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Random theorems in topology

by

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Abstract. Let E and X be Polish spaces and A and B be two disjoint analytic subsets of $E\times X$ with closed vertical sections. We prove the following results.

(i) There is a Borel map $f\colon E\times X\to[0,1]$ such that $f\equiv 0$ on A, $f\equiv 1$ on B and for each $e\in E$, the map $x\to f(e,x)$ is continuous.

(ii) If Z is a retract of finite ór countable product of intervals and if $f\colon A\to Z$ is a Borel map such that for every $e\in E$, the map $x\to f(e,x)$ is continuous then there is a Borel measurable extension $F\colon E\times X\to Z$ of f such that $x\to F(e,x)$ is continuous for each $e\in E$.

(iii) If A is Borel then (ii) holds for all convex subsets Z of a second countable affine space of type m.

1. Notation. For notation and basic results in Descriptive Set Theory we follow Moschovakis [11]. Throughout X is a Polish space with a bounded metric d. For $x\in X$ and positive real number r, $S_r(x)$ (resp. $\bar{S}_r(x)$) denotes the open (resp. closed) ball of X with centre x and radius r. Let E be an arbitrary set and $\mathscr{E}$ a family of subsets of E. A *multifunction* $F\colon E\to X$ is a map with domain E and values non-empty, closed subsets of X. We say that the multifunction $F\colon E\to X$ is *$\mathscr{E}$-measurable* if

$$F^{-1}(U)=\{e\in E\colon F(e)\cap U\neq\emptyset\}$$

belongs to $\mathscr{E}$ for every open set U in X. The set

$$\{(e,x)\in E\times X\colon x\in F(e)\}$$

will be called the *Graph* of F and will be denoted by $G(F)$. We consider a point map also as a multifunction.

Let Z be a topological space and $f\colon G(F)\to Z$ a point map. We call f a *G-Carathéodory* map if

(i) for each $e\in E$, $x\to f(e,x)$ is continuous and

(ii) for every $\mathscr{E}$-measurable selector $s\colon E\to X$ of E, the map $e\to f(e,s(e))$ is $\mathscr{E}$-measurable.

Let $\mathscr{E}$ be a σ-field and $G\subseteq E\times X$. Then a map $f\colon G\to Z$ will be called *$\mathscr{E}$-Carathéodory* or simply *Carathéodory* if for each $e\in E$, the map $x\to f(e,x)$ defined on the section $G(e)$ of G is continuous and f is $\mathscr{E}\times\mathscr{B}_X|G$-measurable, where $\mathscr{B}_X$ is the Borel σ-field of X and $\mathscr{E}\times\mathscr{B}_X$ is the product σ-field.

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Remark. If Z is a metrizable space and $(E, \mathscr{E})$ a measurable space, then for every $\mathscr{E}$-measurable multifunction F: $E \to X$, each G-Carathéodory map f: $G(F) \to Z$ is Carathéodory.

Proof. By [9], fix a sequence $\{s_i\}$ of $\mathscr{E}$-measurable selectors of F such that for every $e \in E$, $\{s_i(e)\}$ is dense in $F(e)$. Let C be a closed set in Z and for each positive integer n, let

$$C_n = \{z \in Z\colon \operatorname{dist}(z', z) < 1/n \text{ for some } z' \in C\}.$$

Then for every $(e, x) \in G(F)$,

$$f(e, x) \in C \Leftrightarrow \forall n \exists i \ (\operatorname{dist}(x, s_i(e)) < 1/n \text{ and } f(e, s_i(e)) \in C_n).$$

The rest of our notation is standard. If E is a metrizable space then unless otherwise mentioned, $\mathscr{E}$ will denote its Borel σ-field.

For concepts in General Topology we follow Dugundji [6].

