 0.6	3.1 ± 0.1	23.746/9
p–p		1.1 ± 0.8	3.0 ± 0.2	26.530/7

Table 2. Numerical values of the fit parameters of power law equation for neutral pion (π^0) production in p–p and p–Be collisions at 530 GeV, $p_T = 1$ to 9 GeV/c.

Collisions	Rapidity ($y_{\rm cm}$)	A	q	n	$\frac{\chi^2}{ndf}$
p–Be	$-0.750 < y_{\rm cm} < -0.625$	997 ± 508	2.7 ± 0.4	21 ± 1	6.50/9
p–p		$(6.9 \pm 0.4) \times 10^{14}$	0.12 ± 0.06	13.2 ± 0.9	0.920/8
p–Be	$-0.500 < y_{\rm cm} < -0.375$	502 ± 118	3.0 ± 0.2	21.2 ± 0.6	1.80/8
p–p		$(7 \pm 0.4) \times 10^{14}$	0.09 ± 0.03	12 ± 1	1.14/8
p–Be	$-0.250 < y_{\rm cm} < -0.125$	685 ± 195	2.8 ± 0.2	20.4 ± 0.8	3.06/9
p–p		$(1.6 \pm 0.6) \times 10^{6}$	0.8 ± 0.2	14.7 ± 0.6	0.18/10
p–Be	$0.000 < y_{\rm cm} < 0.125$	516 ± 139	2.8 ± 0.2	20.4 ± 0.8	2.71/9
p–p		$(0.72 \pm 0.04) \times 10^{5}$	1.1 ± 0.7	15 ± 2	7.74/9
p–Be	$0.250 < y_{\rm cm} < 0.375$	419 ± 164	3.1 ± 0.4	22 ± 1	6.49/9
p–p		$(0.72 \pm 0.34) \times 10^{5}$	1.2 ± 0.9	16 ± 3	14.18/9
p–Be	$0.500 < y_{\rm cm} < 0.625$	69 ± 35	5.0 ± 0.8	28 ± 2	2.39/8
p–p		$(0.15 \pm 0.01) \times 10^{5}$	1.8 ± 0.2	19 ± 2	19.23/9

Table 3. Numerical values of the fit parameters of exponential equation for neutral pion (π^0) production in p–p and p–Be collisions at 800 GeV, $p_T = 1$ to 9 GeV/c.

Collisions	Rapidity ($y_{\rm cm}$)	a	b	$\frac{\chi^2}{ndf}$
p–Be	$-1.000 < y_{\rm cm} < -0.875$	3 ± 2	3.1 ± 0.2	4.980/10
p–p		0.3 ± 0.1	2.7 ± 0.2	5.355/9
p–Be	$-0.750 < y_{\rm cm} < -0.625$	0.2 ± 0.1	2.5 ± 0.2	5.660/10
p–p		0.7 ± 0.6	2.7 ± 0.2	4.806/9
p–Be	$-5.000 < y_{\rm cm} < -0.375$	0.3 ± 0.1	2.5 ± 0.2	3.214/10
p–p		0.5 ± 0.1	2.6 ± 0.2	4.267/9
p–Be	$-0.250 < y_{\rm cm} < -0.125$	1.1 ± 0.5	2.7 ± 0.2	5.826/10
p–p		0.19 ± 0.04	2.4 ± 0.3	4.356/9
p–Be	$0.000 < y_{\rm cm} < 0.125$	0.8 ± 0.3	2.7 ± 0.2	95.186/10
p–p		0.5 ± 0.2	2.60 ± 0.3	30.108/6
p–Be	$0.250 < y_{\rm cm} < 0.375$	0.7 ± 0.4	2.7 ± 0.2	88.560/10
p–p		0.5 ± 0.4	2.6 ± 0.2	30.942/6

Table 4. Numerical values of the fit parameters of power law equation for neutral pion (π^0) production in p–p and p–Be collisions at 800 GeV, $p_T = 1$ to 9 GeV/c.

Collisions	Rapidity ($y_{\rm cm}$)	A	q	n	$\frac{\chi^2}{ndf}$
p–Be	$-1.000 < y_{\rm cm} < -0.875$	2180 ± 850	2.1 ± 0.2	18.3 ± 0.7	2.16/9
p–p		$(2.7 \pm 0.3) \times 10^7$	0.7 ± 0.5	15 ± 2	4.12/10
p–Be	$-0.750 < y_{\rm cm} < -0.625$	714 ± 534	3.0 ± 0.5	21 ± 2	0.14/8
p–p		$(2.7 \pm 0.4) \times 10^7$	0.6 ± 0.1	14 ± 2	2.84/10
p–Be	$-0.500 < y_{\rm cm} < -0.375$	1600 ± 796	2.0 ± 0.2	16.8 ± 0.7	0.27/8
p–p		$(5.6 \pm 0.6) \times 10^6$	0.6 ± 0.1	13 ± 2	1.31/9
p–Be	$-0.250 < y_{\rm cm} < -0.125$	2064 ± 987	1.8 ± 0.2	16.3 ± 0.6	0.24/9
p–p		$(5.50 \pm 0.4)2 \times 10^6$	0.6 ± 0.1	13 ± 2	2.33/10
p–Be	$0.000 < y_{\rm cm} < 0.125$	2137 ± 538	1.8 ± 0.1	16.0 ± 0.3	0.18/9
p–p		$(0.71 \pm 0.07) \times 10^5$	1.0 ± 0.2	14 ± 4	16.87/8
p–Be	$0.250 < y_{\rm cm} < 0.375$	1346 ± 511	2.0 ± 0.2	16.9 ± 0.5	2.50/9
p–p		$(0.71 \pm 0.08) \times 10^5$	1.0 ± 0.6	14 ± 5	67.88/7

Table 5. Numerical values of the fit parameters of exponential equation for eta-meson (η) production in p–p and p–Be collisions at 530 GeV, $p_T = 3.5$ to 7.5 GeV/c.

