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Editorial

S.S. Shrikhande and his work: an appreciation

Professor S.S. Shrikhande is one of those whose research contributions had a tremendous impact on the development of combinatorics in the second-half of this century. It is believed that the most important factor in the sudden spurt in the study of Designs and related areas in the 1960s and 1970s was the sensational result due to R.C. Bose, S.S. Shrikhande and E.T. Parker, on the falsity of a conjecture by Euler on the existence of orthogonal Latin Squares. This conjecture was fondly believed to be true for almost 176 years. The unexpected result created such an excitement that photos of these mathematicians and their discovery found a prominent place on the front page of the New York Times and attracted a large number of young mathematicians into this area. What follows is a modest attempt to look at the work of a great researcher, which not only had high esthetic beauty, but also proved to be quite useful in diverse applications.

Professor Sharadchandra Shankar Shrikhande was born in a middle class family on 19th October 1917 at Sagar, a small town in Madhya Pradesh, close to the geographical centre of India. He received the Bachelor of Science (Honors) degree in Mathematics from Nagpur University in 1939. He joined the Nagpur College of Science in 1942 teaching Mathematics and Statistics and continued there till 1958. During the early 1940s he used to take long trips to Calcutta to meet Professor Raj Chandra Bose (R.C. Bose) who had by then become internationally prominent, while working at the Indian Statistical Institute. This was a period of extreme difficulties as the country, under a colonial regime, was directing all its resources towards supporting the war efforts of the British. There was hardly any support for basic research. Young Shrikhande struggled against all odds: while working at the Science College, he continued to pursue his studies under the guidance of Bose until he left India to take up a Professorship at University of North Carolina, Chapel Hill, USA. Shrikhande joined him at Chapel Hill to complete his Ph.D. in 1950 and thus became the first doctoral student of Bose. The close collaboration between them continued till the last years of Bose's life.

During 1951–53, Shrikhande was an Assistant Professor of Statistics at the University of Kansas, Lawrence, USA and was Associate Professor at Chapel Hill during 1958–60. It was during the later period that the famous result on Euler's conjecture was settled. Instead of staying on in USA, he decided to return to India to take up a Professorship at Banaras Hindu University where he worked till 1963. He then joined the University of Bombay as Professor and Head of the Department of Mathematics until his formal retirement in 1978. During this period, he was also Director of Centre for Advanced Study in Mathematics at Bombay. During 1983–86, he held the position

of Director of Mehta Research Institute at Allahabad, which is a research centre for Mathematics and Mathematical Physics.

Professor Shrikhande had been a Visiting Professor at various US Universities like University of Wisconsin, The Ohio State University, State University of New York, Stanford University and Colorado State University. He has been associated with the Indian Statistical Institute in various capacities. Professor Shrikhande has been a member of a number of learned societies: Indian National Science Academy, Indian Academy of Sciences, Institute of Mathematical Statistics and International Statistical Institute — to mention a few. He has also been on the editorial boards of a number of learned journals.

Combinatorics today has become a rather strange discipline attracting researchers not only from Mathematics and Statistics, but also from diverse areas like Computer Science, Electronics, Management, Biology, Sociology — indeed any application-oriented branch of Science or Technology seems to produce problems of a Combinatorial nature. The foundation for this development was laid in the 1940s and 1950s, when geometers, statisticians, group theorists, number theorists and engineers independently started posing in their own style, interesting combinatorial problems and developing necessary language and tools to solve them. It was decades later, when common themes in these diverse developments were observed and attempts were made to unify them. Among the many early pioneers, some names stand out: Marshall Hall, Herbert Ryser, Alan Hoffman, R.H. Bruck, R.C. Bose, and S.S. Shrikhande, to mention a few. Shrikhande was perhaps one of the youngest among them.