2. Introduction. Motivated by results proved in [1, 4, 7] in [13] we proved, among others, the following two results.

THEOREM 1. *Let $(E, \mathscr{E})$ be a measurable space, F: $E \to X$ an $\mathscr{E}$-measurable multifunction and f: $G(F) \to \mathbf{R}$ a Carathéodory map. Then there is a Carathéodory map g: $E \times X \to \mathbf{R}$ which extends f and which satisfies*

$$g(e, X) \subseteq \operatorname{co}(f(\{e\} \times F(e))), \quad e \in E,$$

where $\operatorname{co}(A)$ *denotes the convex hull of A.*

THEOREM 2. *Let E be a second countable metrizable space, Z a locally convex topological vector space, F: $E \to X$ a measurable multifunction and f: $G(F) \to Z$ a G-Carathéodory map. Then also the conclusions of Theorem* 1 *hold.*

In this paper we give generalizations of these two theorems when E is a Polish space. While proving Theorem 1 we needed some random analogues of the Urysohn Theorem. Here we study this in detail and also show that our random Urysohn theorems are sharp. At the end we prove a random analogue of Lusin's theorem and raise several open problems.

3. Random Urysohn theorems.

THEOREM 3. *If $(E, \mathscr{E})$ is a measurable space and F_0, F_1: $E \to X$ measurable multifunctions with $G(F_0) \cap G(F_1) = \emptyset$ then there is a Carathéodory map f: $E \times X \to [0, 1]$ such that $f \equiv 0$ on $G(F_0)$ and $f \equiv 1$ on $G(F_1)$.*

Proof. By [9], we get sequences $\{f_n^0\}$ and $\{f_n^1\}$ of measurable maps from E into X such that for every $e \in E$, $\{f_n^\varepsilon(e)\}$ is dense in $F_\varepsilon(e)$ where $\varepsilon = 0$ or 1. Now we define

$$f(e, x) = \frac{\operatorname{dist}(x, F_0(e))}{\operatorname{dist}(x, F_0(e)) + \operatorname{dist}(x, F_1(e))}, \quad (e, x) \in E \times X$$

$$= \frac{\inf_n d(x, f_n^0(e))}{\inf_n d(x, f_n^0(e)) + \inf_n d(x, f_n^1(e))}.$$

The map f has the desired properties.

We give an example to show that Theorem 3 cannot be extended to the case $\mathscr{E} = \mathscr{L}_\sigma$ where $\mathscr{L}$ is a field.

EXAMPLE 1. Let $E = \omega^\omega$ and $\mathscr{E} = \Sigma_2^0$. Let A_0 and A_1 be two disjoint Σ_2^0-sets in the space of irrationals ω^ω such that there do not exist disjoint Π_2^0-sets C_0 and C_1 satisfying $A_0 \subseteq C_0$ and $A_1 \subseteq C_1$ [11, p. 205]. Define F_0, F_1: $\omega^\omega \to [0, 1]$ by

$$\begin{aligned} F_0(\alpha) &= [0, 3/4] && \text{if } \alpha \in A_0, \\ F_0(\alpha) &= \{0\} && \text{if } \alpha \in \omega^\omega \setminus A_0, \\ F_1(\alpha) &= [1/4, 1] && \text{if } \alpha \in A_1, \\ F_1(\alpha) &= \{1\} && \text{if } \alpha \in \omega^\omega \setminus A_1. \end{aligned}$$

Then F_0 and F_1 are two Σ_2^0-measurable, compact-valued multifunctions with $G(F_0) \cap G(F_1) = \emptyset$. If possible suppose there is a map f: $\omega^\omega \times [0, 1] \to [0, 1]$ such that $f \equiv 0$ on $G(F_0)$ and $f \equiv 1$ on $G(F_1)$ and such that for every $x \in [0, 1]$, $e \to f(e, x)$ is Σ_2^0-measurable. Now consider

$$C_0 = \{\alpha \in \omega^\omega : f(\alpha, 1/2) = 0\}, \qquad C_1 = \{\alpha \in \omega^\omega : f(\alpha, 1/2) = 1\}.$$

These are disjoint Π_2^0-sets such that $A_i \subseteq C_i$, $i = 0, 1$. This is a contradiction.

For semi-continuous multifunctions we have

LEMMA. *Let E be a metrizable space, X a Polish space and F: $E \to X$ a closed valued upper or lower semi-continuous multifunction. Then for each $x \in X$ the map*

$$e \to \operatorname{dist}(x, F(e))$$

is Σ_2^0-measurable.

