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A Note on an Inventory Model with Different Demand Rates During Stock-in and Stock-out Period

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ABSTRACT

This note is concerned with a single item deterministic inventory model with instantaneous replenishment and different demand rates during the period of availability and the period of shortage. The model is illustrated with a numerical example.

INTRODUCTION :

Single item deterministic inventory models with stock dependent demand rates have been considered in the literature only recently. Gupta and Vart (1986) suggested an EOQ model with stock dependent demand rate, instantaneous replenishment of stocks and shortages not being allowed. However, in their mathematical analysis the dependence of the stock level on the demand rate was not properly taken care of. Mandal and Phaujdar (1989 *a*) reconsidered this model recently and obtained the EOQ based on the criterion of maximization of total profit per unit time when the demand rate increases linearly with the stock level. Mandal

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and Phaujdar (1989 *b*) *in another paper also considered a* general inve tory model for deteriorating items with stock dependent demand ra wherein the replenishment of stock occurs at a finite rate and shortag are allowed to occur.

Conventional deterministic inventory models generally assume th shortages do not affect the demand rate during the period of shortag But in practice, for many consumer goods, the demand rate may chan during the period of shortage (it may decrease or even increase) and th is different from what it was when the item was in stock. This certain affects the EOQ for the model. In this note we consider a single ite deterministic inventory model in which the demand rate has one consta value so long as there is no shortage, but changes to another const during the period of shortage. The calculation of EOQ here is based *the criterion of minimization of total cost per unit time.*

2. THE MODEL AND ITS MATHEMATICAL ANALYSIS :

A single item deterministic inventory model is considered here un the assumption of instantaneous replenishment of stock, shortages be allowed and backlogged, demand rates during the respective period and t_2 of stock-in and shortage being different constants D and *respectively,* k *being a positive constant. Note that* $k=1$ correspond the same constant demand rate during the periods of stock-in a shortage.

Let C_1 be the unit holding cost per unit time, C_2 be the unit short cost per unit time, C_3 be the fixed setup cost for each cycle and indep dent of the order quantity. Then the total cost per unit time during cycle $t=t_1+t_2$ is

$$C'(t_1,\ t_2)=\frac{1}{t_1+t_2}\left[\frac{C_1D}{2}\ t_1^2+\frac{kC_2D}{2}\ t_2^2+C_3+CD(t_1+kt_2)\right]$$

where C *is the unit cost of the item.*

The optimum values t_1^* and t_2^* of t_1 and t_2 respectively follow the criterion of minimum total cost per unit time are obtained fr solutions of the equations

$$C'_{t_1}=0, \quad C'_{t_2}=0$$

provided

$$C'_{t_1t_1}>0,\ C'_{t_2t_2}>0,\ C'_{t_1t_1}\,C'_{t_2t_2}-C'^2_{t_1t_2}>0$$

for these values of t_1 and t_2. The equations (2) give

$$\gamma t_1^2 - \alpha t_2^2 + 2\gamma t_1 t_2 + 2C\beta t_2 = 2C_3$$

$$-\gamma t_1^2 + \alpha t_2^2 + 2\gamma t_1 t_2 - 2C\beta t_1 = 2C_3 \qquad ...(4)$$

ere

$$\alpha = kC_2D, \qquad \beta = CD(1-k), \qquad \gamma = C_1D \qquad ...(5)$$

From the equations (4) we find,

$$(\alpha+\gamma)\, t_1 t_2 + \beta(t_2 - t_1) = 2C_3 \qquad ...(6)$$

l

$$\gamma \frac{t_1}{t_2} + (\gamma - \alpha) - \alpha \frac{t_2}{t_1} + \beta\left(\frac{1}{t_1} + \frac{1}{t_2}\right) = 0 \qquad ...(7)$$

If we write $\eta = \frac{t_1}{t_2}$, then (7) gives

$$t_1 = \frac{\beta\eta}{\alpha - \gamma\eta} \qquad ...(8)$$

that

$$t_2 = \frac{\beta}{\alpha - \gamma\eta} \qquad ...(9)$$

$_1$ and t_2 from (8) and (9) are substituted in (6), it results in a quadratic ation in η whose solution is given by

$$\eta^* = \frac{-2C_1C_2C_3k \pm C(1-k)[kC_1C_2D\{2C_3(C_1+kC_2) - C^2D(1-k)^2\}]^{1/2}}{C_1\{DC^2(1-k)^2 - 2C_1C_3\}} \qquad ...(10)$$

re obviously we have to take positive value.

The EOQ is now given by

$$q^* = D(t_1^* + kt_2^*)$$

$$= \frac{D\beta(\eta^* + k)}{\alpha - \gamma\eta^*} \qquad (11)$$

Optimal total cost per unit time can now be easily obtained from (1) substitution of appropriate values of t_1^* and t_2^* in terms of η^*.

The classical EOQ formula with constant demand rate during the od of stock and shortage can be deduced as follows. In this case l so that

$$\alpha = C_2D,\ \beta = 0,\ \gamma = C_1D$$

$$\eta^* = \frac{C_2}{C_1} \text{ so that } \frac{t_1^*}{C_2} = \frac{t_2}{C_1} = \quad , \text{ say,}$$

where now (6) gives,

$$(C_1+C_2)\ DC_1C_2\lambda^2=2C_3$$

resulting in

$$t_1^*=\left\{\frac{2C_2C_3}{DC_1(C_1+C_2)}\right\}^{1/2}, \qquad t_2^*=\left\{\frac{2C_1C_3}{DC_2\ (C_3+C_2)}\right\}^{1/2}$$

and

$$q^*=\left\{\frac{2DC_3(C_1+C_2)}{C_1C_2}\right\}^{1/2}$$

which are independent of C, the units cost of the item, as it shoul (cf. Naddor (1966)).

For numerical illustration of the model, we take the num values of the different parameters as $C_1=5$ paise per unit per $C_2=20$ paise per unit per week, $C_3=$Rs. 10.00 per order, $C=$Rs. 5.0 unit and $k=0.9$. Then the optimal values of t_1, t_2, q and C' are obta as $t_1^*=1.95$ weeks, $t_2^*=2.68$ weeks, $q^*=35$ units and $C'^*=$Rs. 41.0

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