Collisions	Rapidity (y_{cm})	a	b	$\frac{\chi^2}{ndf}$
p–Be	$-0.750 < y_{\mathrm{cm}} < -0.625$	0.2 ± 0.1	2.8 ± 0.2	0.453/3
p–Be	$-0.500 < y_{\mathrm{cm}} < -0.375$	0.06 ± 0.01	2.5 ± 0.2	1.910/4
p–p	$-0.750 < y_{\mathrm{cm}} < -0.500$	0.008 ± 0.001	2.2 ± 0.4	0.377/2
p–Be	$-0.250 < y_{\mathrm{cm}} < -0.125$	0.12 ± 0.09	2.6 ± 0.1	3.177/4
p–Be	$0.000 < y_{\mathrm{cm}} < 0.125$	0.05 ± 0.02	2.5 ± 0.2	4.156/4
p–p	$-0.250 < y_{\mathrm{cm}} < 0.000$	0.5 ± 0.3	2.9 ± 0.2	0.187/1
p–Be	$0.250 < y_{\mathrm{cm}} < 0.375$	0.07 ± 0.05	2.5 ± 0.2	6.752/4
p–Be	$0.500 < y_{\mathrm{cm}} < 0.625$	0.05 ± 0.03	2.5 ± 0.1	1.131/3
p–p	$0.250 < y_{\mathrm{cm}} < 0.500$	0.2 ± 0.1	2.8 ± 0.2	0.177/1

Table 6. Numerical values of the fit parameters of power law equation for eta-meson (η) production in p–p and p–Be collisions at 530 GeV, $p_T = 3.5$ to 7.5 GeV/c.

Collisions	Rapidity (y_{cm})	A	q	n	$\frac{\chi^2}{ndf}$
p–Be	$-0.750 < y_{\mathrm{cm}} < -0.625$	263 ± 9	4 ± 2	25 ± 6	0.03/4
p–Be	$-0.500 < y_{\mathrm{cm}} < -0.375$	$(1.68 \pm 0.01) \times 10^9$	0.5 ± 0.1	15 ± 4	0.17/4
p–p	$-0.750 < y_{\mathrm{cm}} < -0.500$	1.33×10^7	0.62	14.7	—
p–Be	$-0.250 < y_{\mathrm{cm}} < -0.125$	$(1.27 \pm 0.01) \times 10^4$	1.5 ± 0.3	17 ± 1	0.37/5
p–Be	$0.000 < y_{\mathrm{cm}} < 0.125$	$(5.4 \pm 0.5) \times 10^4$	1.2 ± 0.2	16 ± 7	20.21/5
p–p	$-0.250 < y_{\mathrm{cm}} < 0.000$	1.47×10^5	1.77	20.26	—
p–Be	$0.250 < y_{\mathrm{cm}} < 0.375$	$(3.4 \pm 0.3) \times 10^4$	1.6 ± 0.2	18 ± 8	1.11/5
p–Be	$0.500 < y_{\mathrm{cm}} < 0.625$	4 ± 2	7.5 ± 0.3	30 ± 20	9.72/4
p–p	$0.250 < y_{\mathrm{cm}} < 0.500$	9.24×10^5	1.66	17.57	—

Table 7. Numerical values of the fit parameters of exponential equation for eta-meson (η) production in p–p and p–Be collisions at 800 GeV, $p_T = 3$ to 9 GeV/c.

Collisions	Rapidity (y_{cm})	a	b	$\frac{\chi^2}{ndf}$
p–Be	$-0.750 < y_{\mathrm{cm}} < -0.625$	0.004 ± 0.001	1.9 ± 0.3	4.270/5
p–Be	$-0.500 < y_{\mathrm{cm}} < -0.375$	0.03 ± 0.02	2.2 ± 0.2	2.70/5
p–p	$-1.000 < y_{\mathrm{cm}} < -0.750$	0.006 ± 0.001	2.0 ± 0.4	0.239/1
p–Be	$0.000 < y_{\mathrm{cm}} < 0.125$	0.03 ± 0.02	2.1 ± 0.2	6.09/5
p–p	$-0.750 < y_{\mathrm{cm}} < -0.500$	0.12 ± 0.03	2.5 ± 0.2	0.201/1
p–p	$-0.500 < y_{\mathrm{cm}} < -0.250$	0.02 ± 0.01	2.1 ± 0.2	0.483/1
p–Be	$-0.250 < y_{\mathrm{cm}} < -0.125$	0.002 ± 0.001	1.8 ± 0.2	1.837/5
p–p	$-0.250 < y_{\mathrm{cm}} < 0.000$	0.004 ± 0.003	1.8 ± 0.1	0.303/1
p–p	$0.000 < y_{\mathrm{cm}} < 0.250$	0.004 ± 0.001	1.8 ± 0.3	0.172/1

Table 8. Numerical values of the fit parameters of power law equation for eta-meson (η) production in p–p and p–Be collisions at 800 GeV, $p_T = 1$ to 9 GeV/c.

Collisions	Rapidity ($y_{\rm cm}$)	A	q	n	$\frac{\chi^2}{ndf}$
p–Be	$-0.750 < y_{\rm cm} < -0.625$	$(1.5 \pm 0.2) \times 10^7$	0.40 ± 0.01	12 ± 1	0.86/5
p–Be	$-0.500 < y_{\rm cm} < -0.375$	$(1.49 \pm 0.1) \times 10^7$	0.4 ± 0.2	13 ± 5	0.51/6
p–p	$-1.000 < y_{\rm cm} < -0.750$	822	1.4	14.2	—
p–Be	$0.000 < y_{\rm cm} < 0.125$	$(1.50 \pm 0.06) \times 10^7$	0.47 ± 0.07	13 ± 1	0.09/6
p–p	$-0.750 < y_{\rm cm} < -0.500$	5.56×10^6	0.73	14.8	—
p–p	$-0.500 < y_{\rm cm} < -0.250$	1.0×10^7	0.43	12.09	—
p–Be	$-0.250 < y_{\rm cm} < -0.125$	$(1.5 \pm 0.2) \times 10^7$	0.4 ± 0.3	12 ± 6	0.99/6
p–p	$0.250 < y_{\rm cm} < 0.000$	0.15×10^6	0.50	11	—
p–p	$0.000 < y_{\rm cm} < 0.250$	2.5×10^7	0.37	12	—

Table 9. Calculated values of average transverse momentum $\langle p_T \rangle$ for different rapidity ranges at two different laboratory energies.

Laboratory energy of interaction	Rapidity ranges	Value of $\langle p_T \rangle$ in GeV/c
530 GeV	$-0.750 < y_{\rm cm} < -0.625$	0.30
	$-0.500 < y_{\rm cm} < -0.375$	0.32
	$-0.250 < y_{\rm cm} < -0.125$	0.31
	$-0.000 < y_{\rm cm} < 0.125$	0.32
	$0.250 < y_{\rm cm} < 0.375$	0.33
530 GeV	$0.500 < y_{\rm cm} < 0.625$	0.40
	$-0.750 < y_{\rm cm} < 0.750$	0.43
800 GeV	$-0.750 < y_{\rm cm} < -0.625$	0.33
	$-0.500 < y_{\rm cm} < -0.375$	0.29
	$-0.250 < y_{\rm cm} < -0.125$	0.28
	$-0.000 < y_{\rm cm} < 0.125$	0.28
	$0.250 < y_{\rm cm} < 0.375$	0.29
800 GeV	$-0.000 < y_{\rm cm} < 0.500$	0.34

Table 10. Numerical values of the fit parameters of mixed model for neutral pion (π^0) production in p–p and p–Be collisions at 530 and 800 GeV, $p_T = 1$ to 7 GeV/c, rapidity ranges: $0.000 < y_{\rm cm} < 0.125$.