Shrikhande's first paper, published in 1950, in Annals of Mathematical Statistics, on the impossibility of certain symmetrical balanced incomplete block designs (SBIBD) is path breaking. He extended the style developed by Bruck and Ryser of using the quadratic form associated with a projective plane to all symmetric BIBDs. Results of this paper were independently obtained by M. Schutzenberger and by Chowla and Ryser. This was the first paper to systematically develop a method, using incidence matrix of a statistical design, to study non-existence of such designs. This helped to fill in gaps in the table of Fisher & Yates, created for statisticians. This method has been used later to study similar questions for many other designs by Ogawa, Raghavarao, Vartak, Hall, Ryser, Hughes, etc. Shrikhande himself wrote several papers, jointly with others, applying the method to many different classes of designs — group divisible designs, partially balanced designs, affine designs, cyclic designs and so on.

A pattern is visible in the work of Prof. Shrikhande. His main effort has been in the area of Combinatorial Designs and related topics. Several times, he developed path breaking techniques, generating a lot of activity in the field. He himself continued to pursue them and enjoyed applying them to any new structure he studied. Often he successfully used them to solve well-known problems.

As mentioned earlier, the most well known among them is the historical Euler's conjecture on existence of orthogonal Latin Squares, that a pair of orthogonal Latin Squares of order $4t + 2$ does not exist. Shrikhande together with Bose disproved this

conjecture for a particular case in 1959. Together with E.T. Parker, they showed later that Euler was wrong except for the orders 2 and 6. The methods developed by Bose and Shrikhande in construction of designs helped, not only in the solution of this problem, but gave great impetus to the study of Design theory as such. A significant development, in terms of methods used, was the proof of embedding quasi-residual designs into symmetric BIBDs, jointly done with Bose and Singhi in 1974.

Rather than attempt an exhaustive survey of his work (his collected works are available), we shall explore how some of his methods led to these interesting developments.

In 1959–60, he published two papers on characterisation of association schemes. His paper on triangular association scheme $T(n)$ along with papers of Hoffman and Connor, characterised the scheme $T(n)$ by parameters, ($T(n)$ is the line graph of the complete graph k_n) for all $n \neq 8$. In the second paper, he characterised the association scheme $L_2(n)$ (or the line graph of the complete bipartite graph $K_{n,n}$) for $n \neq 4$. These results show that the $T(n)$ and $L_2(n)$ graphs are characterized by their eigenvalues, the only exceptional cases being $T(8)$ abd $L_2(4)$. Seidel showed these exceptional cases are related, respectively, to the $T(8)$ and $L_2(4)$ graphs by a process now referred to as "Seidel switching". These were among the first papers in this direction. Such characterizations became an important activity in the 1960s and 1970s. Several papers appeared characterizing various graphs of balanced block designs and partially balanced block designs by Hoffman, Hall, Mesner, Connor, Bose, Ray-Chaudhuri, Seidel, Goethals, Bhagwandas, Singhi, etc., among others.

Hoffman and Ray-Chaudhuri characterized line graphs of projective spaces. In a land mark paper, R.H. Bruck characterized graphs arising out of a Net or affine designs. Bruck's paper developed the basic mathematical structure of graphs to study all such graphs. Bose generalized and unified all these characterizations in his famous paper on partial geometries and strongly regular graphs. A decade later, Shrikhande, together with Bose and Singhi extended the whole theory to multigraphs, by defining a very general structure called partial geometric designs.

In this context, Bose had mentioned the following incident to one of the authors. In 1961, Bruck showed to Bose an example (for $n=4$) where $n-3$ mutually orthogonal Latin squares (mols) cannot be extended into a complete set of mols. Shrikhande very soon showed that $n=4$ is the only exception and that for $n \neq 4$, any $n-3$ mols can be completed into a full set of mols. When Bruck came to know of this, he might have been disappointed, but he pursued the matter further resulting in his landmark paper on Nets generalizing Shrikhande's results to any set of $n-d$ mols, for all sufficiently large n, which in turn resulted in Bose's paper on partial geometries in 1963.