Proof. Fix $x \in X$, $e \in E$ and reals $a < b$. We have

$$\operatorname{dist}(x, F(e)) < b$$

$$\text{(i)} \qquad \Leftrightarrow S_b(x) \cap F(e) \neq \emptyset$$

$$\text{(ii)} \qquad \Leftrightarrow (\exists m)\ (\bar{S}_{b-1/m}(x) \cap F(e) \neq \emptyset),$$

$$\operatorname{dist}(x, F(e)) > a$$

$$\text{(iii)} \qquad \Leftrightarrow (\exists m)\ (S_{a+1/m}(x) \cap F(e) = \emptyset)$$

$$\text{(iv)} \qquad \Leftrightarrow (\exists m)\ (\bar{S}_{a+1/m}(x) \cap F(e) = \emptyset).$$

Equivalences (i) and (iii) prove the lemma when F is lower semi-continuous. For upper semi-continuous F we use (ii) and (iv).

We now have

THEOREM 4. *Let E be a metrizable space and F_0, F_1: $E \to X$ be lower or upper semi-continuous multifunctions such that $F_0(e) \cap F_1(e) = \emptyset$ for every $e \in E$. Then there is a map f: $E \times X \to [0, 1]$ such that*

(i) $f \equiv 0$ *on* F_0, *and* $f \equiv 1$ *on* F_1,
(ii) $x \to f(e, x)$ *is continuous for every* $e \in E$, *and*
(iii) $e \to f(e, x)$ *is* Σ_2^0*-measurable, for each* $x \in X$.

Before we proceed to prove our next theorem we present an example.

EXAMPLE 2. Let A be a Σ_1^1 but non-Borel subset of $[0, 1]$. Fix a metric ϱ on ω^ω an let α, β be two distinct points of ω^ω. Let U be a non-empty clopen subset of $S_{(1/2)\varrho(\alpha,\beta)}($ Let B be a Borel subset of $[0, 1] \times U$ with closed sections such that $A = \text{proj}(B)$. L

$$F = B \cup ([0, 1] \times \{\beta\}).$$

Then F is a Borel subset of $[0, 1] \times \omega^\omega$ with non-empty closed sections. If possit suppose the map $e \to \text{dist}(\alpha, F(e))$ defined on $[0, 1]$ is Borel. Then, as

$$e \in A \Leftrightarrow \text{dist}(\alpha, F(e)) \leqslant \tfrac{1}{2}\varrho(\alpha, \beta),$$

A is Borel. Therefore, the map $e \to \text{dist}(\alpha, F(e))$ is not Borel.

The above example shows that the simple-minded arguments contained in t proofs of Theorems 3 and 4 do not work for our main random Urysohn theor mentioned in the abstract. Instead we shall use the following three results.

THEOREM A (Saint-Raymond, [12]). *Let E and X be two Polish spaces and A and B two Σ_1^1-subsets of $E \times X$ such that for every $e \in E$, $\overline{A(e)} \cap B(e) = \emptyset$. Then there is a B set C in $E \times X$ such that for every $e \in E$, $C(e)$ is closed and $A \subseteq C \subseteq (E \times X) \backslash B$.*

THEOREM B (Dellacherie, [5]). *If E and X are Polish spaces and $B \subseteq E \times X$ is a B set with $B(e)$ open for every $e \in E$ then*

$$B = \bigcup_{n \in \omega} (B_n \times U_n)$$

where B_n is Borel in E and U_n open in X.

THEOREM C (Miller, [10]). *Let E be a second countable metrizable space. Denote $\mathcal{T}$ the topology on E. Then given any sequence $\{B_n\}$ of Borel sets in E there is a sec countable metrizable topology $\mathcal{T}'$ on E such that*

(i) *each of $B_n \in \mathcal{T}'$, and*

(ii) *the σ-fields generated by $\mathcal{T}$ and $\mathcal{T}'$ are the same.*

Actually this is a simpler case of Miller's theorem and a proof of it is also presen in ([13, Theorem 5]).

From now on E will be a Polish space.

