Indian Economic Review, Vol. XXXII, No. 1, 1997, pp. 107-115

On the Relationship Between Price and Output Seasonality in Backward Agriculture

ABHIRUP SARKAR*
Indian Statistical Institute, Calcutta

ABSTRACT

The purpose of the paper is to explain, in terms of a theoretical model, why low (high) price seasonality coexists with high (low) seasonality in market arrival in some backward agricultural markets. The explanation is provided in terms of differences in the degree of monopoly across markets.

1. INTRODUCTION

The purpose of this paper is to look at the relationship between output and price seasonalities in backward agricultural markets. In Bhaduri (1983) an extremely interesting though puzzling observation was made about such relationships in agricultural markets in India. It was observed that markets with higher seasonality in market arrival also exhibit lower price seasonality. Thus, for example, it was observed that in the state of Punjab most of the output arrived in the market just after the harvest (*i.e.* during the busy season) but intertemporal prices, over the busy and the slack periods, exhibited little seasonality. On the other hand, in the state of Bihar, the proportion of total output coming to the market just after the harvest was observed to be relatively low while price seasonality was observed to be much higher. Earlier Lele (1971) made similar observations. The authors made those observations for both rice and wheat markets and over a number of harvesting years.

The observation is puzzling because apparently it violates the principles of demand and supply. Suppose, for the sake of expositional simplicity, that there are two periods,

* This paper was written when the author was visiting the International Centre For Economic Research, Turin, Italy. Support from that Centre, helpful discussions with Sanjit Bose and Mihir Rakshit and comments from an anonymous referee are gratefully acknowledged. The usual disclaimer applies.

busy and slack. It is reasonable to assume that market demand is uniform in the two periods. Then, higher is the fluctuations in supply between the two periods, the higher should be the fluctuations in prices. Since the data indicates otherwise, Bhaduri has concluded that the demand-supply mechanism is not a useful tool to explain the behaviour of prices in backward agricultural markets.

In this paper, we argue that it is possible to explain the observed puzzle within the framework of demand-supply. In backward agricultural markets, the price is usually controlled by a number of large traders. We show that the extent of price seasonality depends crucially upon the extent of intertemporal arbitrage these traders engage themselves in. It is also shown that the extent of intertemporal arbitrage, in turn, depends upon the number of traders operating in the market, i.e. upon the degree of imperfect competition. The smaller is the number of traders in the market, the higher is the extent of price seasonality.

The seasonality in market arrival of output, in turn, depends upon the behaviour of the farmers who are assumed to be price takers. Given that these farmers have a high cost of holding stocks, it is worthwhile for them to carry stocks to the lean season only if the lean season price is high relative to the busy season price, i.e. only if there is sufficient price seasonality. As mentioned above, price seasonality will be high in markets where the number of traders is low. It follows that in these markets seasonality in market arrival will be low because more stocks will be carried to the lean season.

It is reasonable to believe that the number of large traders is higher in Punjab than in Bihar. From our analysis it is then implied that higher price seasonality will coexist with lower seasonality in market arrival in Bihar as compared to Punjab.

In what follows, we develop the basic model in Section 2 and prove the main propositions of the paper. In Section 3 we introduce government intervention in the form of public procurement and show that our results get strengthened if government intervention in this form is taken into account. Section 4 contains some concluding remarks.

2. THE MODEL

The empirical relationship between seasonality in price and market arrival that we are trying to explain has been observed for wholesale markets. Consumers, on the other hand, typically buy from retail markets. Consequently, we consider a single agricultural good for which there are two markets—— a wholesale market and a retail market. There are three types of economic agents—— consumers, producers and traders. Our focus of analysis is on the relationship between price and market arrival in the wholesale market.

Output is seasonal and is obtained at discrete points in time. It is stored during the interval which lies between two harvests to meet a continuous demand. The length of time lying between two harvests is denoted by the interval $[O, T]$. The producers, whom we call farmers, sell their stocks in the wholesale market which are bought by the traders. The traders, in turn, sell these stocks to the consumers in the retail market. We allow the possibility that some traders, apart from acquiring stocks from the wholesale market, might have stocks of their own. That is, some traders might also be producers.