Collisions	$E_{\rm lab}$ (GeV)	B	α	β	κ	ν	$\frac{\chi^2}{ndf}$
p–Be	530 GeV	4.2 ± 0.1	0.05 ± 0.02	$-(0.7 \pm 0.2)$	1.6 ± 0.2	13.9 ± 0.2	5.732/11
p–Be	800 GeV	20.1 ± 0.2	0.07 ± 0.01	$-(0.8 \pm 0.1)$	1.3 ± 0.1	12.0 ± 0.2	0.847/11
p–p	530 GeV	5.5 ± 0.2	0.850 ± 0.003	$-(6.40 \pm 0.02)$	1.240 ± 0.002	11.7 ± 0.2	5.558/8
p–p	800 GeV	5.0 ± 0.3	0.96 ± 0.02	$-(7.4 \pm 0.2)$	1.27 ± 0.01	11.3 ± 0.4	26.881/8

Table 11. Numerical values of the fit parameters of mixed model for eta-meson (η) production in p–p and p–Be collisions at 530 and 800 GeV, $p_T = 3$ to 7.5 GeV/c, rapidity ranges: $0.000 < y_{\rm cm} < 0.125$.

Collisions	$E_{\rm lab}$	B	α	β	κ	ν	$\frac{\chi^2}{ndf}$
p–Be	530 GeV	1.5 ± 0.1	0.90 ± 0.03	$-(7.0 \pm 0.2)$	1.45 ± 0.01	13 ± 2	15.790/5
p–Be	800 GeV	2.2 ± 0.1	0.11 ± 0.01	$-(1.25 \pm 0.04)$	1.360 ± 0.004	12 ± 1	0.600/6

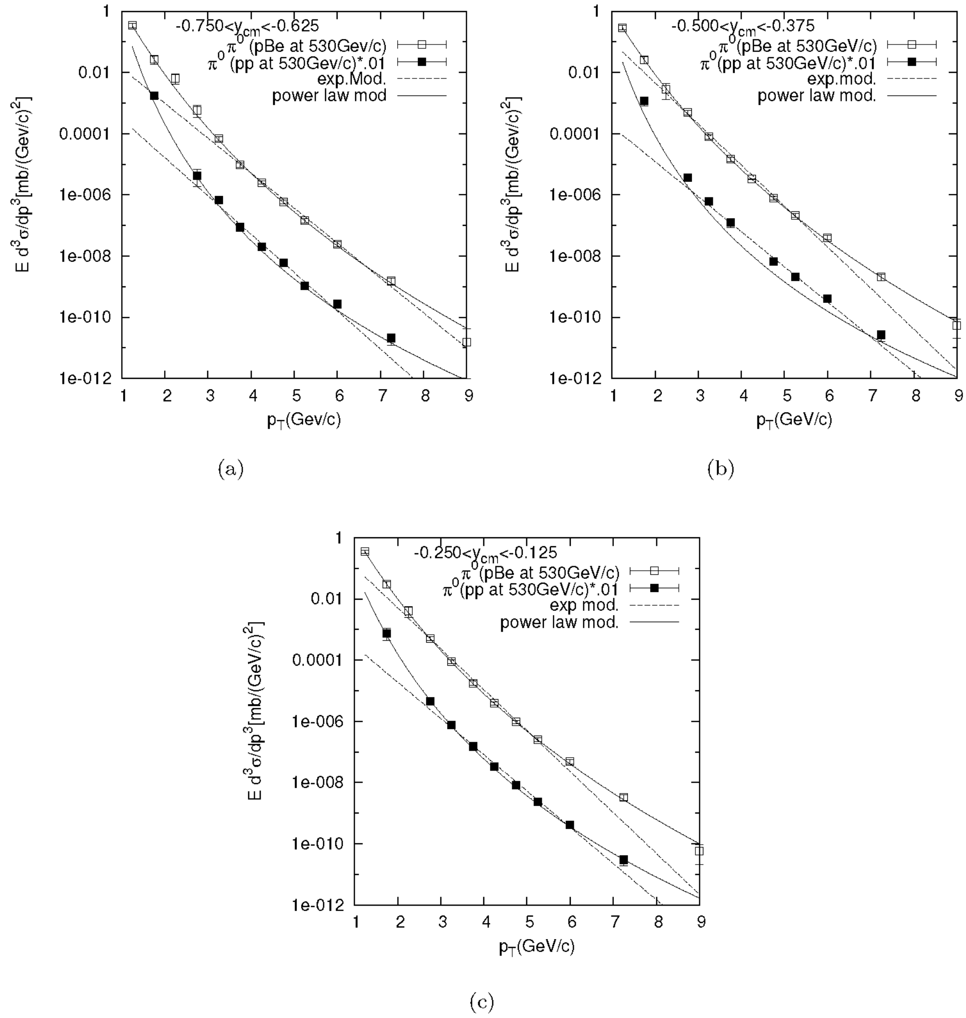


Fig. 1. Transverse momentum spectra for production of neutral pions in pp and p–Be collisions at $E_{\rm lab} = 530$ GeV/c at three negative rapidity regions. The experimental data are taken from Ref. 1. The solid curves are fits for power law model while the dashed ones are for exponential model.

in total, of which one is the arbitrary normalization constant and the other two are q and n. The values of q and n are given in Tables 2, 4, 6 and 8. Obviously, they depend on the nature of secondaries and the center-of-mass (c.m.) energies of the basic interactions and also on the rapidity range in which the studies are made. And the nature of fit for both neutral pions and eta mesons are shown in Figs. 1–6. Figure 7 is exclusively for presentation of the results on production of neutral pions in some proton-induced non-beryllium collisions as is indicated in the plot. With