THEOREM 5. *Let F_0 and F_1 be two disjoint Σ_1^1 sets in $E \times X$ such that for each $e \in E$, sections $F_0(e)$ and $F_1(e)$ are closed. Then there is a Carathéodory map $f\colon E \times X \to [0,$ such that $f \equiv 0$ on F_0 and $f \equiv 1$ on F_1.*

Proof. By applying Theorem A twice, we get two disjoint Borel sets C_0 and C_1 $E \times X$ with $C_0(e)$ and $C_1(e)$ closed, $F_0 \subseteq C_0$ and $F_1 \subseteq C_1$. By Theorem B, we wr

$$(E \times X) \backslash C_i = \bigcup_{n \in \omega} (B_n^i \times U_n^i), \quad i = 0 \text{ or } 1$$

with B_n^i Borel in E and U_n^i open in X. Denote the topology on E by $\mathcal{T}$. By Theorem let $\mathcal{T}'$ be a second countable metrizable topology on E such that

(i) $B_n^i \in \mathscr{T}'$, $n \in \omega$, $i = 0$ or 1, and

(ii) the σ-fields generated by $\mathscr{T}$ and $\mathscr{T}'$ are the same.

Now C_0 and C_1 are disjoint closed sets in $E \times X$ when E is equipped with $\mathscr{T}'$ and X has its own Polish topology, say $\mathscr{T}''$. By Urysohn's theorem there is a $\mathscr{T}' \times \mathscr{T}''$-continuous map f: $E \times X \to [0, 1]$ such that $f \equiv 0$ on C_0 and $f \equiv 1$ on C_1. Since the σ-fields generated by $\mathscr{T}$ and $\mathscr{T}'$ are the same, this f has all the desired properties.

Our next example shows that Theorem 5 does not hold if F_0, F_1 are Π_1^1.

EXAMPLE 3. In Example 1 take A_0 and A_1 to be two disjoint Π_1^1 sets such that there do not exist disjoint Borel sets C_0 and C_1 with $A_0 \subseteq C_0$ and $A_1 \subseteq C_1$. Define F_0 and F_1 exactly the same way. The same arguments show that there does not exist a Carathéodory map f: $E \times X \to [0, 1]$ such that $f \equiv 0$ on F_0 and $f \equiv 1$ on F_1.

Remark 1. It is worth noting that the following generalization of Theorem 5 also olds.

THEOREM 6. *Let E and X be Polish spaces, F_0, F_1 be two disjoint Σ_1^1 sets in $E \times X$ uch that for all $e \in E$, $F_0(e)$ and $F_1(e)$ are Π_ξ^0. Then there is a Borel map : $E \times X \to [0, 1]$ such that*

(i) *$f \equiv 0$ on F_0, $f \equiv 1$ on F_1 and*

(ii) *for every $e \in E$, $x \to f(e, x)$ is Σ_ξ^0-measurable.*

Proof. For $\xi = 1$ this is Theorem 5. Let $1 < \xi < \omega_1$. Embed X in a recursively presented Polish space H, say the Hilbert cube. We now invoke a result of R. Barua ([2]) (which, in fact, is a simple extension of a result of A. Louveau [8]) and get a Borel set B in $E \times X$ such that

(i) $F_1 \subseteq B \subseteq E \times X \backslash F_0$, and

(ii) $B(e)$ is Δ_ξ^0 for every $e \in E$.

We take $f = I_B$, the indicator function of B.

Remark 2. The argument above also works when $\xi = 1$ and X a zero-dimensional Polish space. In this case embed X in and as a closed subspace of ω^ω.

4. Random extension theorems. Using the ideas contained in the proof of Theorem 5 we prove

THEOREM 7. *Let A be a Borel set in $E \times X$ such that the sections $A(e)$ are closed for every $e \in E$. Suppose Z is a second countable convex subspace of an affine space of type m and f: $A \to Z$ a Carathéodory map. Then there is a Carathéodory map g: $E \times X \to Z$ which extends f.*

Proof. Fix a countable base $W_1, W_2, \ldots$ of Z. Let $A_0 = A$ and

$$A_n = A \backslash f^{-1}(W_n), \quad n = 1, 2, \ldots$$

By the arguments contained in the proof of Theorem 5 we get a finer second countable metrizable topology $\mathscr{T}'$ such that each of A_i is closed when E is equipped with $\mathscr{T}'$ and the Borel σ-field of E remains the same. This makes A closed and f continuous when E has the new topology. By the extension theorem of Dugundji ([6], p. 188) there is a continuous extension g: $E \times X \to Z$ of f. This g is a Carathéodory map when E has the original topology.