Two types of trade are going on in our model — spatial and intertemporal. The former involves buying in the wholesale market and selling instantaneously in the retail market. The latter involves buying in the wholesale market at some date t_1 and selling in the retail market at a future date t_2. We assume that there is free entry as far as spatial trade is concerned and each unit of output can be transported from the wholesale to the retail market at a constant cost ε[1]. It then follows that at each instant the difference between the retail and the wholesale price is ε.

Not everybody is capable of undertaking intertemporal trade. Those who are, can enjoy some market power i.e. can influence the market price[2]. In what follows, we shall keep the spatial traders in the background because their only role in the model is to equate the wholesale price with the retail price net of transport cost. Consequently, the intertemporal traders are called simply traders. The farmers, on the other hand, are assumed to be price-takers[3]. They are also assumed to have higher costs of holding stocks than the traders[4]. Finally, the consumers are assumed to be passive in the sense that they do not store. They buy from the retail market and have a uniform demand at each point in time. For the sake of simplicity, we assume that the demand function is linear and takes the form

$$q(t) = a - c(t) \tag{2.1}$$

where $q(t)$ is the price and $c(t)$ is demand in the retail market at time t. Let $z(t)$ be the amount sold by farmers in the wholesale market at time t and let $y(t)$ be the amount sold (if positive) or bought (if negative) by the traders at t.[5] Then (2.1) may be rewritten as

$$p(t) = a - \varepsilon - z(t) - y(t) \tag{2.2}$$

where $p(t)$ is the wholesale price at t and $p(t) = q(t) - \varepsilon$.

Let us now consider the traders' problem formally. There are n traders in the market. A representative trader i solves the following problem

$$\max \int_0^T [p(t)\, y_i(t) - \delta\, x_i(t)]\, dt \tag{2.3}$$

$$\text{s.t. } x_i(0) = x_i,\ x_i(T) = 0$$

1. ε can also be interpreted as the constant trading margin.

2. A person can undertake intertemporal trade because he has the power to hold stocks.

3. Basically we assume that the farmers are small. The implication is that those who are large has also the power of holding stocks and are classified as traders.

4. The higher cost of holding stocks will guarantee that the farmers have no incentive to buy from the wholesale market.

5. The traders buy at the wholesale market and sell in the retail market at different points in time. Also, a little reflection will convince the reader that they cannot buy and sell at the same point in time.

where $y_i(t) = -\dot{x}_i(t)$, i.e. sales (purchase if $y_i(t)$ is negative) by trader i at t is equal to the rate of decrease (increase) in stocks; δ is the constant per unit cost of holding stocks per unit of time[6]. The ith trader maximizes (2.3) given the sequence of sales of the farmers, i.e. $\{z(t)\}$, the sequence of sales (and purchase) of other traders, i.e. $\{y_j(t)\}$, $j = 1,...n$, $j \neq i$, and the demand equation (2.2).

Let us, for the time being, assume that the sequence $\{z(t)\}$ is exogenously given. Given $\{z(t)\}$, (Nash) equilibrium is defined as a collection of sequences $\{y^*_i(t)\}$, $i = 1,...n$, such that $\{y^*_i(t)\}$ maximizes profits for the ith traders given $\{y^*_j(t)\}$, $j=1,...n$, $j\neq i$, $i=1,...n$. Presently, we shall find out this equilibrium. The Euler equation representing the first order condition of maximizing (2.3) is given by

$$\dot{m}_i(t) = \delta, \quad i = 1,...n. \tag{2.4}$$

where $m_i(t)$ represents the marginal revenue of the ith trader at time t. If trader i is a net buyer at t, then $m_i(t)$ represents marginal cost. Equation (2.4) means that marginal revenues (marginal costs), net of storage cost δ, are equalized over time. Using the expression for $m_i(t)$ from (2.2) we can simultaneously solve (2.4) to get

$$\begin{aligned} \dot{y}_1(t) &= -\frac{1}{n+1}(\delta + \dot{z}(t)) \\ \dot{y}(t) &= -\frac{n}{n+1}(\delta + \dot{z}(t)) \\ \dot{p}(t) &= \frac{1}{n+1}(n\delta - \dot{z}(t)) \end{aligned} \tag{2.5}$$