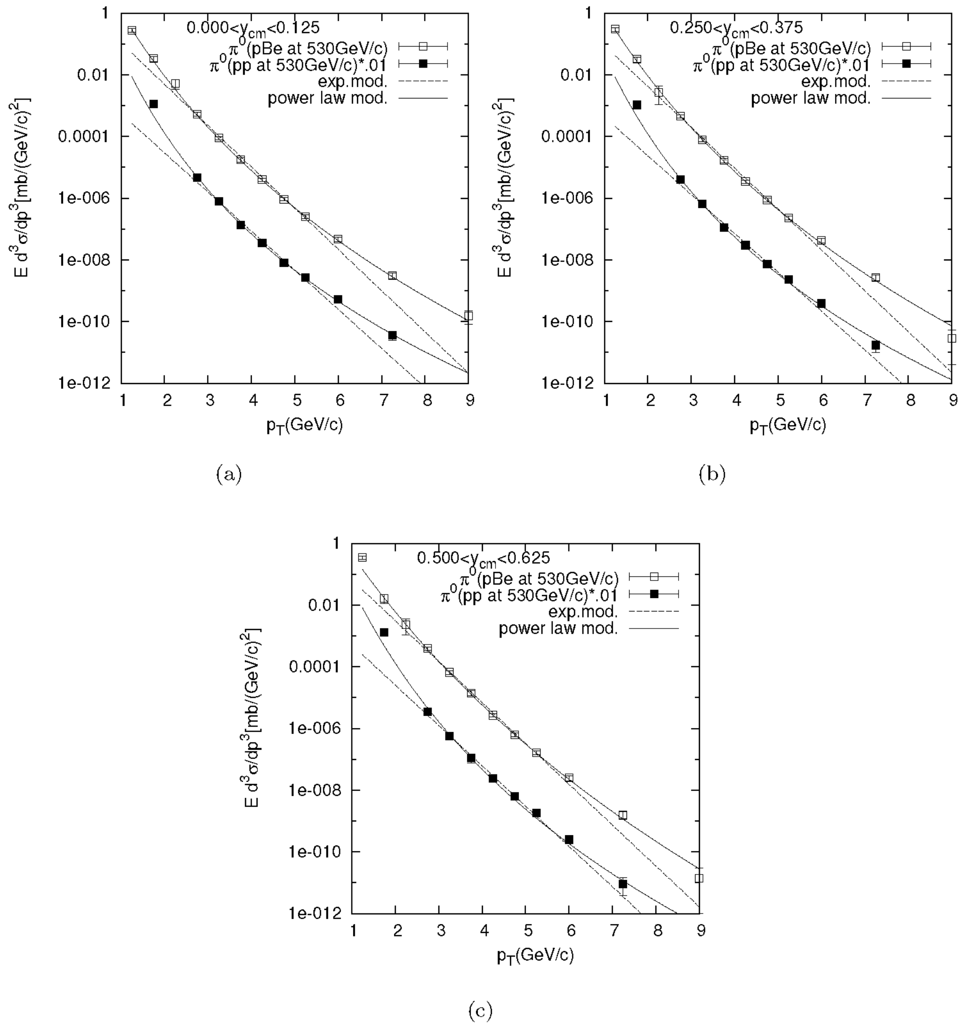


Fig. 2. Plots of transverse momentum spectra for π^0 produced in three positive rapidity regions of pp and p–Be collisions at $E_{\mathrm{lab}} = 530$ GeV/c. The experimental data are taken from Ref. 1. The solid curves provide fits on the basis of power law model while the dashed ones are for exponential model.

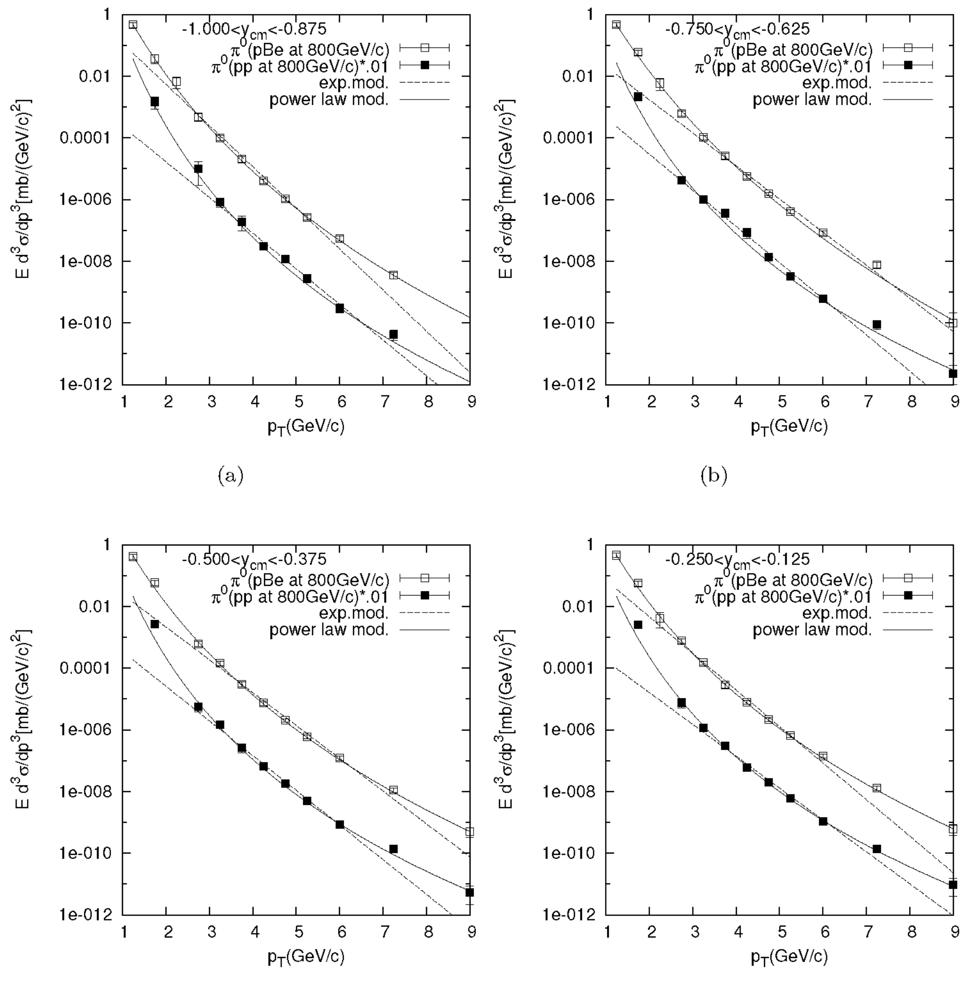
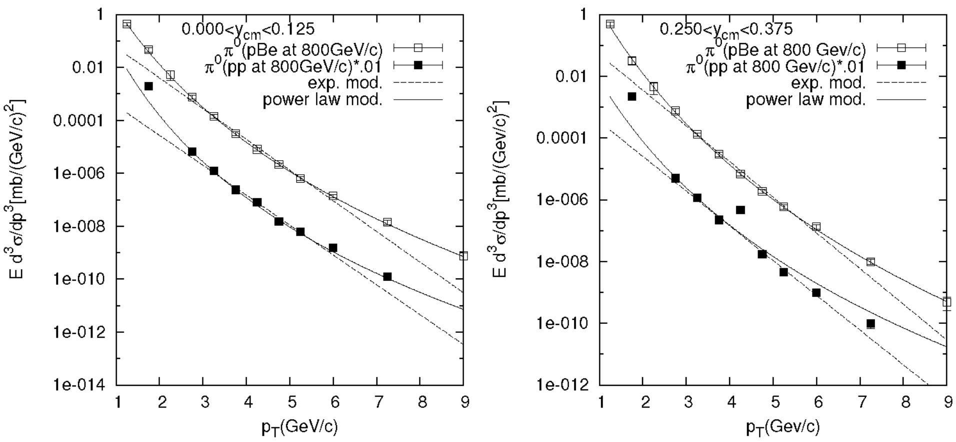