THEOREM 8. *Let A be a Σ_1^1 set in $E \times X$ and Z a retract of a finite or countable produ[ct] of intervals in $\mathbf{R}$. Let $f\colon A \to Z$ be a Borel measurable Carathéodory map. Then there i[s] a Carathéodory map $g\colon E \times X \to Z$ which extends f.*

Proof. *Case* 1. $Z = [-1, 1]$.

We define a sequence of Carathéodory maps $g_i\colon E \times X \to [-1, 1]$, $i = 0, 1, \ldots$ suc[h] that for every i

(i) $|g_i(e, x)| \leqslant \frac{1}{3}\cdot(\frac{2}{3})^i$, for all (e, x), and

(ii) $|f(e, x) - g_0(e, x) - \ldots - g_i(e, x)| \leqslant (\frac{2}{3})^i$, for all $(e, x) \in A$.

To see that such a sequence can be defined we proceed inductively. Let

$$F_0^0 = \{(e, x) \in A\colon f(e, x) \leqslant -\tfrac{1}{3}\} \quad \text{and} \quad F_1^0 = \{(e, x) \in A\colon f(e, x) \geqslant \tfrac{1}{3}\}.$$

By Theorem 5 we get a Carathéodory $g_0\colon E \times X \to [-1/3, 1/3]$ having the requir[ed] properties. Having defined $g_0, g_1, \ldots, g_i$ satisfying (i)–(iii), we let

$$F_0^{i+1} = \{(e, x) \in A\colon f(e, x) - g_0(e, x) - \ldots - g_i(e, x) \leqslant -\tfrac{1}{3}\cdot(\tfrac{2}{3})^i\},$$

$$F_1^{i+1} = \{(e, x) \in A\colon f(e, x) - g_0(e, x) - \ldots - g_i(e, x) \geqslant \tfrac{1}{3}\cdot(\tfrac{2}{3})^i\}.$$

By Theorem 5, we get a Carathéodory map $g_{i+1}\colon E \times X \to [-\frac{1}{3}\cdot(\frac{2}{3})^i, \frac{1}{3}\cdot(\frac{2}{3})^i]$ such th[at] $g_{i+1} \equiv -\frac{1}{3}\cdot(\frac{2}{3})^i$ on F_0^{i+1} and is $\equiv \frac{1}{3}\cdot(\frac{2}{3})^i$ on F_1^{i+1}.

We define

$$g(e, x) = \lim_i g_i(e, x), \quad (e, x) \in E \times X.$$

Case 2. $Z = (-1, 1)$

Using case 1, we get a Carathéodory map $h\colon E \times X \to [-1, 1]$ which extends f. Le[t]

$$B = \{(e, x) \in E \times X\colon |h(e, x)| = 1\}.$$

Then A and B are two disjoint Σ_1^1-sets with closed sections. By Theorem 5, we g[et] a Carathéodory map

$$u\colon E \times X \to [0, 1]$$

such that $u \equiv 1$ on A and $\equiv 0$ on B. Put $g = u \cdot h$.

Remaining cases. It is now clear that the result is true for all intervals. When Z [is] a finite or countable product of intervals we extend each of the coordinate function[s]. Finally, let Z' be a finite or countable product of intervals and Z a retract of Z'. Fi[x] a retraction $r\colon Z' \to Z$. If $f\colon A \to Z$ is a given Carathéodory map, first get a Cara[-]théodory map $h\colon E \times X \to Z'$ which extends f and then take $g = r \circ h$. This complete[s] the proof.

In Theorems 1 and 2 we get extensions satisfying

$$g(e, X) \subseteq \operatorname{co}\big(f(\{e\} \times F(e))\big), \quad e \in E.$$

our next example shows that we cannot have this in Theorems 7 and 8 even whe[n] $Z = \mathbf{R}$.