In 1960, Shrikhande published a simple proof of a well-known result of Hall and Connor on embedding quasi-residual designs with $\lambda = 2$ into symmetric BIBDs, using characterizations of Triangular scheme. These methods were generalized in 1974 by Shrikhande and Singhi to show that embedding is possible for $\lambda = 3$ and for some higher values of λ. Finally Shrikhande, jointly with Bose and Singhi developed the general concept of partial geometric designs to prove the embedding results for any λ, and sufficiently large block sizes. An important aspect of this work is the generalization

to multigraphs of the methods of Bruck and Hoffman in connection with the study of Nets using Claws and Cliques.

In 1959, Shrikhande, jointly with Bose devised an ingenious technique of using pairwise balanced designs (PBD) with varying block sizes to construct regular designs like BIBDs and latin squares, by pasting together smaller PBDs. Apart from its immediate application in disproving Euler's conjecture on Latin Squares, these techniques led to tremendous activity among design theorists. Some of those who contributed significantly to developing these techniques include Hanani, Ray-Chaudhuri, M. Hall, Wilson, Connor, Mullin and Stanton. These developments led to the solution of Kirkman's school girl problem by Ray-Chaudhuri and Wilson, as well as to the proof of existence of t-designs for $t = 2$ (BIBDs) for sufficiently large number of points, by Wilson. Existence conjecture Steiner for t-designs for $t > 2$ is one of the central problems around which much of the theory of designs is developed, but is yet to be settled.

While working on the central areas of the theory of designs, he also wrote several interesting papers on other problems. In late 1960s, he developed with Bhat, a method of using good intersection patterns in Hadamard designs and affine designs to generate a large number of non-isomorphic solutions. Later Singhi extended their methods to classify all (19, 9, 4) designs. V.C. Mavron generalized these results to develop a complete theory to study matroids and groups related to affine designs.

Professor Shrikhande's paper with Bhagwandas on Seidel equivalence of graphs is quite exceptional. They showed that all interesting examples of strongly regular graphs with the same parameters are equivalent in this sense. Seidel, Goethals, Hoffman and Ray-Chaudhuri extended their results considerably.

One of Shrikhande's well-known work is the joint paper with Singhi published in 1974 on the λ-design conjecture of Ryser. They showed that the conjecture is true for all primes. Their result is still the best result so far known.

Erdös used to like a joint paper of Shrikhande and Singhi published in 1986, in which they showed that if a conjecture of Erdös and Larson on pairwise balanced designs is true, then the projective plane conjecture is false.

A remarkable fact about Professor Shrikhande, not usually mentioned, is that while in India, he did active research, although holding a good deal of administrative responsibilities. Head of a major department, Director of Centre for Advanced Study in Mathematics, member of several learned societies, member of governing council of the Indian Statistical Institute, member of various committees constituted by these bodies as well as many Indian Universities, it would have been quite tough for an ordinary person to find enough time to do his own research, let alone excel in his work. One admirable quality of Professor Shrikhande is that in spite of his great achievements he continued to be kind and courteous and genuinely helpful. We have often heard that among the famous people, Professor Shrikhande may be one of the few with a large circle of friends and admirers. Many among today's Mathematicians in India working in Combinatorics, like V.N. Bhat Nayak, S.S. Sane, N.M. Singhi, K.S. Vijayan, S.B. Rao, A.R. Rao, Vijayakumar have sweet memories of help and guidance, Prof. Shrikhande had provided, at crucial moments of their careers. On the initiative of some

of these, the Indian Statistical Institute organised an international seminar in his honour in 1982, on the occasion of his 65th birthday. Some of the best known names in Combinatorics from around the world were present on that occasion. After the passing away of his wife Shakuntala almost a decade ago, Professor Shrikhande divides his time between his ancestral home and USA where his children Mohan, Asha and Anil are well settled. We hope and pray that he would continue to enjoy good health and happiness for many many years to come.

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Further reading

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