Fig. 3. Invariant spectra as a function of transverse momenta of neutral pions produced in pp and p–Be collisions at $E_{\text{lab}} = 800$ GeV/c at four negative rapidity regions. The experimental data are taken from Ref. 1. The solid curves are fits for power law model while the dashed ones are for exponential model.

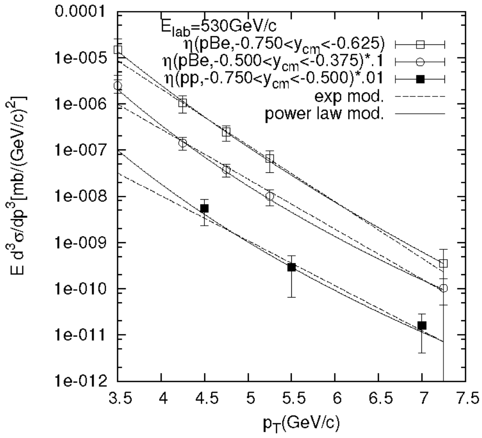
a view to investigating the correlation between q and n, we proceed in a manner indicated first by Hagedorn[7] who showed first that the parameters in this type of power law do essentially reflect the ranges of average transverse momentum of the produced secondary. And we have chosen to study these aspects as well in checking whether the power law fits depicted by us are just some coincidences or they do really merit some special attentions. And we observe finally that the values of q and n chosen by us for the fit of p_T-spectra do also represent the range of the average



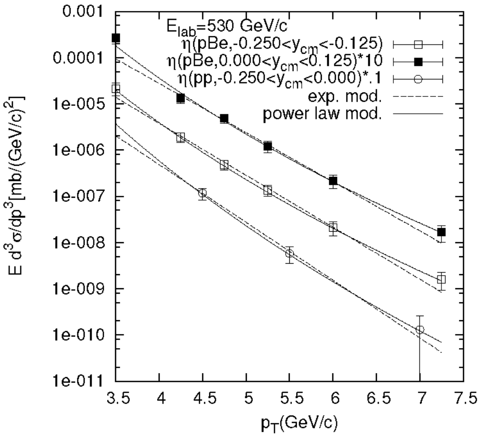
(a) (b)

Fig. 4. Transverse momentum spectra for production of neutral pions in pp and p–Be collisions at $E_{\rm lab} = 800$ GeV/c at two positive rapidity regions. The experimental data are taken from Ref. 1. The solid curves are fits for power law model while the dashed ones are for exponential model.

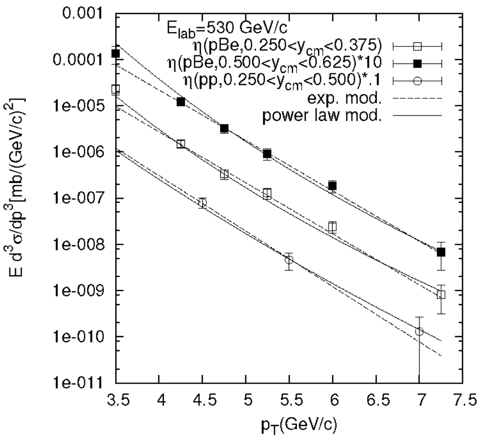
transverse momenta as reported by some other experimental measurements. The ranges of average transverse momenta that we obtain here lie within 0.28 GeV/c to 0.43 GeV/c. These values have been shown in Table 9. All these values tally with the similar ranges arrived at by experimental measurements.[14–16] This helps to obtain for us a consistency check-up of the parameter values used for getting fits to the data on p_T-spectra. However this type of empirical distribution suffers from another well-diagnosed disease called its non-uniqueness property. But there are some limitations which should not and could not be overlooked. The data in this experiment were measured for two separate energies. But we cannot make any meaningful comment or predict about energy-dependence on the basis of data sets, as they were done for absolutely different rapidity ranges for each of the energy. Of course, one has to accept the fact that no valid or meaningful predictions could be made depending on data available at just two energies. But, due to the difference in the rapidity intervals at two distinct energies, we can hazard even no guess. Lastly, values of the parameters used in the mixed model (MM) are shown in Tables 10 and 11. It is seen that the plots based on MM agrees very well with the measured data for range of p_T-values, from very low to quite large. The plots of results depicted Figs. 8(a) and 8(b) demonstrate this grand success of the MM for production of neutral pion and eta meson in two sets of interaction. The plots of Figs. 9(a) and 9(b) present a close comparison of the performances of the power law model and the mixed model. The four diagrams in Fig. 10 illustrate the nature of agreement between the measured data on eta-to-pion ratios and the PLM-based results. It is found that the PLM traces the nature of data on the ratios quite well.



(a)



(b)



(c)

Fig. 5. Plots of transverse momentum spectra for production of η in pp and p–Be collisions at $E_{\text{lab}} = 530$ GeV/c at different rapidity regions. The experimental data are taken from Ref. 1. The solid curves are drawn on the basis of for power law model while the dashed ones are from exponential model.