we may now solve for $y_i(t)$, $y(t)$ and $p(t)$ in the following way. First note that

$$y_i(t) = y_i(0) + \int_0^t \dot{y}_i(\tau)\, d\tau \tag{2.6}$$

Using the value of $\dot{y}_i(\tau)$ from (2.5), and integrating over $[0, T]$ (2.6) may be written as

$$y_i(0) = \frac{x_i}{T} - \frac{\delta}{n+1}\frac{T}{2} - \frac{l}{n+1}[\bar{z} - z(0)] \tag{2.7}$$

6. If δ is very high, a trader might sell off all his stocks before T. We assume that this does not happen, *i.e.* δ is not very high.

where $\bar{z}$ is the average market arrival defined as H/T, H being the total amount sold by farmers over the entire horizon. Combining (2.6) with (2.7) the final solution for $y_i(t)$, $y(t)$ and $p(t)$ are given by

$$y_i(t) = \frac{1}{n+1}(\bar{z} - z(t)) + \frac{X_i}{T} + \frac{\delta}{n+1}\left(\frac{T}{2} - t\right)$$

$$y(t) = \frac{n}{n+1}(\bar{z} - z(t)) + \frac{X}{T} + \frac{n\delta}{n+1}\left(\frac{T}{2} - t\right) \qquad (2.8)$$

$$p(t) = (a - \varepsilon) - \frac{n}{n+1}\bar{z} - \frac{I}{n+1} z(t) - \frac{X}{T} - \frac{n\delta}{n+1}\left(\frac{T}{2} - t\right)$$

where $X = \sum_{i=1}^{n} X_i$.

We may now try to interpret equations (2.5) and (2.8). It is clear from (2.5) that seasonal fluctuations in prices depend upon three things : the degree of seasonal fluctuations in market arrival (given by $\dot{z}(t)$), the degree of monopoly in the market (given by n) and storage cost (given by δ). In particular, if the number of traders become very large ($n \to \infty$) then whatever be the fluctuations in market arrival, intertemporal prices net of storage costs are equalized, i.e. there is perfect arbitrage. The intuition is that for large n each trader becomes a price taker and therefore each would reallocate his purchase or sales as long as intertemporal prices, net of storage costs, are different. The degree of arbitrage would go down as the number of traders become smaller. With a smaller number of traders, each would have sufficient market power to affect the market price and hence intertemporal marginal revenues, net of storage costs, will be equated. Intertemporal prices in this case will be different and from this difference the traders will be able to earn positive profits.

The other point to note is that the traders, in order to be on their optimal purchase and sales paths, would have to know only the total output (or equivalently the average output) the farmers will be selling in the market over the entire time horizon. But a trader neither has to know the distribution of market arrival of output brought by the farmers, nor how much output other traders have as initial stocks. His purchase or sales policy is quite simple : he would distribute sales from his own stocks according to the second two terms on the right hand side of the first equation of (2.8). Indeed, this would give his optimal sales path if the farmers did not bring anything to the market (see Sarkar (1993)). With farmers bringing in output to the market, the trader would sale an additional amount if market arrival $z(t)$, which is observable at time t, falls short of the average market arrival $\bar{z}$. If $z(t)$ exceeds the average, he would sell less. If $z(t)$ is sufficiently larger than $\bar{z}$ at some t, then at that t a trader would accumulate stocks by buying from the market. The extent to which a trader will buy and sell in response to the difference between $z(t)$ and z depends on the number of traders in the market.

Next we consider the behaviour of the farmers. The farmers, by assumption, are price takers. More specifically, they maximize their profits for a given sequence of expected prices. The sequence of market arrival $z(t)$ is obtained as a result of their maximization exercise. Formally, the farmers solve

$$\max \int_0^{T_z} [\, p^e(t)\, z(t) - C(H(t)) \,]\, dt \tag{2.9}$$

$$\text{s.t. } H(0) = H,\ H(T_z) = 0\ ,\ T_z \leq T$$

where $p^e(t)$ denotes expected price at t, $C(H(t))$ is the cost of storage at t as a function of the stock held at t, T_z is the point in time till which the farmers stay in the market and $z(t) = -\dot{H}(t)$.