EXAMPLE 4. Let A_0 and A_1 be two Σ_1^1-sets in $[0, 1]$ such that $A_0 \cup A_1 = [0, 1]$ but there does not exist a Borel set B such that $B \subseteq A_0$ and $B^c \subseteq A_1$. Let I_0 be the space of all irrationals contained in $[0, 1/3]$ whereas I_1 is the space of all those irrationals which are contained in $[2/3, 1]$. Let C_i be a Borel set in $[0, 1] \times I_i$ whose sections are closed in I_i and such that $\text{Proj}(C_i) = A_i$, $i = 0$ or 1. Let $C = C_0 \cup C_1$. Let X be the set of all irrationals in $[0, 1]$. Then we have a Borel set in $[0, 1] \times X$ whose sections are closed in X. Define $f\colon C \to \boldsymbol{R}$ by

$$f(e, x) = x, \quad (e, x) \in C.$$

If possible suppose there is a Carathéodory map $g\colon [0, 1] \times X \to \boldsymbol{R}$ which extends f and which satisfies

$$g(e, X) \subseteq \text{co}(f(\{e\} \times C(e))), \quad e \in [0, 1].$$

Let

$$B = \{e \in [0, 1]\colon g(e, 1/\sqrt{2}) \leqslant 1/2\}.$$

Then B is Borel, $B \subseteq A_0$ and $B^c \subseteq A_1$. Contradiction.

5. A random Lusin theorem.

THEOREM 9. *Let $f\colon E \times X \to [0, 1]$ be a Borel map. Let $\mu(e, B)$ be a transition function on $E \times \mathscr{B}_X$. Then for every $\varepsilon > 0$ there exists a Carathéodory map $g\colon E \times X \to [0, 1]$ such that for every $e \in E$*

$$\mu(e, \{x \in X\colon g(e, x) \neq f(e, x)\}) < \varepsilon.$$

Proof. Define a sequence $\{E_n\}$ of subsets of $E \times X$ as follows:

$$E_n = \Big\{(e, x) \in E \times X\colon \frac{2k-1}{2^n} \leqslant f(e, x) < \frac{2k}{2^n} \text{ for some }$$

$$k = 1, 2, \ldots, 2^{n-1} \text{ or } f(e, x) = 1\Big\}.$$

Then $f = \sum_{n=1}^{\infty} (1/2^n) I_{E_n}$.

By [3], get Borel sets F_n and U_n in $E \times X$ such that

(i) $F_n \subseteq E_n \subseteq U_n$, $n = 1, 2, \ldots$;

(ii) $\mu(e, F_n(e) \backslash U_n(e)) < \varepsilon/2^n$ for $n = 1, 2, \ldots$ and $e \in E$; and

(iii) $F_n(e)$ and $X \backslash U_n(e)$ are compact for each n and e.

Now, $e \to F_n(e)$ and $e \to X \backslash U_n(e)$ are measurable, closed-valued multifunctions for each n. Hence by Theorem 3, there exist Carathéodory maps $g_n\colon E \times X \to [0, 1]$ such that

$$g_n(e, x) = \begin{cases} 0 & \text{if } (e, x) \in X \backslash U_n, \\ 1 & \text{if } (e, x) \in F_n. \end{cases}$$

Put $g = \sum_{n=1}^{\infty} (1/2^n) g_n(e, x)$.

6. Open problems.

PROBLEM 1. In Theorem 5 suppose we take F_0, F_1 to be Borel but E an arbitrar second countable metrizable or even a Π_1^1-set. Do the conclusions of Theorem 5 hold i this case?

PROBLEM 2. Does Theorem 7 hold for a Σ_1^1-set A? We do not know the answer eve when Z is a convex subset of $\boldsymbol{R}^2$.

PROBLEM 3. Can Theorem 8 be extended for Π_1^1-sets A? We do not know the answe even when $Z = \boldsymbol{R}$.

A question related to Problem 3 is the following:

PROBLEM 4. Let C_0 and C_1 be two disjoint Π_1^1-sets in $E \times X$ such that for every $e \in l$ the sections $C_0(e)$ and $C_1(e)$ are closed. Further assume that there is a Borel se B containing C_0 but disjoint from C_1. Do there exist disjoint Borel sets B_0 and B_1 suc that $C_0 \subseteq B_0$, $C_1 \subseteq B_1$ and for every $e \in E$, the sections $B_0(e)$ and $B_1(e)$ are closed in X

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