4. Total Supremacy of Power Laws in High Energy Physics?

The most important observable in the domain of particle production at high energies is the average multiplicity of the various secondaries. One of us deduced[17] some very workable power laws for the average multiplicity of pions, kaons, baryon–antibaryons with c.m. energies; for pions it was shown that $\langle n\rangle_\pi \sim s^{\frac{1}{5}}$ and for other two varieties the multiplicity varies as $\langle n\rangle_{\text{av}} \sim s^{\frac{1}{4}}$. Long ago, Landau's

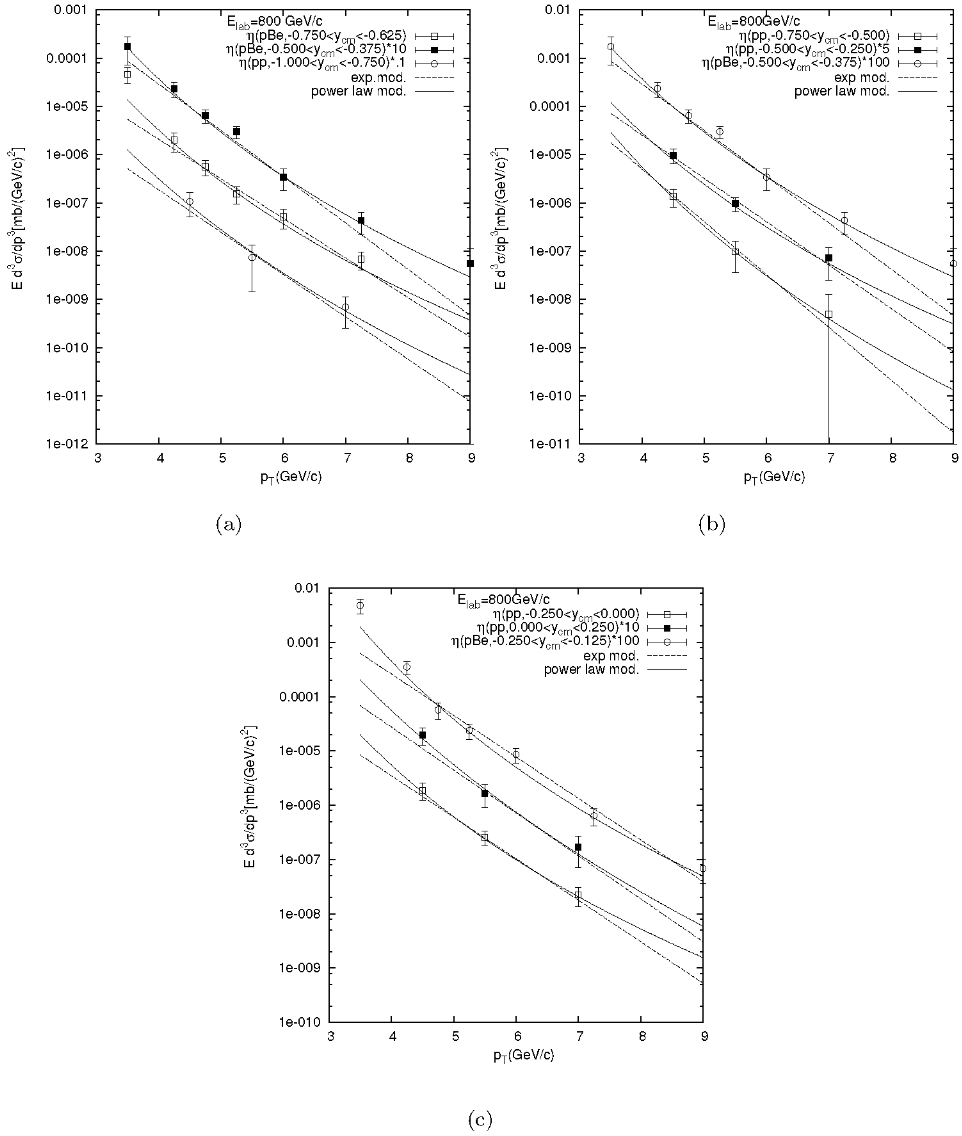


Fig. 6. Plots of inclusive spectra as function of p_T for production of η in pp and p–Be collisions at $E_{\rm lab} = 800$ GeV/c at different rapidity regions. The experimental data are taken from Ref. 1. The solid curves are drawn on the basis of for power law model while the dashed ones are from exponential model.

hydrodynamic extension of Fermi's theory of multiple production of hadrons also led to propound the power law of average multiplicity in the form $\langle n\rangle \sim s^{\frac{1}{4}}$.[18] In a separate work one of us[19] showed that even the average transverse momenta of all these secondaries gave a good description of the experimental data obtained by

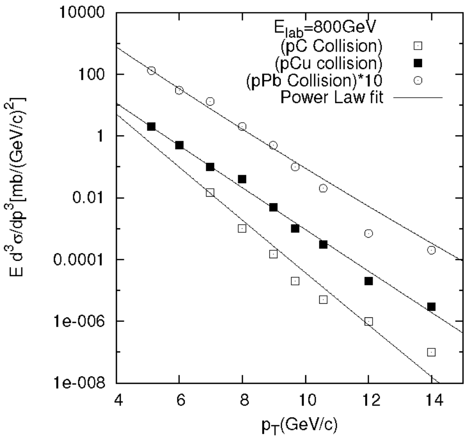
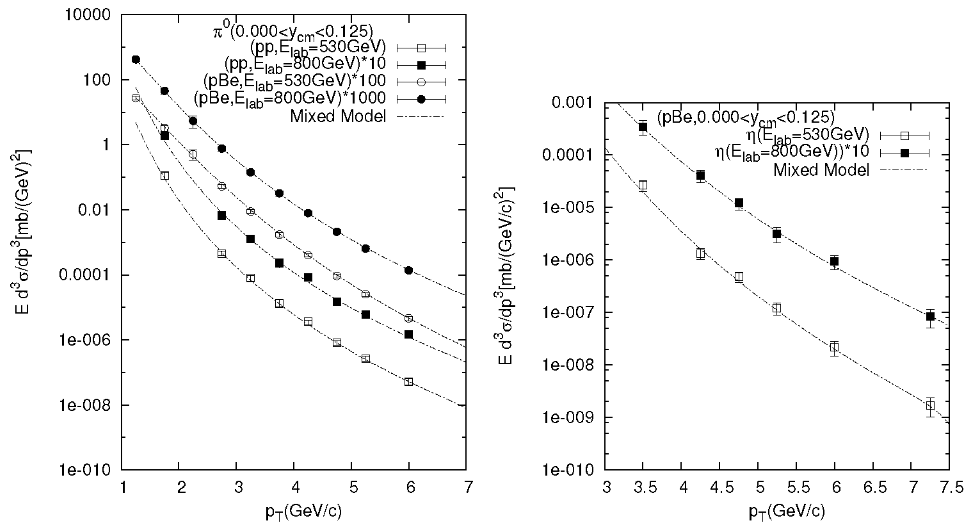