We may now define an equilibrium in the following way : the collection $[\{y_1^*(t)\}, \{z^*(t)\}, \{p^*(t)\}]$ is an equilibrium if the following holds :

(*i*) $\{y_i^*(t)\}$ maximizes (2.3) given $\{y_j^*(t)\}_{i \neq j}$ for all i,j and given $z^*(t)$;

(*ii*) $\{z^*(t)\}$ maximizes (2.9) given $\{p^e(t)\}$

(*iii*) $p^e(t) = p^*(t) = (a - \varepsilon) - y^*(t) - z^*(t)$ for all t.

The first two conditions represent profit maximization by the traders and farmers respectively; the third condition states that in equilibrium price expectations by the farmers are fulfilled.

We make two further assumptions before proceeding to determine the equilibrium. Firstly, we assume that the storage cost of the farmers is quadratic in $H(t)$ and takes the form

$$C(H(t)) = \beta H(t) + \frac{\mu}{2}[H(t)]^2 \tag{2.10}$$

where β and μ are known positive constants and by assumption $\beta > \delta$.[7] Secondly, we assume that price expectations of the farmers satisfy[8]

$$p^e(t) = Ae^{-r(t)t} \tag{2.11}$$

7. This means that at all levels of H, the marginal cost of storage for the farmer is greater than that of the trader.

8. There may be other forms of price expectations which are consistent with equilibrium.

We have to solve for A and $r(t)$ which satisfy the equilibrium conditions (*i*), (*ii*) and (*iii*). From (2.10) and (2.11), the Euler equation for the maximization of (2.9) is given by

$$Ae^{-r(t)t} = \beta + \mu H(t) \tag{2.12}$$

Differentiating (2.12) and nothing that $z(t) = -\dot{H}(t)$ we get

$$z(t) = \frac{A}{\mu} r(t)\, e^{-r(t)t} \tag{2.13}$$

and differentiating once again

$$\dot{z}(t) = -\frac{A}{\mu} [\, r(t)\,]^2 \; e^{-r(t)t} \tag{2.14}$$

From the boundary condition $H(0) = H$ we get

$$A = B + \mu H \tag{2.15}$$

Similarly T_z can be obtained from $H(T_z) = 0$ by solving

$$e^{r(T_z)\, T_z} = \frac{A}{\beta} = \frac{\mu}{\beta} H + 1 \tag{2.16}$$

once $r(T_z)$ is known. To solve for $r(t)$, we combine (2.11), (2.14) and (2.5) to get

$$\frac{n}{n+1} \cdot \frac{\delta}{A} e^{r(t)\, t} = 1 - \frac{[\, r(t)\,]^2}{(n+1)} \tag{2.17}$$

From (2.17) , $r(t)$ may be solved. In particular, note that the left hand side is a rising function of $r(t)$ and the right hand side is a falling function of $r(t)$. Hence $r(t)$ is unique. Also, as t goes up, at any given $r(t)$, the right hand side of (2.17) remains unchanged while the left hand side increases. Hence the value of $r(t)$ goes down as t goes up. This completes our determination of equilibrium[9]. It follows from (2.11) that $\dot{p}(t)$ is positive. A straight forward differentiation of (2.11) and (2.17) yields that $\dot{p}(t)$ falls over time, *i.e.* the rate of increase in price goes down over time. The equilibrium price path is upward rising and concave with respect to time.

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9. There is a jump in $Y(t)$ at T_z. Note that $z(t)$ goes discontinuously to zero at T_z. Since, because of arbitrage, there cannot be a price jump at T_z, i.e. the traders will allocate sales in such a way that $p(t)$ is smooth, there must be a jump in $Y(t)$ at T_z.