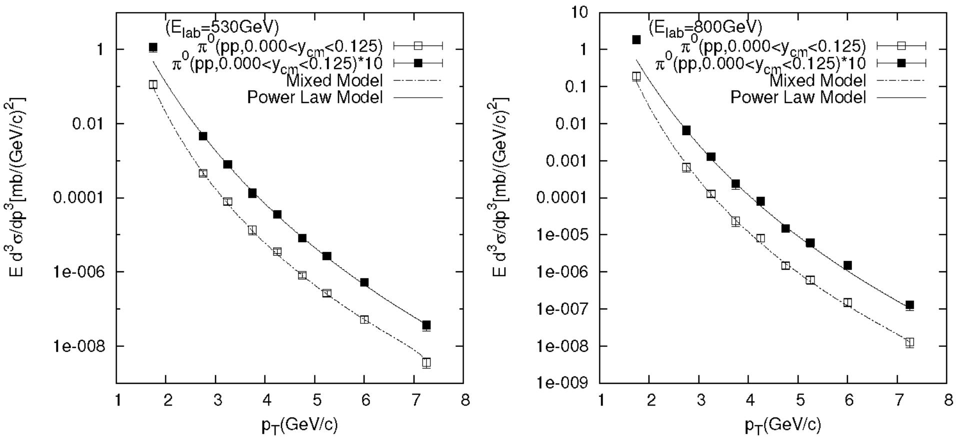
Fig. 7. Transverse momentum spectra for production of pions in some proton-induced reactions at $E_{\mathrm{lab}} = 800$ GeV/c. The experimental data are from Ref. 2. The solid curves depict the power law based fits.



(a) (b)

Fig. 8. Transverse momentum spectra for (a) production of neutral pions in pp and p–Be collisions at $E_{\mathrm{lab}} = 530$ GeV/c and $E_{\mathrm{lab}} = 800$ GeV/c in a specific rapidity range. (b) Production of η mesons in p–Be collision in the same rapidity range and same energies. The experimental data are taken from Ref. 1.The curves are fits to the data based on the mixed model.

the high energy measurements. Along with others, the nature of elastic and total cross-sections were also found by De *et al.*[20] to be in accord with the power laws. The expressions for the inclusive cross-sections of the various secondaries produced



(a) (b)

Fig. 9. Comparison of the fits based on the power law model and the mixed model to the data on neutral pion productions in pp collisions in the same rapidity range at (a) $E_{\text{lab}} = 530$ GeV/c and (b) $E_{\text{lab}} = 800$ GeV/c.

in PP collisions the power law nature was originally proposed by G. Arnison *et al.*[4] which was later adopted by many others. Even the production of particles in deep inelastic scattering on nuclei also shows a remarkable agreement with the power laws on A-dependence of the inclusive cross-sections.[21] Thus, in so far as the direct evidences are concerned, we find an overwhelming support to the power law in nature in almost all the sectors. There is yet another striking evidence in favor of the power laws and that comes from the intermittency studies[22–26] in particle physics. Besides the generalized fractal[27–31] behavior of nature also follows some power laws. In cosmic ray physics the primary spectra of the nucleons are invariably assumed to be of the power law form.[32–34] So, it is not only for pA collisions, but in almost all the sectors of particle physics and of cosmic ray physics, power laws have become the strongly winning candidate.

5. Concluding Remarks

Our findings from this work are quite simple and straightforward:

(a) The efficacy of the exponential model is limited, in general, to a very small range of p_T values. But it cannot be rejected altogether by considering and calling it obsolete, especially after the resurrection of it by d'Enterria and by BRAHMS Collaboration.[35] Very recently BRAHMS Collaboration[35] has shown that even for RHIC-BNL experiments involving Au–Au collisions at $\sqrt{s_{NN}} = 200$ GeV the spectra of the secondary kaons, protons and antiprotons spectra could be accommodated in terms of either or a sum of two exponential functions.