Next we consider an increase in n. Differentiating (2.17) we find that for any t, $\frac{dr(t)}{dn} > 0$ if and only if $A e^{-r(t)t} > \delta$. Since $p(t) = A e^{-r(t)t}$ and since $p(t)$ is falling in time, if we can guarantee that $A e^{-r(T_z)T_z} > \delta$, then the condition holds for all $t < T_z$. But from (2.16), $A e^{-r(T_z)T_z} = \beta$. Hence by our earlier assumption that $\beta > \delta$, we get

$$\frac{dr(t)}{dn} > 0 \text{ for all } t.$$

How does an increase in $r(t)$ affect price and market arrival seasonalities? Since $\dot{p}(t) = A e^{-r(t)t}$, a rise in $r(t)$ reduces $\dot{p}(t)$. On the other hand, from (2.13) we get $\frac{dz(t)}{dr(t)} = A[1-r(t)\,t]\,e^{-r(t)t}$ for any t. Thus $\frac{dz(t)}{dr(t)}$ is positive for $t < \frac{1}{r(t)}$ and negative for $t > \frac{1}{r(t)}$. Hence as $r(t)$ goes up more is sold towards the beginning of the horizon and less towards the end. In other words, as $r(t)$ goes up market arrival becomes more seasonal.

We are now in a position to explain the puzzling empirical observation we had started with. Our analysis suggests that a higher n leads to a lower price seasonality but a higher seasonality in market arrival. Now, it is reasonable to assume that n, the number of large traders, is higher in Punjab than in Bihar. Consequently, lower price seasonality coexists with higher seasonality in market arrival in Punjab as compared to Bihar.

3. GOVERNMENT PROCUREMENT

Our result gets strengthened if we introduce government procurement. Let us assume that at time 0 the government announces a procurement price $\bar{p}$ at which it is willing to buy any amount of stocks from the wholesale market. Also assume, for the sake of simplicity, that the stocks thus procured by the government (in the current year or in the past) are not sold to the consumers (let us say, they are imported)[10]. Thus the market demand function remains unaffected. With government procuring stocks at a constant price, a trader has to take two kinds of decisions : first, he has to decide how much to sell in the (retail) market and how much to the government; second, he has to decide how he should distribute his open market sales over time. Since there is a positive storage cost δ and since the government purchases at a constant price through out the year, sales to the government will be completed immediately after the harvest, *i.e.* at time 0. What remains after selling to the government will be sold in the open market according to the Euler condition $\dot{m}_i(t) = \delta$, like before. A little reflection will convince the reader that this means that government procurement will affect the level of prices and not their change over time. Thus $p(t)$ will go up but $\dot{p}(t)$ will remain unaffected.

10. Suppose the government releases stocks uniformly at a price less than the market price. Then the market demand uniformly goes down which is shown by a fall in the intercept term 'a' in the demand function. This has the effect of raising the price level uniformly, but the degree of seasonality remains unaffected.

Next consider the question : what determines the division of sales between the open market and the government? Clearly, for each trader these sales will be determined at the point where marginal revenue from selling to the market is equal to the procurement price, *i.e.* $m_i(0) = \bar{p}$. The condition implies, among other things, that traders who are selling both to the government and to the open market must be having the same marginal revenue $m_i(0)$ in equilibrium and hence must be selling the same amount to the market. The residual will be sold to the government. Thus larger the initial stock of the trader, the larger will be his sales to the government. A corollary is that if a trader has very low initial stocks, he will not sell to the government at all. The intuition is that a large seller will sell part of his stocks to the government because if he did not, he would have to sell them in the open market which in turn would depress prices. A small seller, on the other hand, will prefer to sell to the market because even if he sells his entire stock to the market his marginal revenue will remain above the procurement price. It follows, therefore, that farmers, who are small price takers, will be selling only to the open market.

What is then the effect of government procurement on price and market arrival seasonalities? As we pointed out above, government will keep price seasonality unchanged, but will increase seasonality in market arrival. This is simply because the large sellers will sell, immediately after the harvest, part of their stocks to the government. The higher the number of large sellers in the market, greater will be the effect of increased seasonality in market arrival due to government procurement. Therefore, on this count also, Punjab should exhibit higher seasonality in market arrival than Bihar.

4. CONCLUDING REMARKS

The paper tried to give an explanation as to why low (high) output seasonality coexists with high(low) price seasonality in some agricultural markets in India. The basic explanation was provided in terms of differences in the degree of monopoly across markets. It was also shown that government procurement tends to strengthen the result.

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