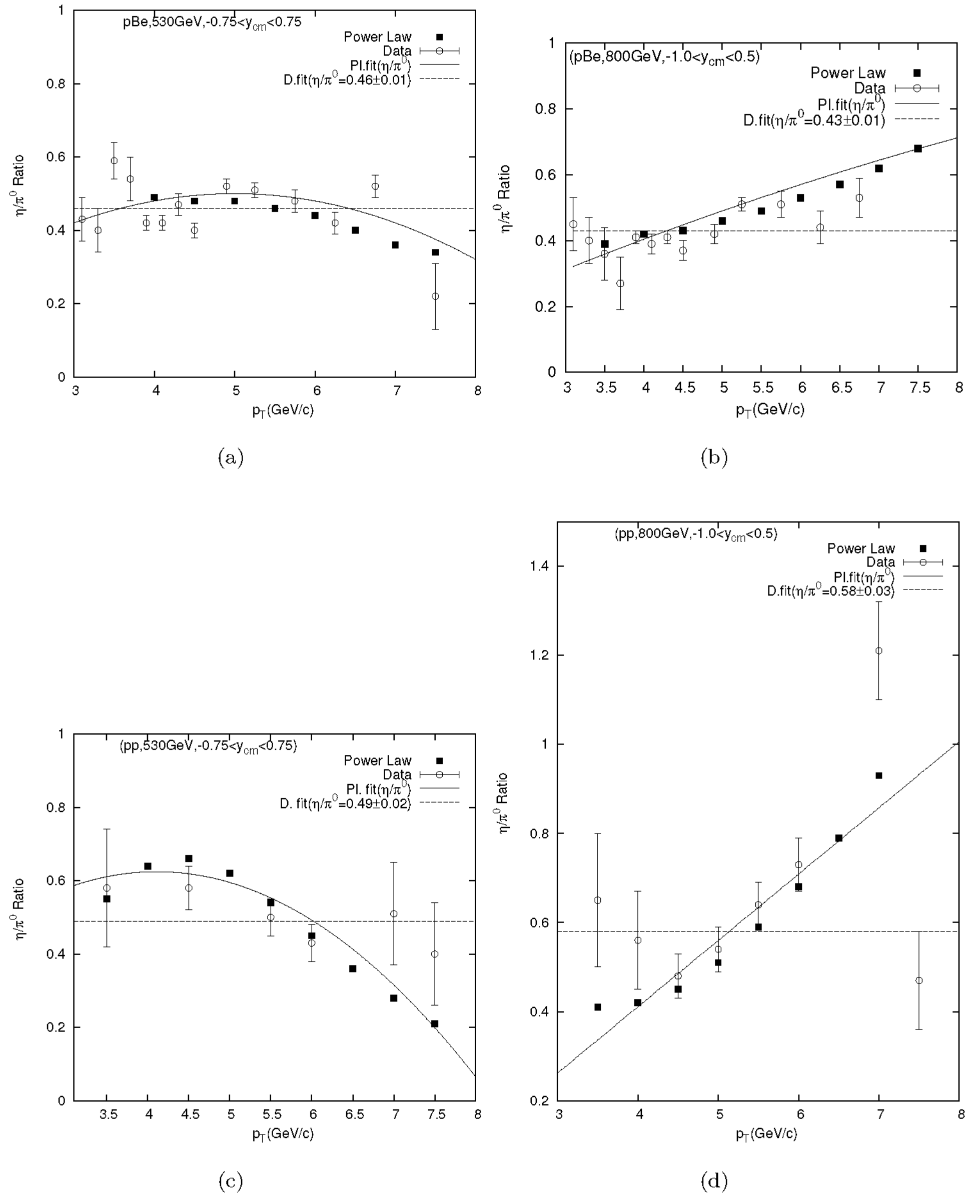


Fig. 10. Transverse momentum-dependence of η/π^0 for p–Be and pp collisions at 530 GeV/c and 800 GeV/c. The data type-points are taken from Ref. 1. The solid curves or straight lines are drawn on the basis of power law model.

(b) Power law model stands in the forefront in confronting the up-to-date data on not only proton–Beryllium (p–Be) or general proton–nucleus (pA) interactions,[2] but almost all collisions at high energies and large transverse momenta. But it might not be the be-all and end-all, as shown by d'Enterria.[10]

(c) The combination of the exponential and the power law model, former for the low-p_T (soft) and latter for the large-p_T (hard) sectors, might play the most pivotal role in understanding the general trends of measured data in all high energy interactions involving particle–particle, particle–nucleus and nucleus–nucleus reactions.

(d) We have amply illustrated in the several figures presented in this work that the generalized form of power law equation reigns supreme to interpret not only the invariant spectra for neutral pi and eta mesons production, but also to describe the ratio behaviors of eta-to-pion.

(e) The behaviors of eta-to-pi ratios with regard to p_T-studies in both the measured values and in the model-based results do not depict or manifest any clear nature of dependence; they are simply erratic. But the power law fits have reproduced or demonstrated even this erratic trends of data with a modest degree of success. But a pattern might be obtained later when more data would be available. The nature of the ratios emerge to resemble each other, when the concerned rapidity range remains same; so the ratios seem to depend more on the specific rapidity-range in which the measurements are done than on the nature of the specific interacting particle.

References

1. Fermilab E706 Collab. (L. Apanasevich *et al.*), *Phys. Rev. D* **68**, 052001 (2003).
2. Fermilab E557 Collab. (C. Stewart *et al.*), *Phys. Rev. D* **42**, 1385 (1990), preprint: FERMILAB-90/22-E (1990).
3. M. Gaździcki and M. I. Gorenstien, *Phys. Lett. B* **517**, 250 (2001), and the references therein.
4. UA1 Collab. (G. Arnison *et al.*), *Phys. Lett. B* **118**, 167 (1982).
5. UA1 Collab. (G. Bocqet *et al.*), *Phys. Lett. B* **366**, 434 (1996).
6. WA80 Collab. (R. Albrecht *et al.*), *Eur. Phys. J. C* **5**, 255 (1998).
7. R. Hagedorn, *Rev. Nuovo Cimento* **6**, 46 (1983), CERN-TH 3684 (1983).
8. T. Peitzmann, *Phys. Lett. B* **450**, 7 (1999).
9. UA1 Collab. (C. Albajar *et al.*), *Nucl. Phys. B* **335**, 261 (1990).
10. D. d'Enterria, *J. Phys. G* **31**, S491 (2005).
11. E. Wang and X.-N. Wang, *Phys. Rev. C* **64**, 034901 (2001).
12. X.-N. Wang, *Phys. Rev. C* **61**, 064910 (2000).
13. P. Darriulat, *Ann. Rev. Nucl. Part. Sci.* **30**, 159 (1980).
14. UA5 Collab. (G. J. Alner *et al.*), *Z. Phys. C* **33**, 1 (1986).
15. CDF Collab. (F. Abe *et al.*), *Phys. Rev. Lett.* **61**, 1819 (1988).
16. A. Bhadra and B. Kunwar, *Int. J. Mod. Phys. A* **18**, 2647 (2003).
17. S. Bhattacharyya, *J. Phys. G* **14**, 9 (1988).
18. A. Ramkrishnan, *Elementary Particles and Cosmic Rays* (Pergamon Press, 1962), pp. 400–423.
19. S. Bhattacharyya, *Il Nuovo Cimento C* **11**, 51 (1988).
20. B. De, S. Bhattacharyya and P. Guptaroy, *J. Phys. G* **27**, 2389 (2001).
21. A. Accardi *et al.*, hep-ph/0502072.
22. A. Bialas and R. Peschanski, *Nucl. Phys. B* **273**, 703 (1986).
23. I. Sarcevic and H. Satz, *Phys. Lett. B* **233**, 259 (1989).

24. C. P. Singh, *Phys. Rep.* **236**, 147 (1993), and references therein.
25. R. Peschanski, *Int. J. Mod. Phys. A* **6**, 3681 (1991).
26. Y.-F. Wu and L.-S. Liu, *Phys. Rev. Lett.* **70**, 3197 (1993).
27. B. B. Mandelbrot, *The Fractal Geometry of Nature* (Freeman, San Francisco, 1983).
28. J. Feder, *Fractals* (Plenum Press, New York, 1988).
29. P. Sloot, 1994 CERN School of Computing, p. 204, CERN 95-01 (20 January 1995).
30. L. M. Sander, *Nature* **322**, 789 (1986).
31. L. M. Sander, *Sci. Am.* **256**, 82 (1987).
32. T. W. Atwater and P. S. Freier, *Phys. Rev. Lett.* **56**, 1350 (1986).
33. JACEE Collab. (T. H. Burnett *et al.*), *Phys. Rev. Lett.* **51**, 1010 (1983).
34. JACEE Collab. (T. H. Burnett *et al.*), in *Proc. 21st Int. Cosmic Ray Conf.*, Adelaide, Australia, 6–19 January 1990 (Adelaide University, 1990), Vol. 3, p. 101.
35. BRAHMS Collab. (I. Arsene *et al.*), nucl-ex/0503010.